From direct to inverse analysis in flexible multibody dynamics

Olivier Brüls

Department of Aerospace and Mechanical Engineering (LTAS)

University of Liège, Belgium

Annual GAMM Conference

Darmstadt, March 27, 2012

High-fidelity system-level simulation

1. Overview and added-value in mechanical applications

2. Potential and challenges in design optimization

3. Emerging formalisms for inverse analysis
   (Lie group approach)

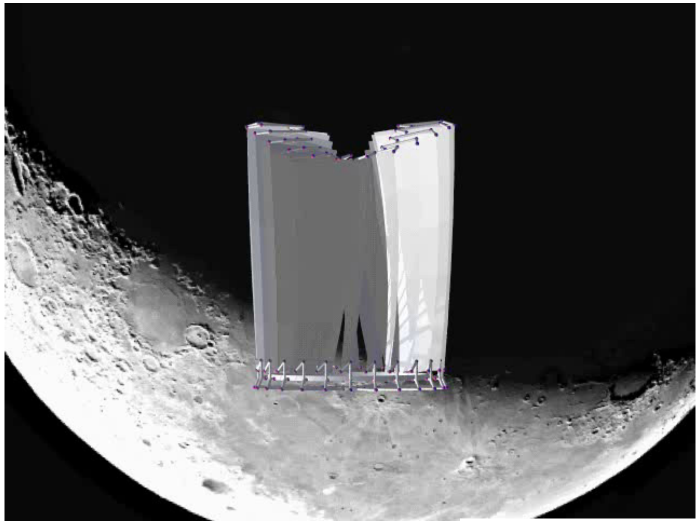
High-fidelity system-level simulation

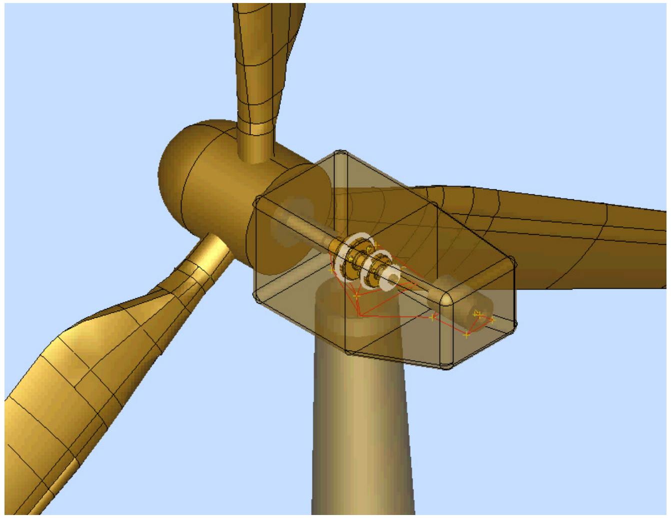
1. Overview and added-value in mechanical applications
   - The FE method in flexible multibody dynamics
   - Simulation of vehicle driveline
   - Simulation of compliant deployable structures

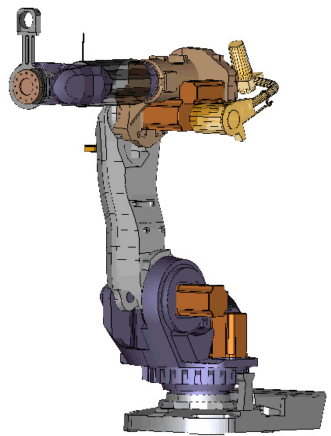
2. Potential and challenges in design optimization

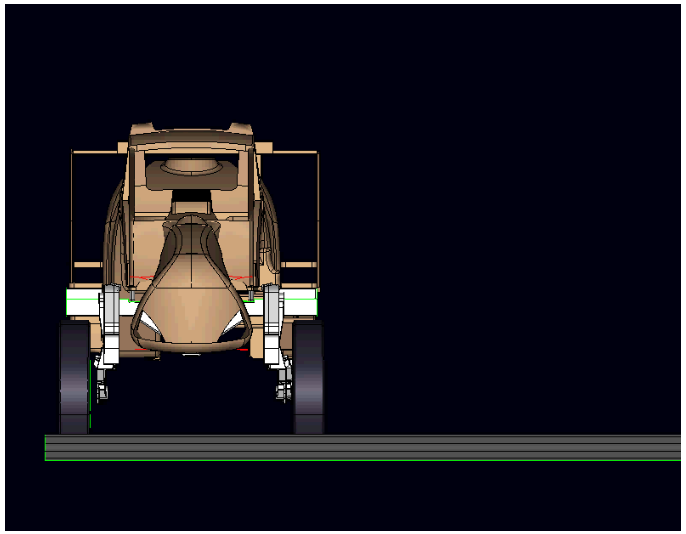
3. Emerging formalisms for inverse analysis (Lie group approach)

Examples of application fields



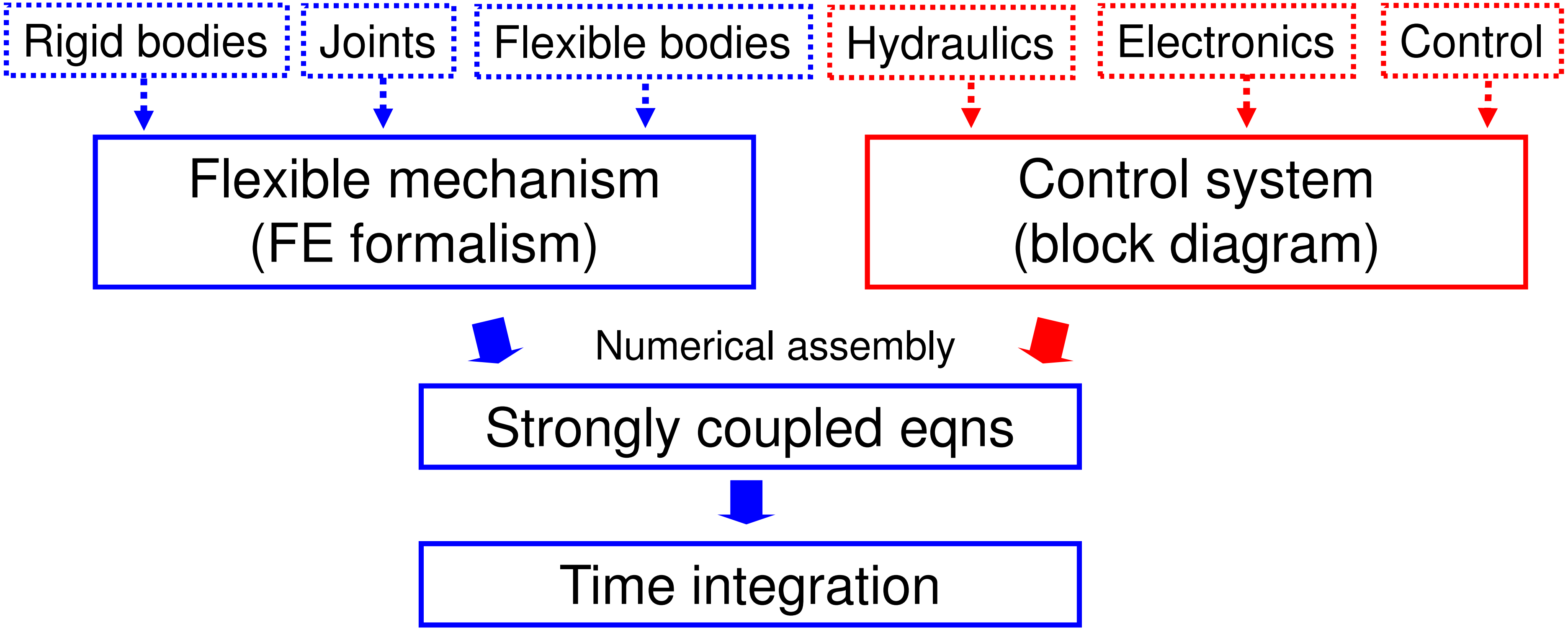






Mecano models (Courtesy: LMS Samtech)

Integrated simulation approach



FE method in flexible multibody dynamics

Technical system

Model

Equations of motion

Time integration

Results

Dynamic performance?

Mechanical loads & stresses?

FE method in flexible multibody dynamics

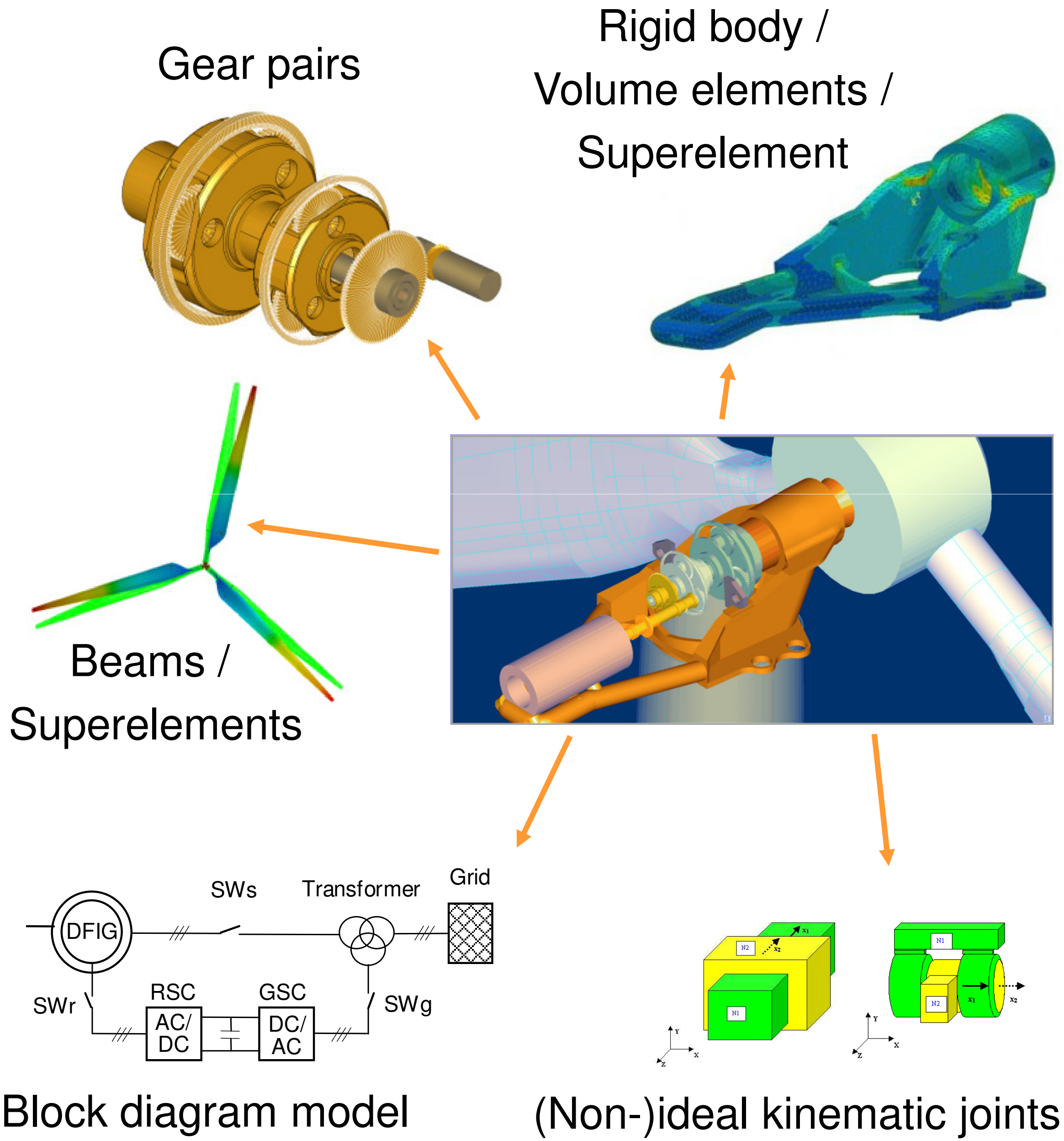
Technical system

Model

Equations of motion

Time integration

Results



FE method in flexible multibody dynamics

(Géradin & Cardona 2001)

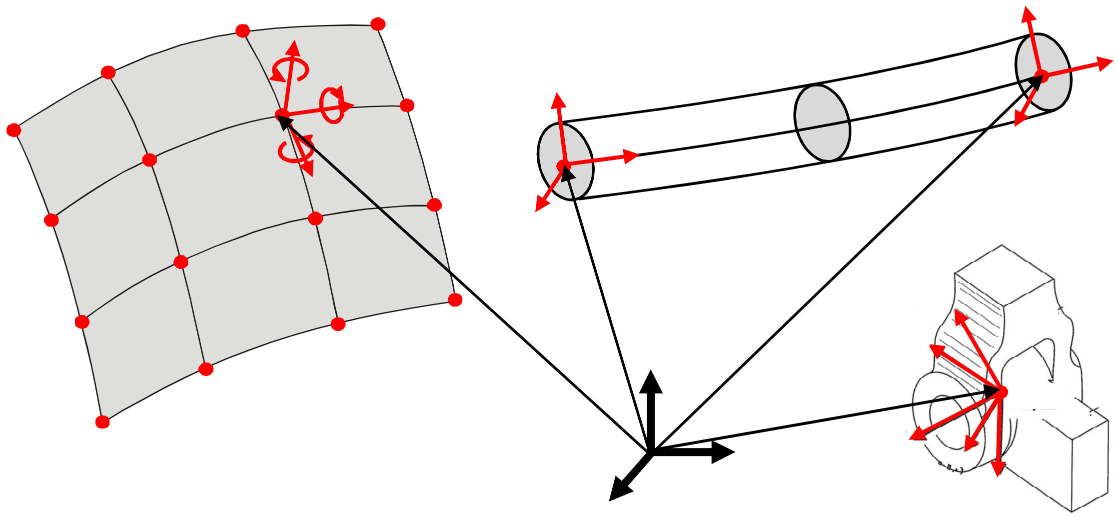
Technical system

Model

Equations of motion

Time integration

Results



If rotations are parameterized:

$$\begin{aligned} \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} &= \mathbf{g}(\mathbf{q}, \dot{\mathbf{q}}, t) - \mathbf{\Phi}_{\mathbf{q}}^T \boldsymbol{\lambda} + \mathbf{L}\mathbf{y} \\ \mathbf{0} &= \mathbf{\Phi}(\mathbf{q}, t) \\ \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, \boldsymbol{\lambda}, \mathbf{x}, \mathbf{y}, t) \\ \mathbf{y} &= \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, \boldsymbol{\lambda}, \mathbf{x}, \mathbf{y}, t) \end{aligned}$$

FE method in flexible multibody dynamics

Technical system

Model

Equations of motion

Time integration

Results

Numerical solution of coupled
1st and 2nd order DAEs with index 3

Generalized-α method
(Chung & Hulbert, 1993)

FE method in flexible multibody dynamics

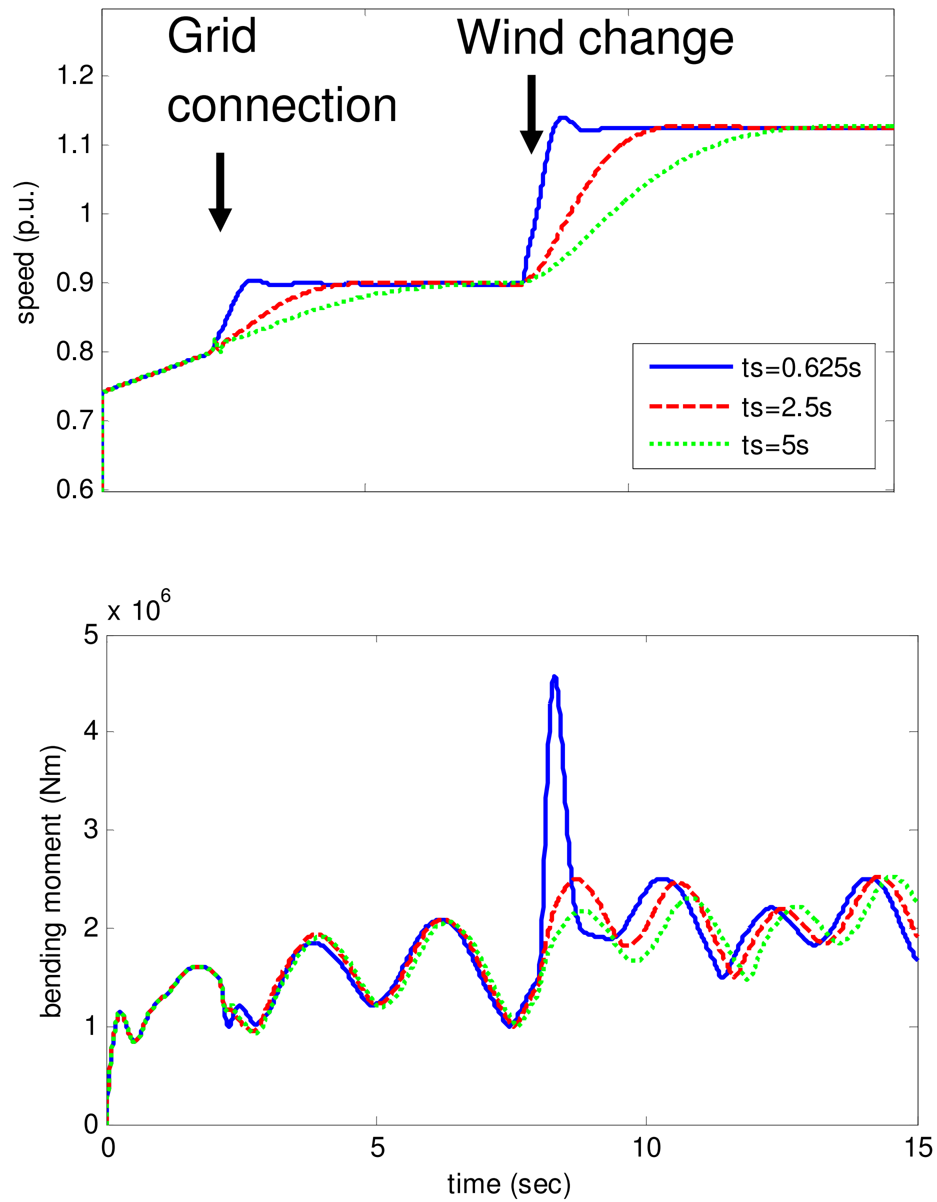
Technical system

Model

Equations of motion

Time integration

Results



Generalized-α time integrator

Stiff & large ODEs in structural dynamics (Chung & Hulbert 1993)

- Second-order accuracy
- Unconditional stability (A-stability)
- Controllable numerical damping (tuning parameter ρ_∞)
- Newmark and HHT are special cases
- Equivalent to a multistep method (Erlicher et al 2002)

Direct integration of index-3 DAEs

- Linear stability (Cardona & Géradin 1989)
- 2nd order accuracy (Arnold & B. 2007)
- Mechatronics (B. & Golinval 2008)

Index reduction methods

(Lunk & Simeon 2006, Jay & Negrut 2007, Arnold 2009, Arnold et al 2011)

Generalized-α time integrator

Newmark implicit formulae:

$$\begin{aligned}
\mathbf{q}_{n+1} &= \mathbf{q}_n + h\dot{\mathbf{q}}_n + h^2(0.5-\beta)\mathbf{a}_n + h^2\beta\mathbf{a}_{n+1} \\
\dot{\mathbf{q}}_{n+1} &= \dot{\mathbf{q}}_n + h(1-\gamma)\mathbf{a}_n + h\gamma\mathbf{a}_{n+1} \\
\mathbf{x}_{n+1} &= \mathbf{x}_n + h(1-\theta)\mathbf{w}_n + h\theta\mathbf{w}_{n+1}
\end{aligned}$$

Generalized-α method (Chung & Hulbert, 1993)

$$\begin{aligned}
(1-\alpha_m)\mathbf{a}_{n+1} + \alpha_m\mathbf{a}_n &= (1-\alpha_f)\ddot{\mathbf{q}}_{n+1} + \alpha_f\ddot{\mathbf{q}}_n \\
(1-\delta_m)\mathbf{w}_{n+1} + \delta_m\mathbf{w}_n &= (1-\delta_f)\dot{\mathbf{x}}_{n+1} + \delta_f\dot{\mathbf{x}}_n
\end{aligned}$$

Equations of motion at time t_{n+1}

$$\begin{aligned}
\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} &= \mathbf{g}(\mathbf{q},\dot{\mathbf{q}},t) - \mathbf{\Phi}_{\mathbf{q}}^T\boldsymbol{\lambda} + \mathbf{L}\mathbf{y} \\
\mathbf{0} &= \mathbf{\Phi}(\mathbf{q},t) \\
\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}},\boldsymbol{\lambda},\mathbf{x},\mathbf{y},t) \\
\mathbf{y} &= \mathbf{h}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}},\boldsymbol{\lambda},\mathbf{x},\mathbf{y},t)
\end{aligned}$$

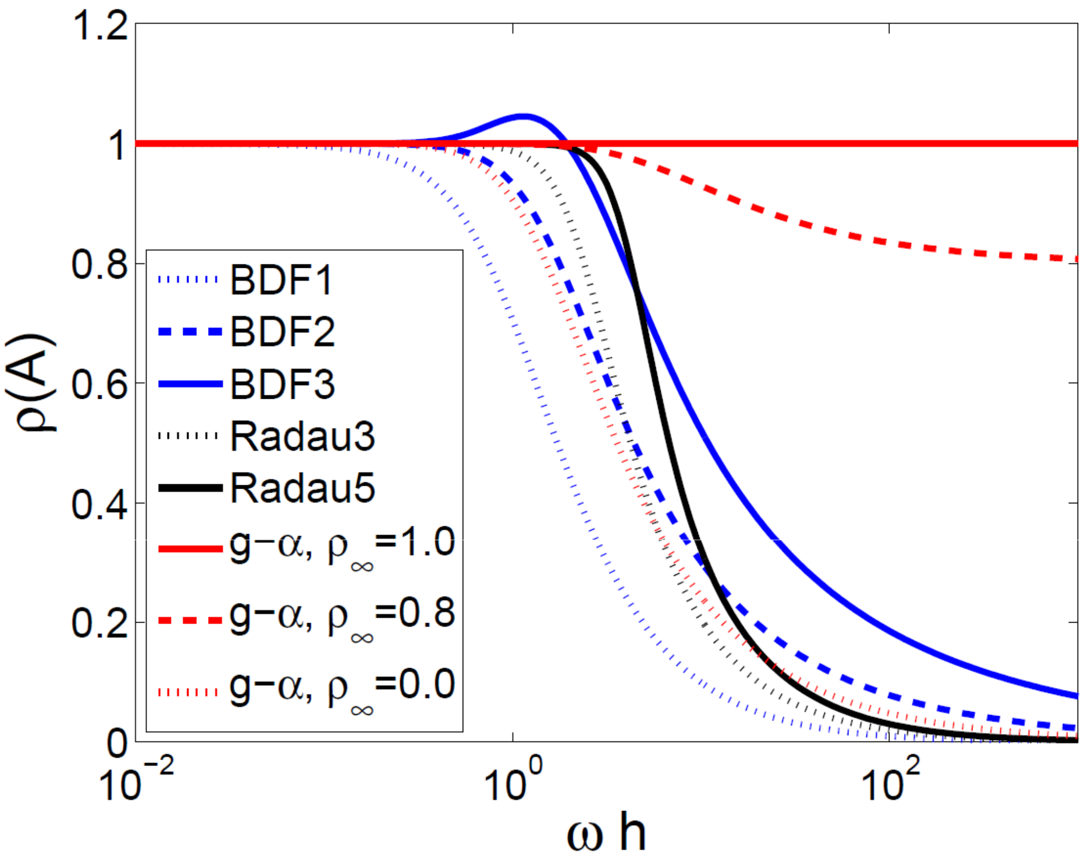
Comparison with other DAE solvers

Test equation:

$$\ddot{q} + \omega^2 \, q = 0$$

Numerical solution:

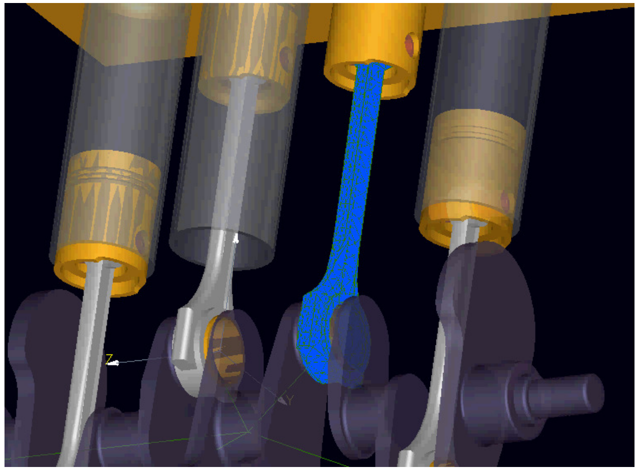
$$\mathbf{x}_n = (\mathbf{A}(\omega h))^n \, \mathbf{x}_0$$



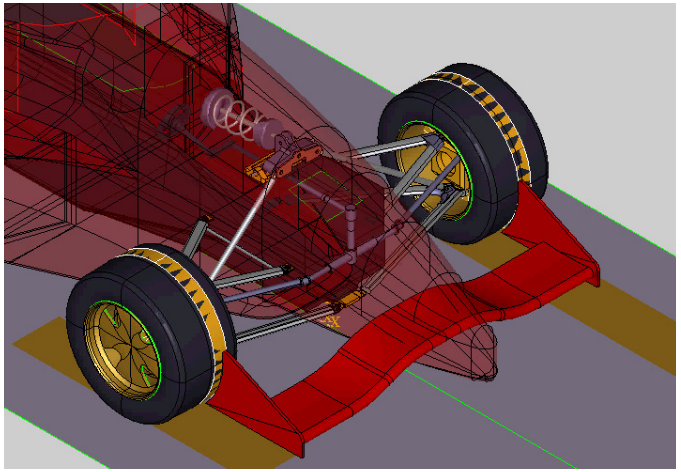
- ➢ Order of unconditionally stable BDF ≤ 2
- ➢ Less numerical dissipation with generalized-α method

Modelling of vehicle drivelines

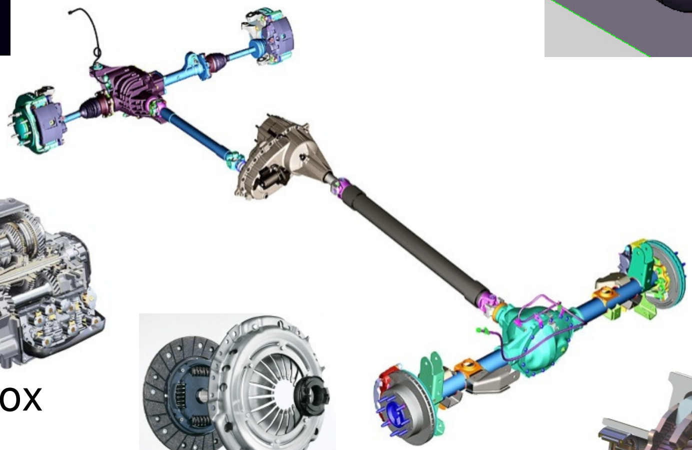
(Courtesy: LMS Samtech)



Motor

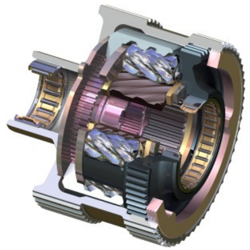


Vehicle dynamics



Gear box

Clutch

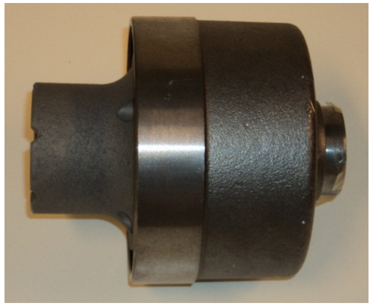


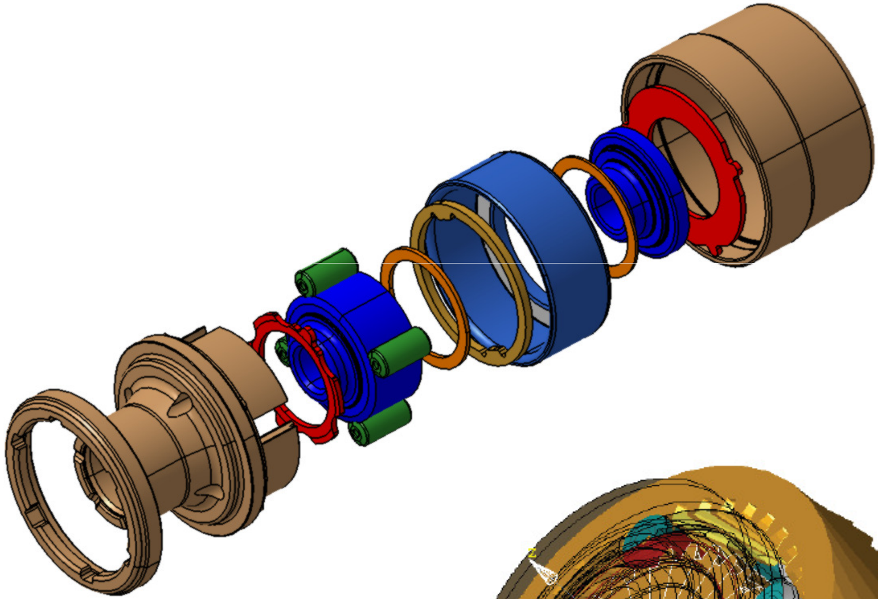
Differential

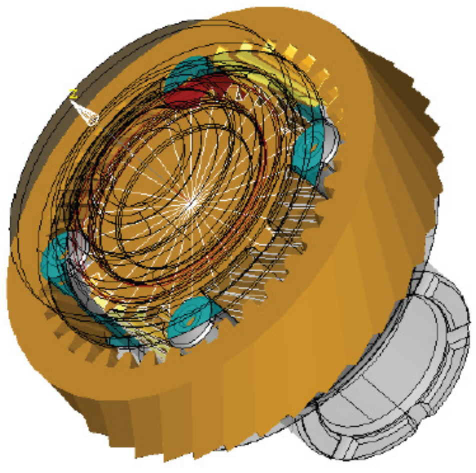
Modelling the components in their environment

Torsen limited slip differential

- Variable torque distribution between the output shafts
- Locking by friction between gear pairs & thrust washers
- 4 working modes

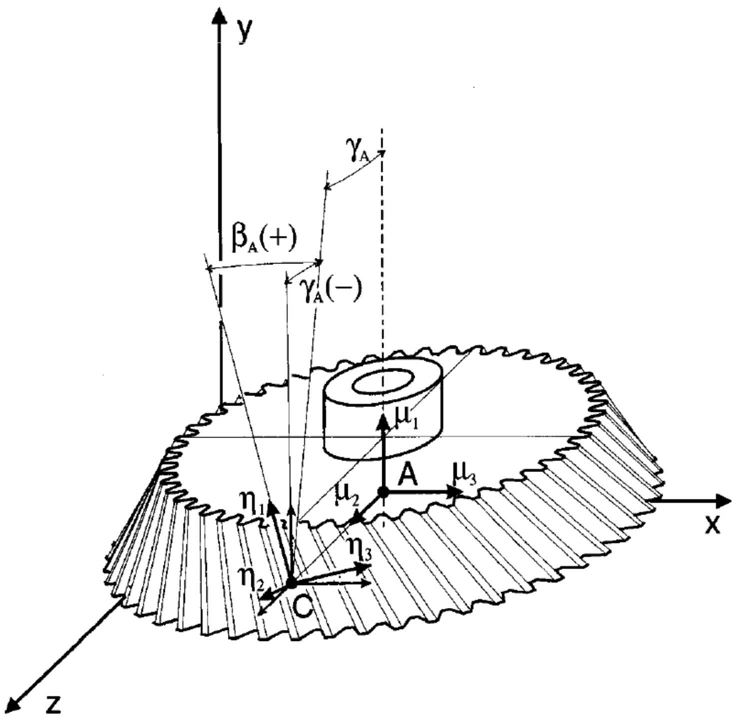


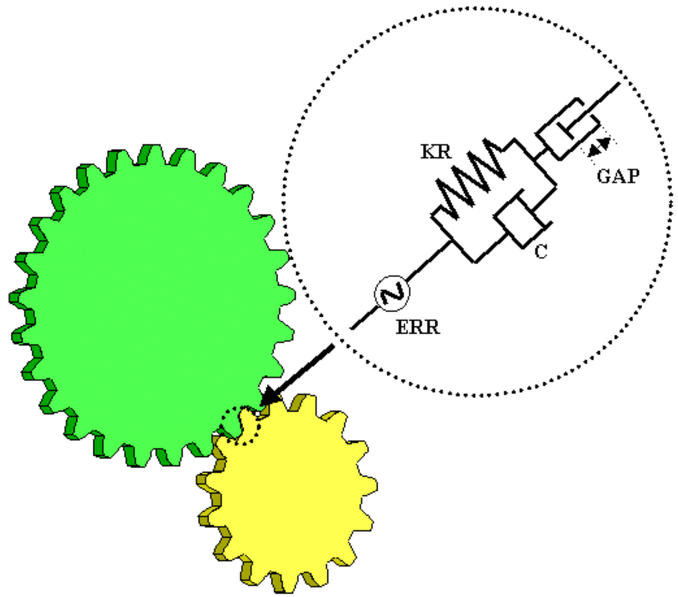




Gear pair element

- Connection between two wheels modelled as rigid bodies
- Local flexibility : spring (KR) and damper (C)
- Time fluctuation of mesh stiffness (ISO 6336)
- Backlash (GAP), load transmission error (ERR), misalignment

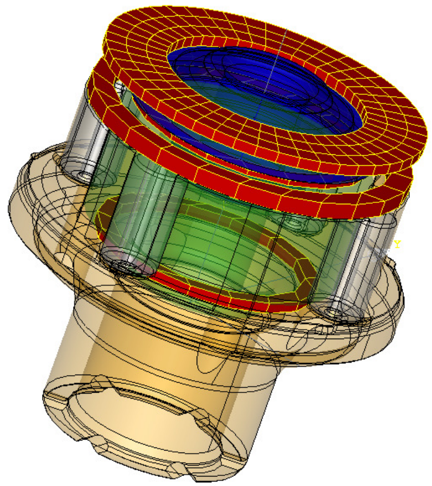


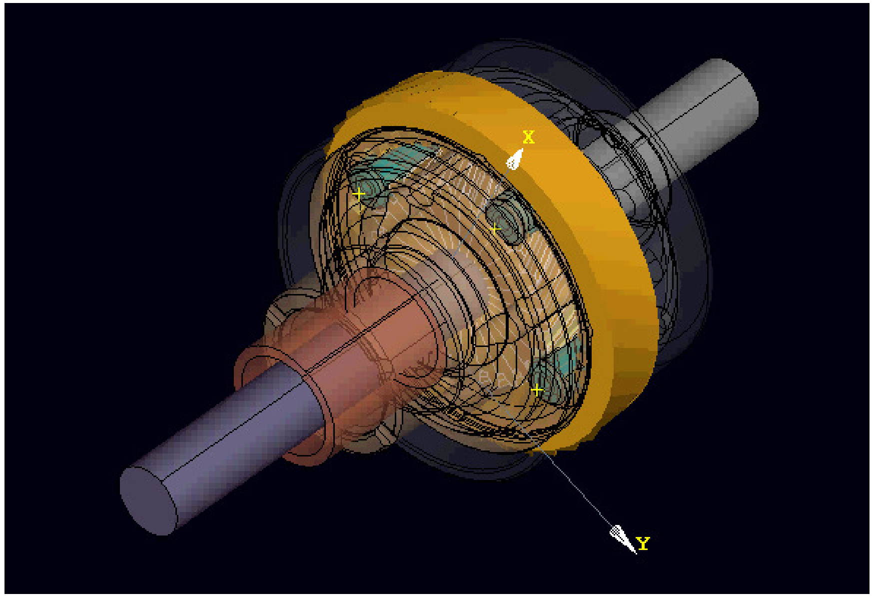


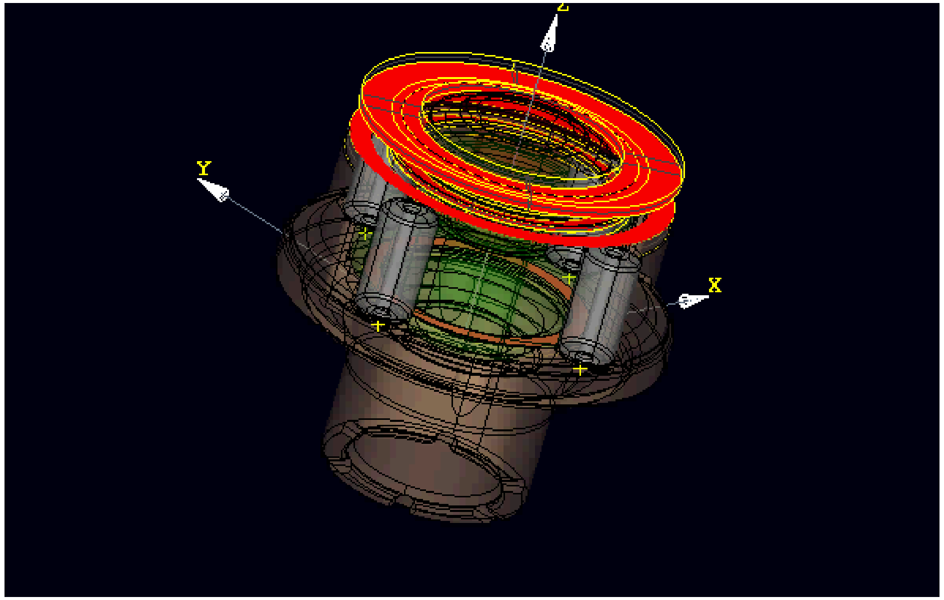
Case 1: flexible washers

- Rigid/flexible contact conditions
- 8000 dofs

TDR1 numerical :	3.90
TDR1 experimental (Torsen) :	4.02

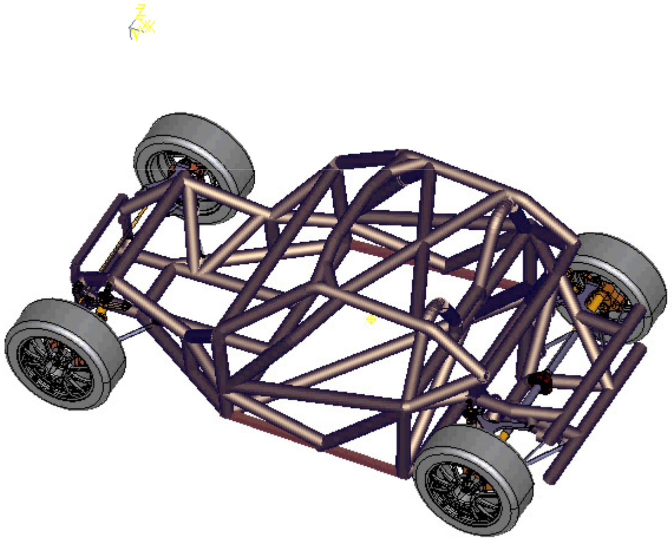


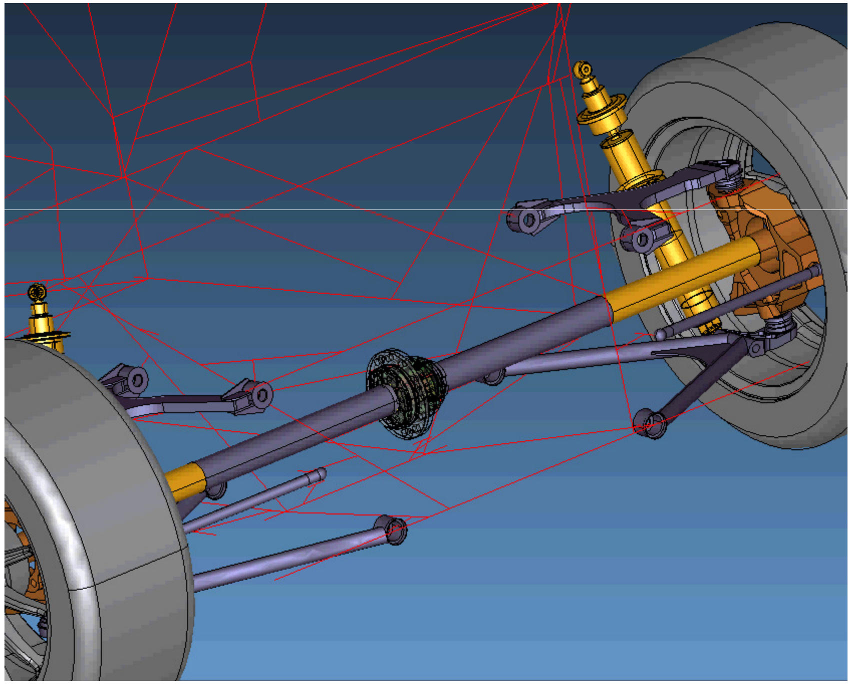




Case 2: rigid washers

Rigid/rigid contact condition: continuous impact model with a coefficient of restitution (Lankarani & Nikravesh 1994)



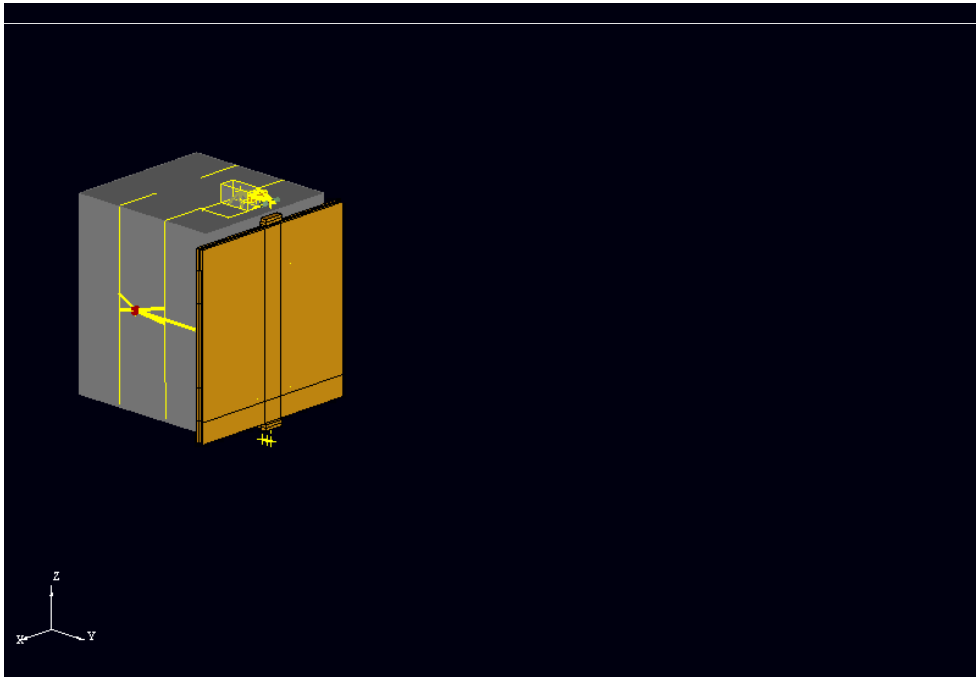


Alternative: nonsmooth description and time-stepping algorithm

Tape-spring hinge

MAEVA hinge (METRAVIB & CNES)

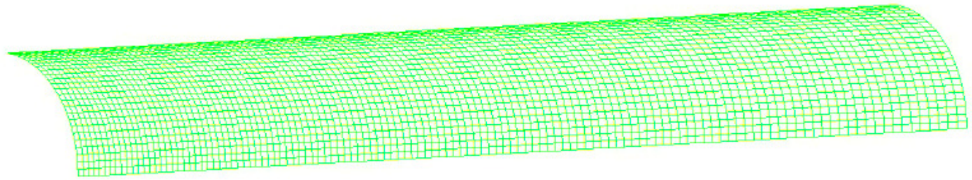
- ➢ Guiding, driving and locking functions
- ➢ No contact between sliding surfaces



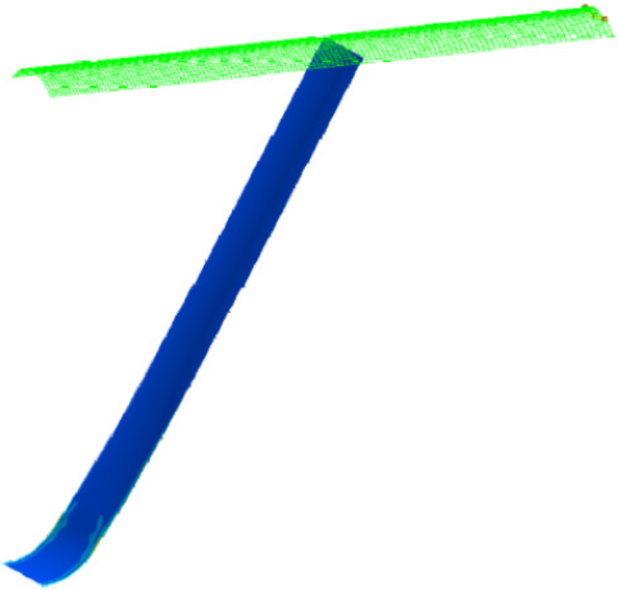
First model: ideal hinge

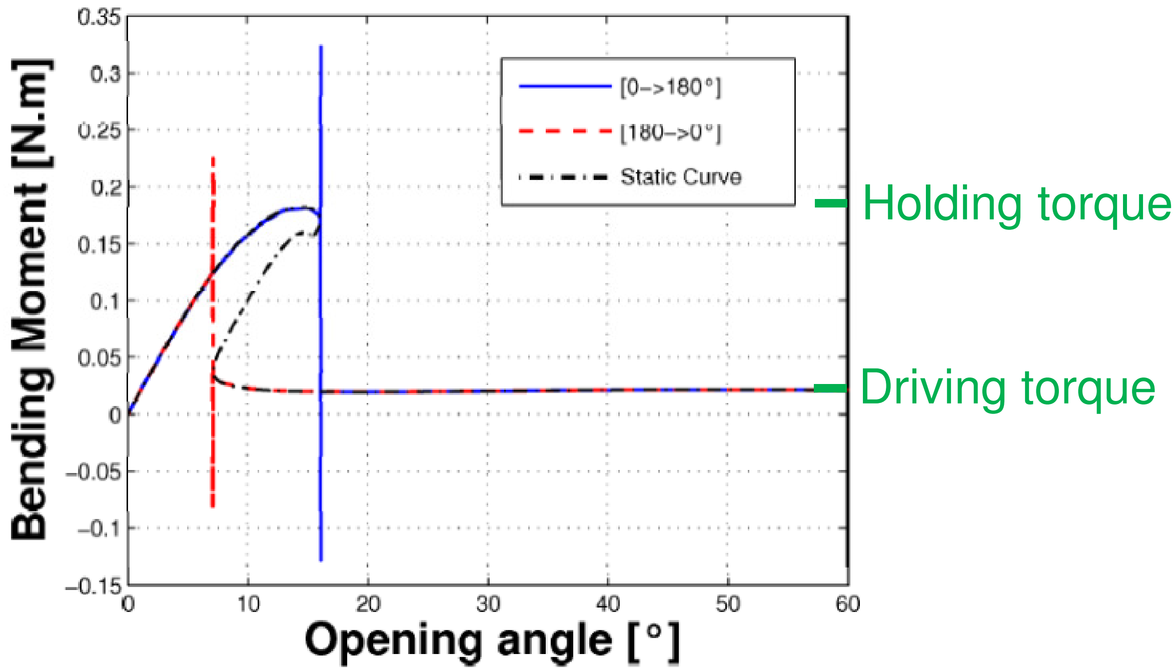
- ➢ No 3D behaviour
- ➢ No self-locking

Static analysis of a single tape-spring

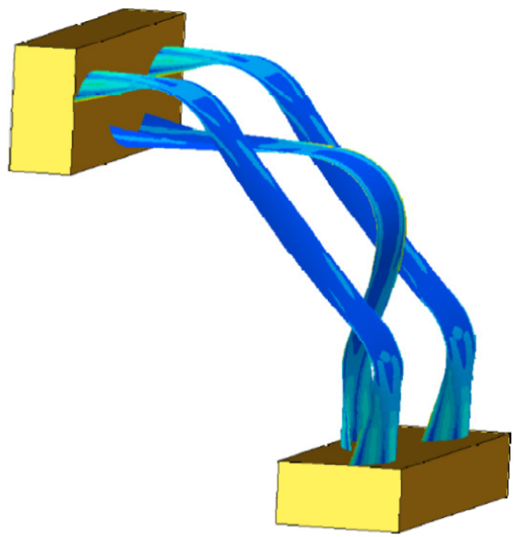


- Fine mesh with second order Mindlin shell elements
- Symmetry is exploited
- Continuation vs. pseudo-dynamic methods





Static behaviour of a full hinge



Numerical results

Driving torque : 0.152 Nm

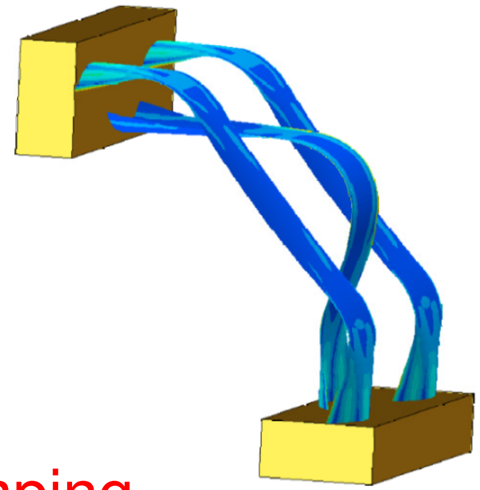
Holding torque : 6.67 Nm

Experimental tests (Metravib):

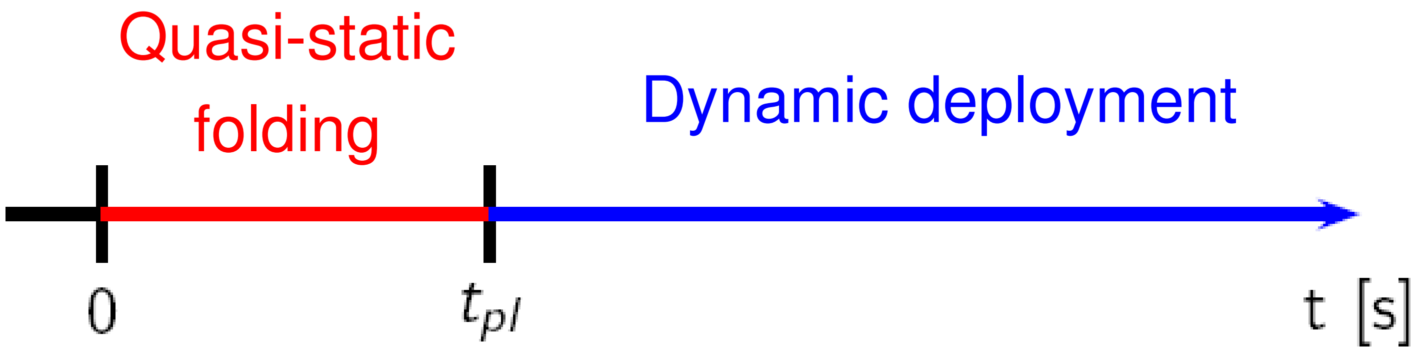
Driving torque > 0.15 Nm

Holding torque > 4.5 Nm

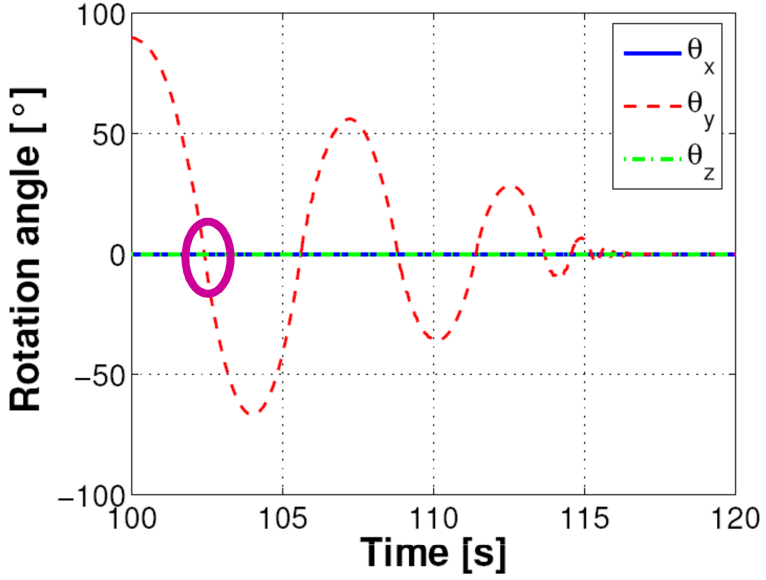
Dynamic behaviour of a full hinge

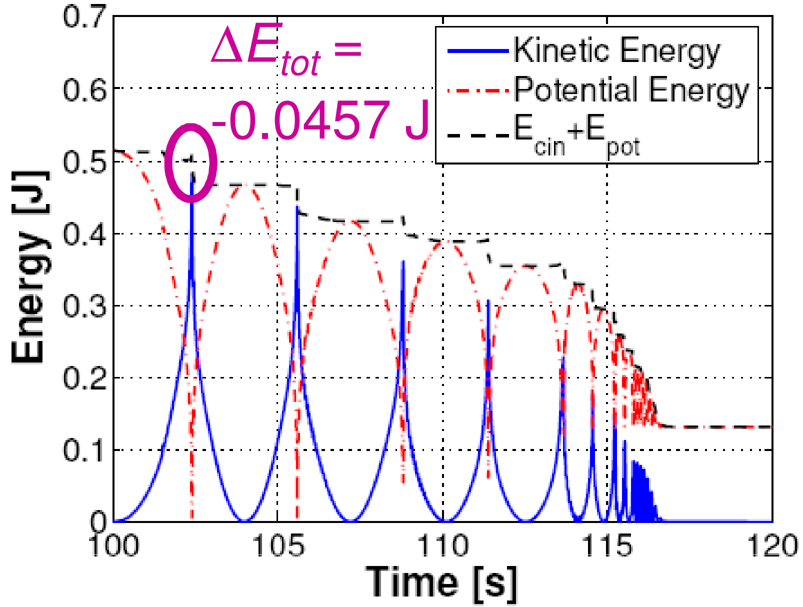


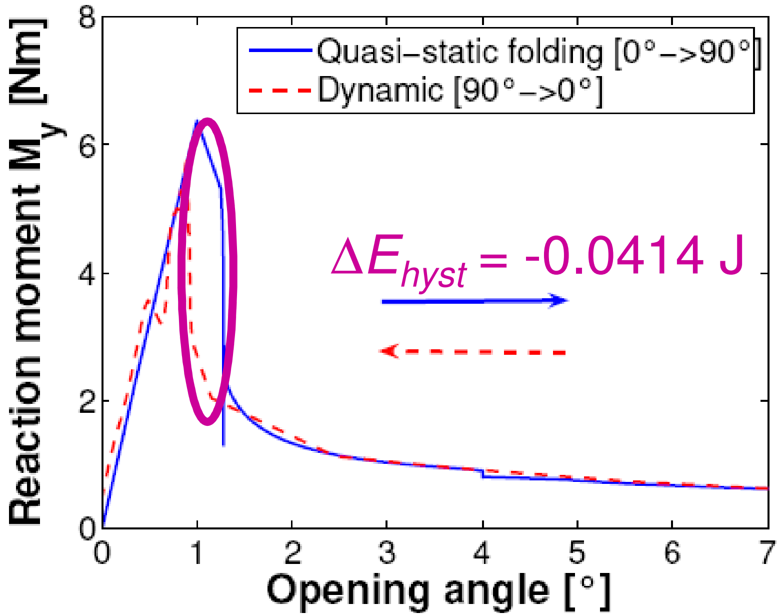
- Inertia of the rigid appendix (solar panel)
- No structural damping but numerical damping



Full hinge - Torsional mode blocked

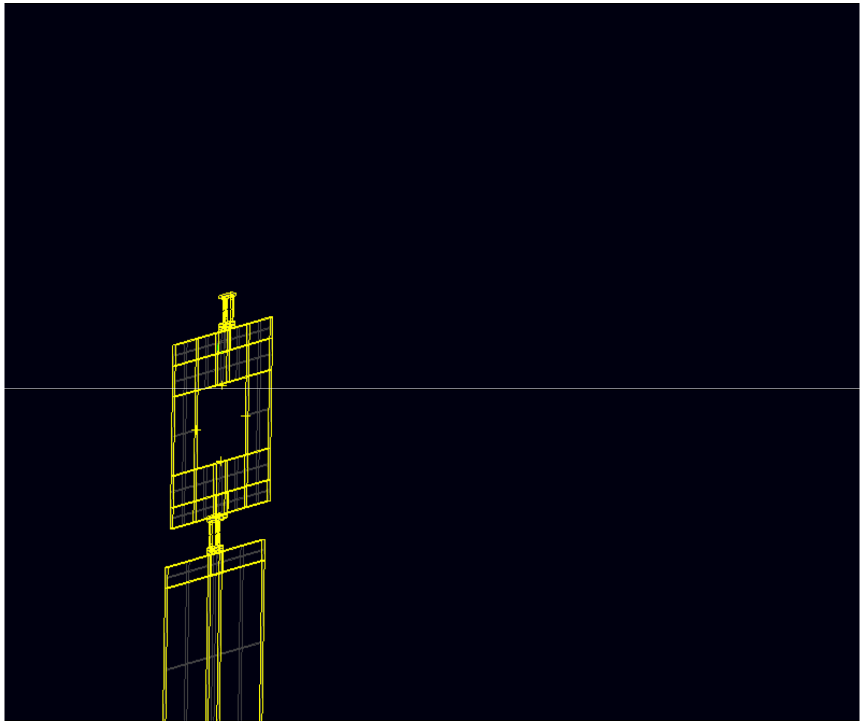


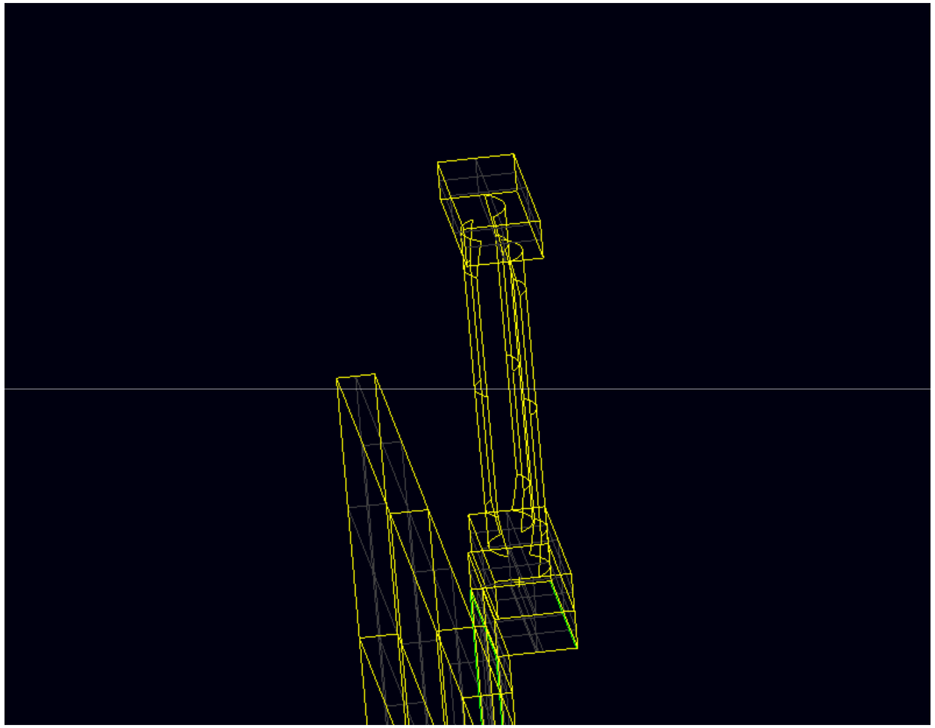




Full hinge - Torsional mode free

(Hoffait et al 2010)





- Self-locking is caused by the hysteresis phenomenon
- The global dynamic response is acceptable
  even though the physical dissipation is not modelled!

Summary

Fully integrated approach in flexible multibody dynamics

- Nonlinear finite element method
- Block diagram language
- Monolithic generalized-α time integration

Added-value in applications:

- Motion, vibration & control analysis
- Stress computation with accurate dynamic loadings
- Analysis of compliant systems

Can we use these simulation tools for inverse analysis?

The unknowns may be

- the externally applied loads
- the mechanical design

High-fidelity system-level simulation

1. Overview and added-value in mechanical applications

2. Potential and challenges in design optimization
   - Inverse dynamics
   - Structural optimization
   - Sensitivity analysis

3. Emerging formalisms for inverse analysis
   (Lie group approach)

Inverse dynamics of flexible MBS

$$
\begin{aligned}
\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{g}(\mathbf{q}, \dot{\mathbf{q}}, t) + \mathbf{\Phi}_{\mathbf{q}}^T \boldsymbol{\lambda} &= \mathbf{A}\mathbf{u} \\
\mathbf{\Phi}(\mathbf{q}) &= \mathbf{0} \\
\mathbf{y}(\mathbf{q}) &= \mathbf{y}_d(t)
\end{aligned}
$$

Flexibility $\Rightarrow$ underactuated system

Forward integration for differentially flat or minimum-phase systems (Blajer & Kolodziejczyk 2004, Seifried 2010)

Stable inversion for systems in nonlinear I/O normal form (Seifried & Eberhard 2009)

Optimal control for flexible MBS in DAE form

Inverse dynamics of flexible MBS

$$\min_{\mathbf{q}(t),\boldsymbol{\lambda}(t),\mathbf{u}(t)} G(\mathbf{q}(t_f),\dot{\mathbf{q}}(t_f)) + \int_{t_0}^{t_f} F(\mathbf{q},\dot{\mathbf{q}},\boldsymbol{\lambda},\mathbf{u})\, d\tau$$

s.t.

$$\begin{aligned} \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{g}(\mathbf{q},\dot{\mathbf{q}},t) + \boldsymbol{\Phi}_{\mathbf{q}}^T\boldsymbol{\lambda} &= \mathbf{A}\mathbf{u} \\ \boldsymbol{\Phi}(\mathbf{q}) &= \mathbf{0} \\ \mathbf{y}(\mathbf{q}) &= \mathbf{y}_d(t) \end{aligned}$$

+ other constraints

Direct collocation method ⇒ large but sparse NLP problem

$$\mathbf{x} = (\mathbf{q}_1, \dot{\mathbf{q}}_1, \ddot{\mathbf{q}}_1, \mathbf{a}_1, \boldsymbol{\lambda}_1, \mathbf{u}_1, \ldots, \mathbf{q}_N, \dot{\mathbf{q}}_N, \ddot{\mathbf{q}}_N, \mathbf{a}_N, \boldsymbol{\lambda}_N, \mathbf{u}_N)$$

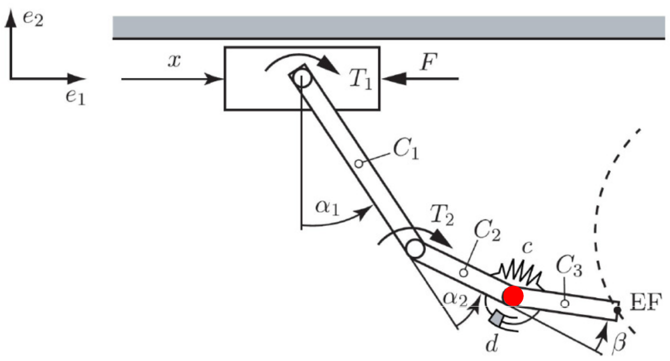
$$\min_{\mathbf{x}} G(\mathbf{q}_N, \dot{\mathbf{q}}_N) + \sum_{n=2}^{N} h\, F(\mathbf{q}_n, \dot{\mathbf{q}}_n, \boldsymbol{\lambda}_n, \mathbf{u}_n)$$

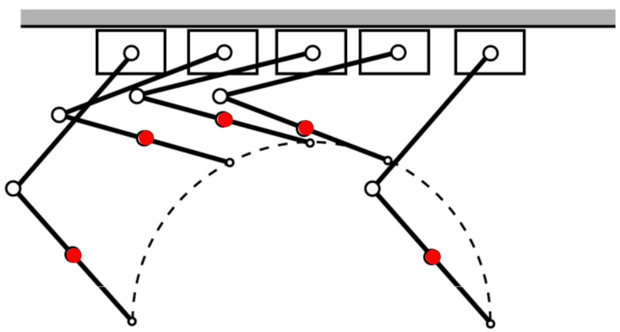
s.t. { equations of motion, integration formulae } at each step

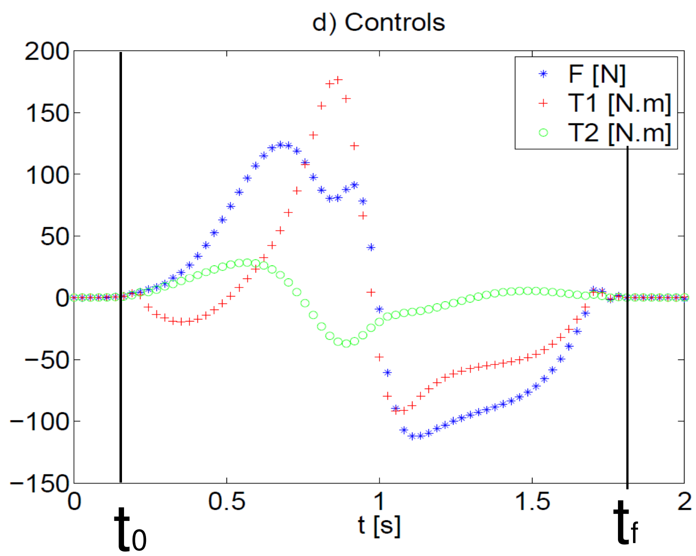
Inverse dynamics of flexible MBS

Manipulator with one passive joint (non-minimum phase)

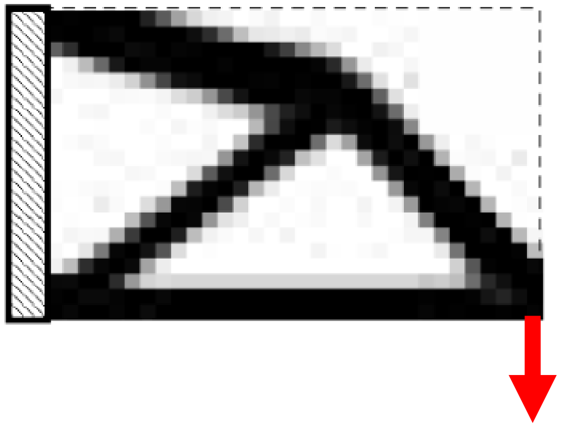
(Bastos, Seifried & B. 2011)







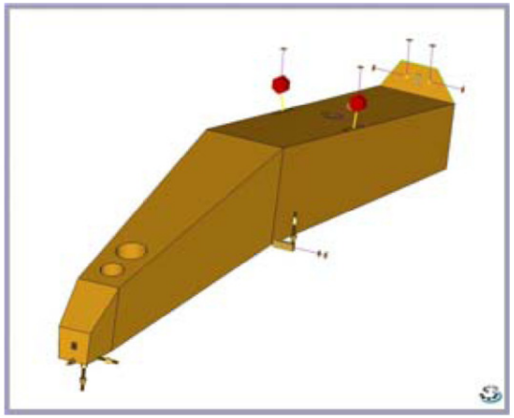
Structural optimization



Topology optimization

(Bendsøe & Kikuchi 1988, Sigmund, 2001)

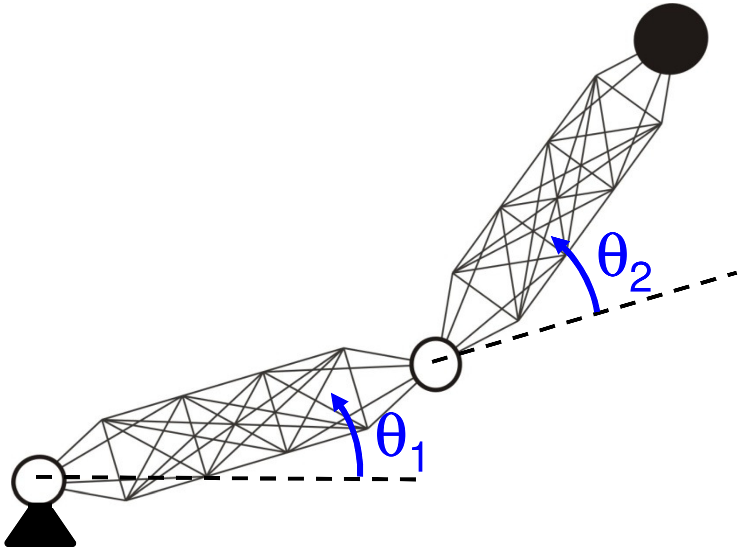
Complex structures can be optimized w.r.t. static loads:

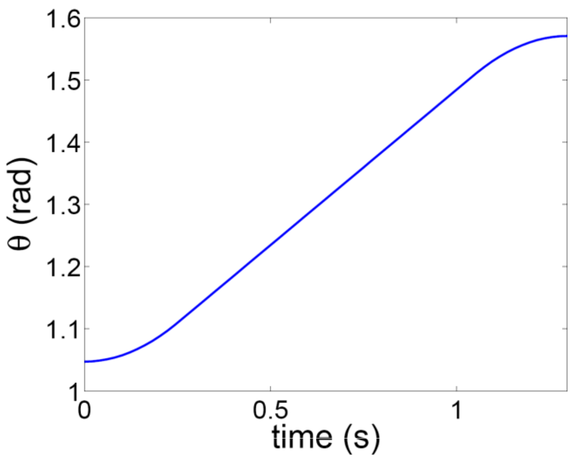


Optimization of articulated systems with dynamic load cases?

Topology optimization of a planar robot arm

(B. et al 2007)





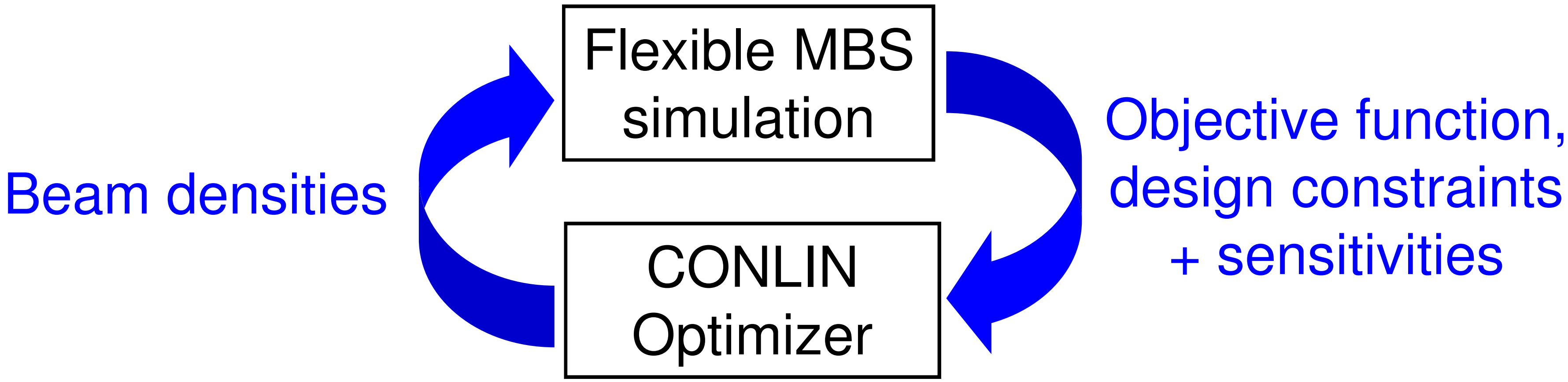
Point-to-point joint trajectory

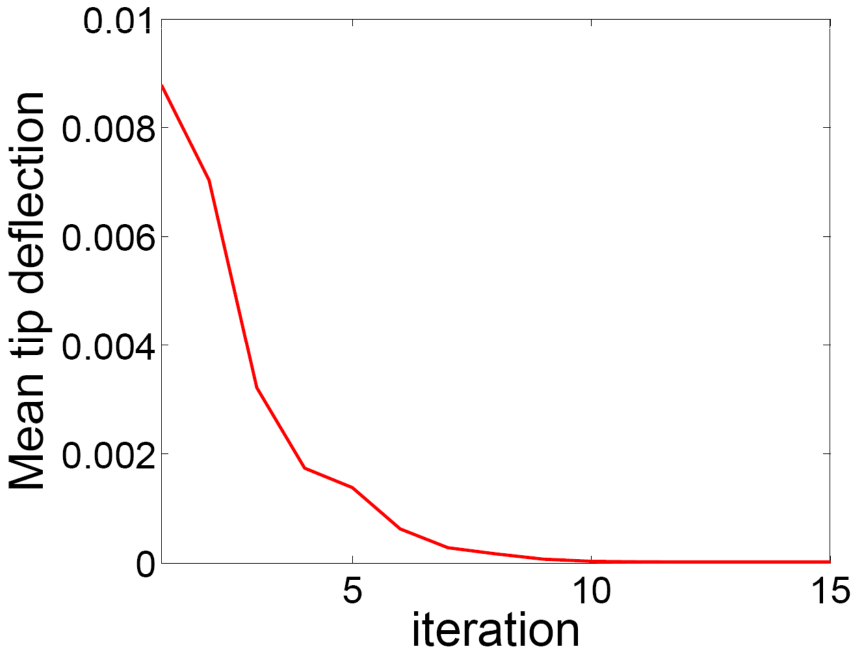
One topology variable per beam (SIMP penalization)

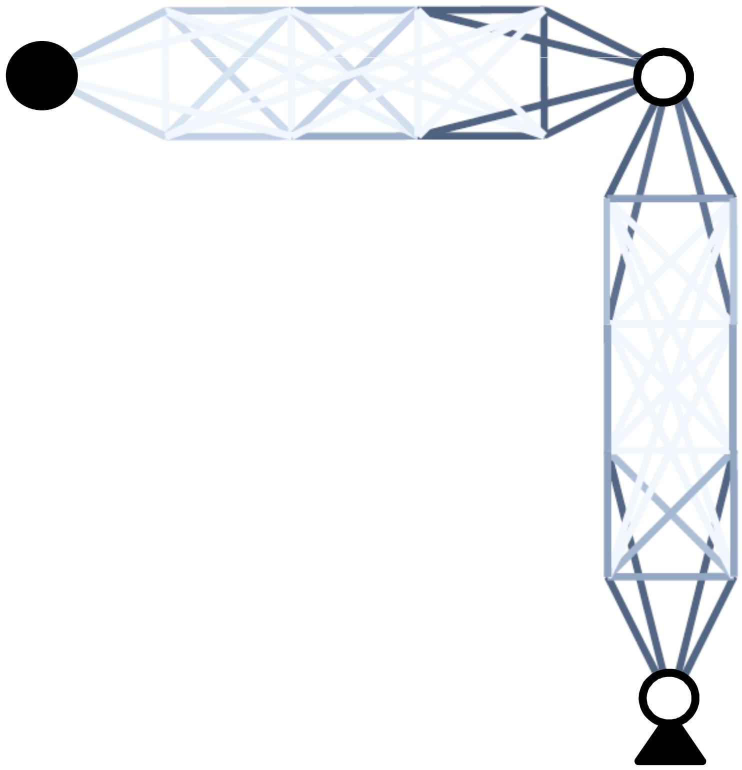
min $$\frac{1}{t_f}\int_0^{t_f} \left\|\mathbf{r} - \mathbf{r}_{rigid}\right\|^2 dt$$

subject to $V_{(i)} \leq 0.4\, V_{full,(i)}$

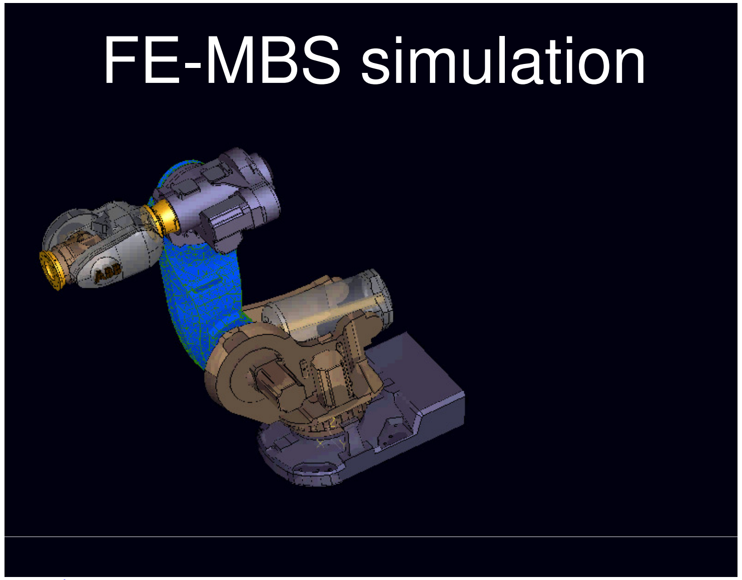
Topology optimization of a planar robot arm







Optimization of full-scale 3D MBS?



Parameters:
- Control
- Properties
- Shape
- Topology

Optimization algorithm

Criteria:
- Dyn. performance
- Deformations
- Vibration levels
- Stresses

Gradient-based & sparse methods (SQP, IP, CONLIN, MMA, etc)

- The problem should be carefully formulated
- An efficient evaluation of the **sensitivities** is needed

Methods for sensitivity analysis

High cost of finite differences for large scale problems

- n_p additional simulations for fwd/bwd differences (order 1)
- $2\ n_p$ additional simulations for central differences (order 2)

Automatic differentiation

- High reliability but suboptimal code, so that a manual post-processing of the code is often required

Semi-analytical methods (direct differentiation / adjoint variable)

- Optimized but manual implementation
- Tend to amplify the intricacy of a simulation code
- Feasible for flexible MBS?

Large rotations in 3D: the whole story

$$\mathbf{q} = (\mathbf{x}_1, \boldsymbol{\alpha}_1, \ldots, \mathbf{x}_{k_n}, \boldsymbol{\alpha}_{k_n})$$

Rotational equilibrium of a free body: $\mathbf{M}(\boldsymbol{\alpha})\,\ddot{\boldsymbol{\alpha}} + \mathbf{g}(\boldsymbol{\alpha}, \dot{\boldsymbol{\alpha}}) = \mathbf{0}$

$$\mathbf{M}(\boldsymbol{\alpha}) = \mathbf{T}^T(\boldsymbol{\alpha})\,\mathbf{J}\,\mathbf{T}(\boldsymbol{\alpha})$$

$$\mathbf{g}(\boldsymbol{\alpha}, \dot{\boldsymbol{\alpha}}) = \mathbf{T}^T(\boldsymbol{\alpha})(\mathbf{J}\,\dot{\mathbf{T}}(\boldsymbol{\alpha}, \dot{\boldsymbol{\alpha}}) + (\mathbf{T}(\boldsymbol{\alpha})\,\dot{\boldsymbol{\alpha}}) \times \mathbf{J}\,\mathbf{T}(\boldsymbol{\alpha})\,\dot{\boldsymbol{\alpha}})$$

Updated Lagrangian strategy (Cardona & Géradin 1989)

$$\mathbf{R}(t_{n+1}) = \mathbf{R}(t_n)\,\mathbf{R}_{inc}(t_{n+1})$$

- Only the incremental rotation is parameterized
- Geometrically exact and singularity-free approach
- Equivalent to a reparameterization at each time step

Successful for simulation codes but challenging for SA!

(B. & Eberhard 2008)

Summary

Dynamic response optimization

- The FEM in flexible multibody dynamics can be exploited for inverse dynamics & structural optimization
- This leads to large scale optimization problems involving transient analyses
- More efficient transient/sensitivity analyses are needed for the optimization of full-scale 3D systems

High-fidelity system-level simulation

1. Overview and added-value in mechanical applications

2. Potential and challenges in design optimization

3. Emerging formalisms for inverse analysis
   - Lie group approach
   - Sensitivity analysis

Lie group formulation

- The configuration of a MBS is described as an element of a matrix Lie group.
- The equations of motion are formulated on the Lie group
- Numerical solution is computed on the Lie group

Properties:

- parameterization-free (geometric) approach
- simpler formulations and numerical procedures

Lie group description of a MBS

Example: $\mathbf{R}(t) \in SO(3)$

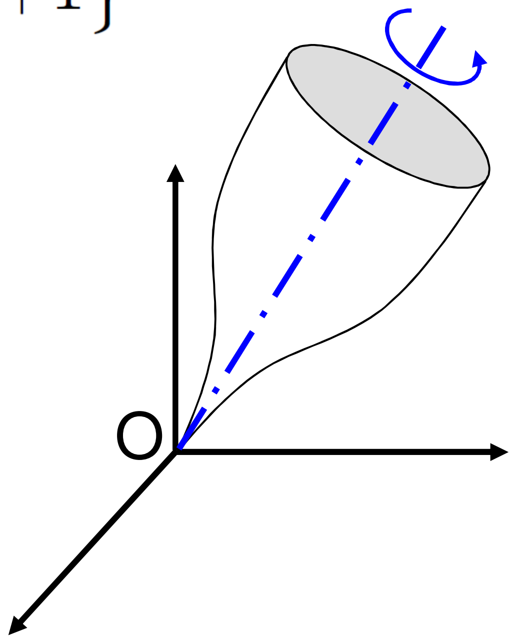
$$SO(3) = \left\{\mathbf{R} \in \mathbb{R}^{3\times 3} | \mathbf{R}^T\mathbf{R} = \mathbf{I}_3, \det\mathbf{R} = +1\right\}$$

$$\dot{\mathbf{R}} = \mathbf{R}\widetilde{\boldsymbol{\Omega}}$$

$$\boldsymbol{\Omega} = [\Omega_1 \ \Omega_2 \ \Omega_3]^T \in \mathbb{R}^3$$

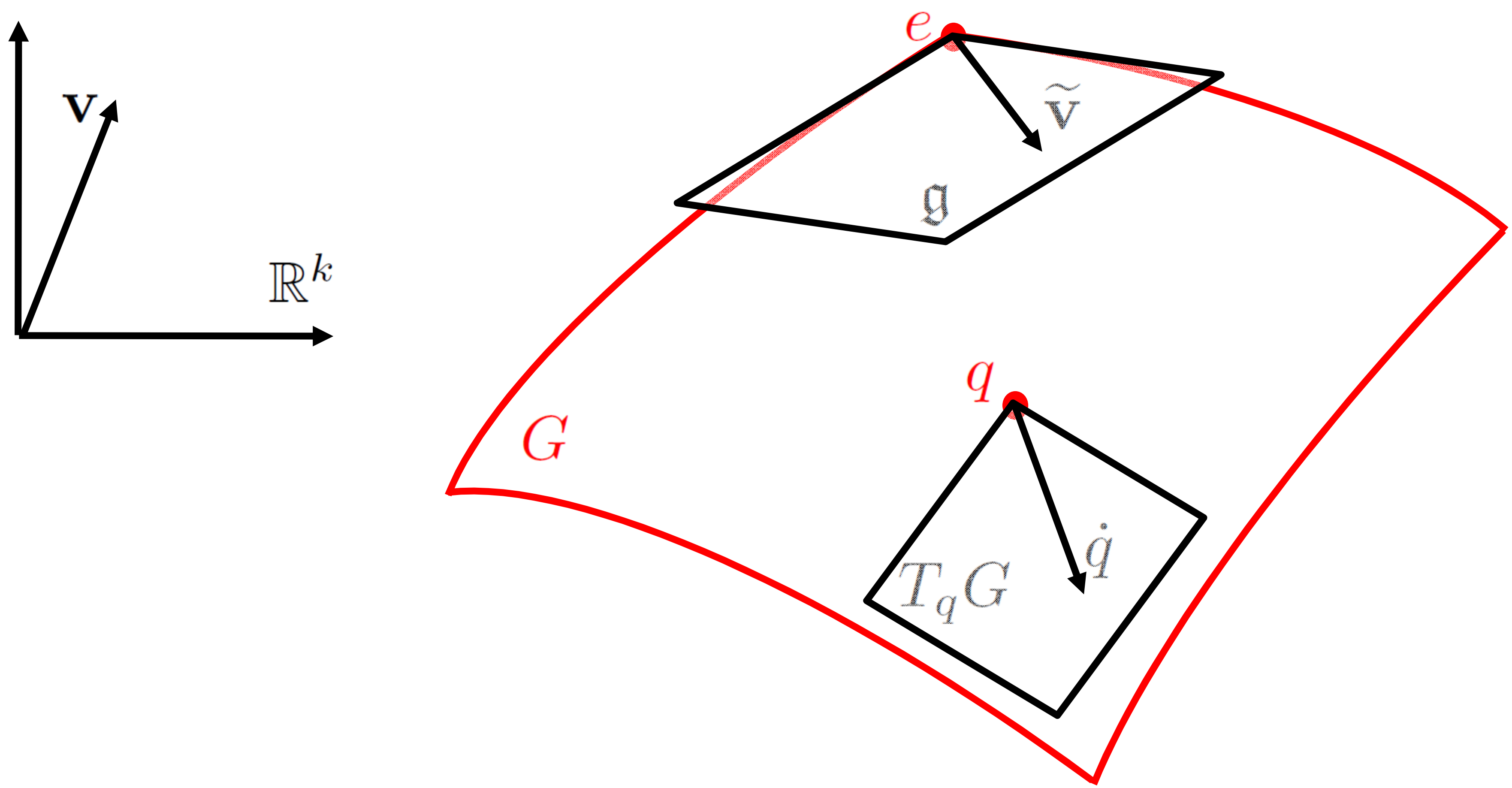
$$\widetilde{\boldsymbol{\Omega}} = \begin{bmatrix} 0 & -\Omega_3 & \Omega_2 \\ \Omega_3 & 0 & -\Omega_1 \\ -\Omega_2 & \Omega_1 & 0 \end{bmatrix}$$

$$\in \mathfrak{so}(3) = \{\widetilde{\boldsymbol{\Omega}} : \ \widetilde{\boldsymbol{\Omega}} + \widetilde{\boldsymbol{\Omega}}^T = \mathbf{0}\}$$



A Lie group is not a linear space!

Lie group, tangent vectors and Lie algebra



Kinematic compatibility equation (left translation map): $\dot{q} = q\widetilde{\mathbf{v}}$

Lie group description of a nodal variable

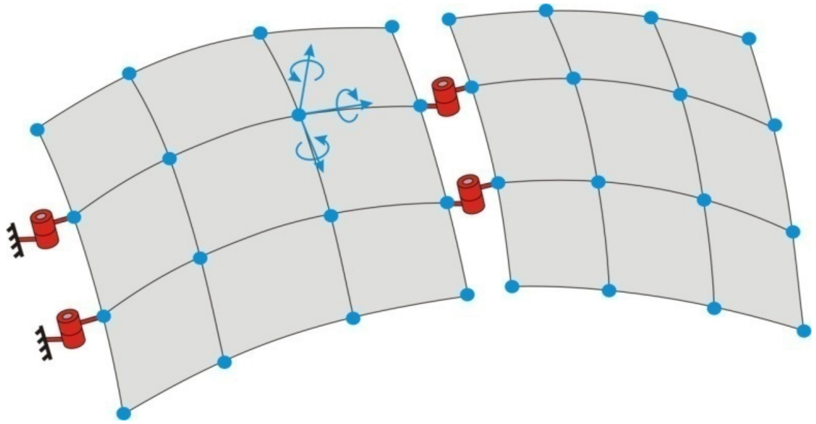
R^3 x SO(3): $q = (\mathbf{x}, \mathbf{R})$

- Composition: $(\mathbf{x}_1, \mathbf{R}_1) \circ (\mathbf{x}_2, \mathbf{R}_2) = (\mathbf{x}_1 + \mathbf{x}_2, \mathbf{R}_1\mathbf{R}_2)$
- Velocity vector: $\mathbf{v} = \begin{bmatrix} \mathbf{u} \\ \boldsymbol{\Omega} \end{bmatrix}$ with $\dot{\mathbf{x}} = \mathbf{u}$

SE(3): $q = \begin{bmatrix} \mathbf{R} & \mathbf{x} \\ \mathbf{0}_{1\times 3} & 1 \end{bmatrix}$

- Composition: product of 4x4 homogenous transf. matrices
- Velocity vector: $\mathbf{v} = \begin{bmatrix} \mathbf{U} \\ \boldsymbol{\Omega} \end{bmatrix}$ with $\dot{\mathbf{x}} = \mathbf{R}\mathbf{U}$

Configuration of a multibody system



- $q \in G$ is a collection of nodal variables, so that,

$$G = \mathbb{R}^3 \times SO(3) \times \ldots \times \mathbb{R}^3 \times SO(3)$$

or $$G = SE(3) \times \ldots \times SE(3)$$

- m kinematic constraints $\mathbf{\Phi}(q)$

Equations of motion on a Lie group

Hamilton principle $\Rightarrow$ Index-3 DAE on a Lie group

$$\begin{aligned}\dot{q} &= q\tilde{\mathbf{v}} \\ \mathbf{M}\dot{\mathbf{v}} - \hat{\mathbf{v}}^T\mathbf{M}\mathbf{v} + \mathbf{g}(q,t) + \mathbf{B}^T(q)\boldsymbol{\lambda} &= \mathbf{0} \\ \boldsymbol{\Phi}(q) &= \mathbf{0}\end{aligned}$$

- The configuration is described by the matrix q
- The velocity is described by a vector $\mathbf{v}$, related with the matrix $\tilde{\mathbf{v}}$
- The mass matrix is constant, inertia forces are quadratic
- Parameterization-free formulation!

Overview of Lie group integration methods

Local (incremental) parameterization of the equations of motion

- Cardona & Géradin (1989): HHT method for flexible MBS
- Munthe-Kaas (1995, 1998): RK method for ODEs
- Bottasso & Borri (1998): RK and EC methods for flexible MBS

Integration formulae on a Lie group using the exponential map

- Simo (1988, 1991): Newmark and EC scheme for NL structures
- Crouch & Grossman (1993): RK and multistep methods for ODEs
- B. et al (2010, 2011): Generalized-α method for flexible MBS

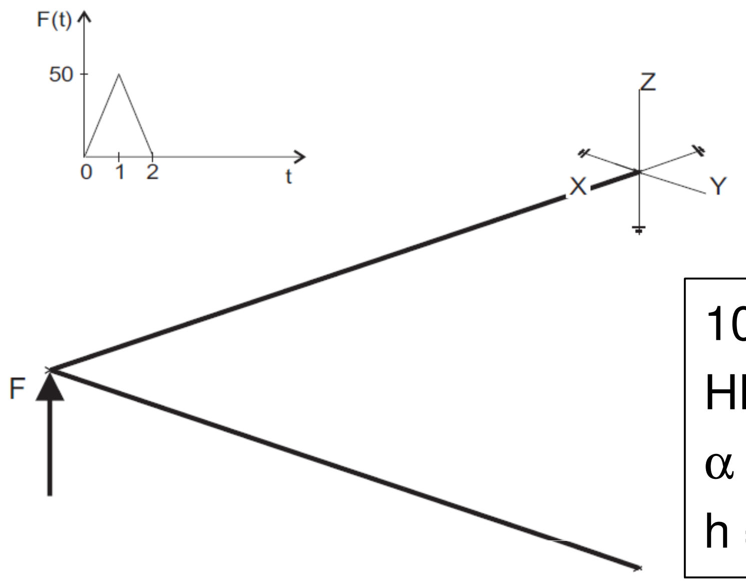
Lie group generalized-α method

Solution of DAEs on a Lie group (B. & Cardona 2010)

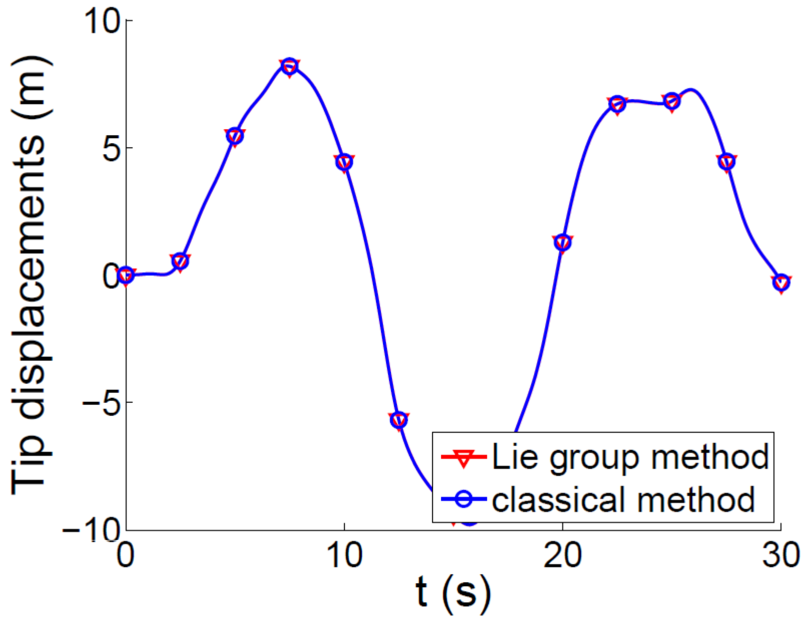
$$
\begin{aligned}
\mathbf{M}\dot{\mathbf{v}}_{n+1} - \hat{\mathbf{v}}_{n+1}^T \mathbf{M}\mathbf{v}_{n+1} &= -\mathbf{g}(q_{n+1}, t_{n+1}) - \mathbf{B}(q_{n+1})^T \boldsymbol{\lambda}_{n+1} \\
\boldsymbol{\Phi}(q_{n+1}) &= \mathbf{0} \\
q_{n+1} &= q_n \exp(\widetilde{\Delta\mathbf{x}}_{n+1}) \\
\Delta\mathbf{x}_{n+1} &= h\mathbf{v}_n + (0.5-\beta)h^2\mathbf{a}_n + \beta h^2 \mathbf{a}_{n+1} \\
\mathbf{v}_{n+1} &= \mathbf{v}_n + (1-\gamma)h\mathbf{a}_n + \gamma h \mathbf{a}_{n+1} \\
(1-\alpha_m)\mathbf{a}_{n+1} + \alpha_m \mathbf{a}_n &= (1-\alpha_f)\dot{\mathbf{v}}_{n+1} + \alpha_f \dot{\mathbf{v}}_n
\end{aligned}
$$

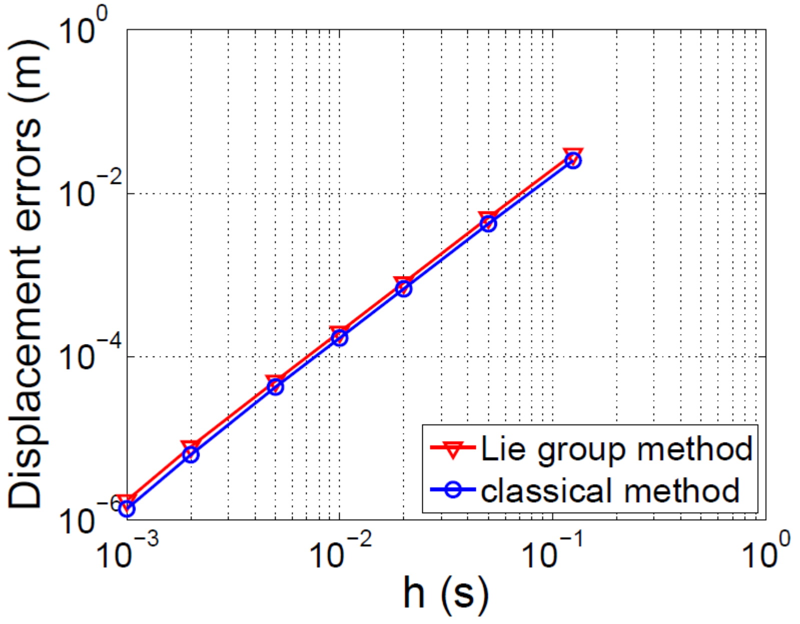
- Inspired by Newmark / generalized-α methods
- Analytical form of the exponential map
- Newton iterations for vector unknowns (not matrices)
- Second-order convergence (B., Arnold, Cardona 2011)
- Reduced-index formulation (Arnold et al 2011)

Rightangle flexible beam

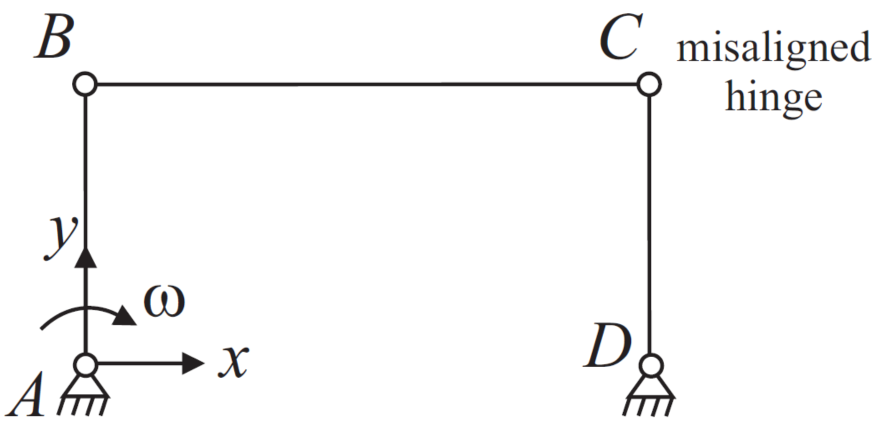


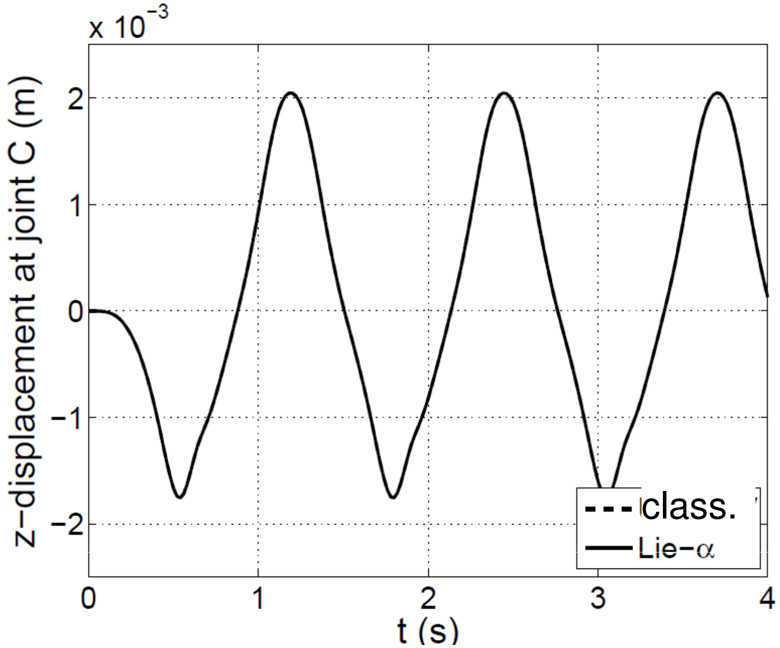
10 elements
HHT method
$\alpha = 0.05$
$h = 0.125$ s

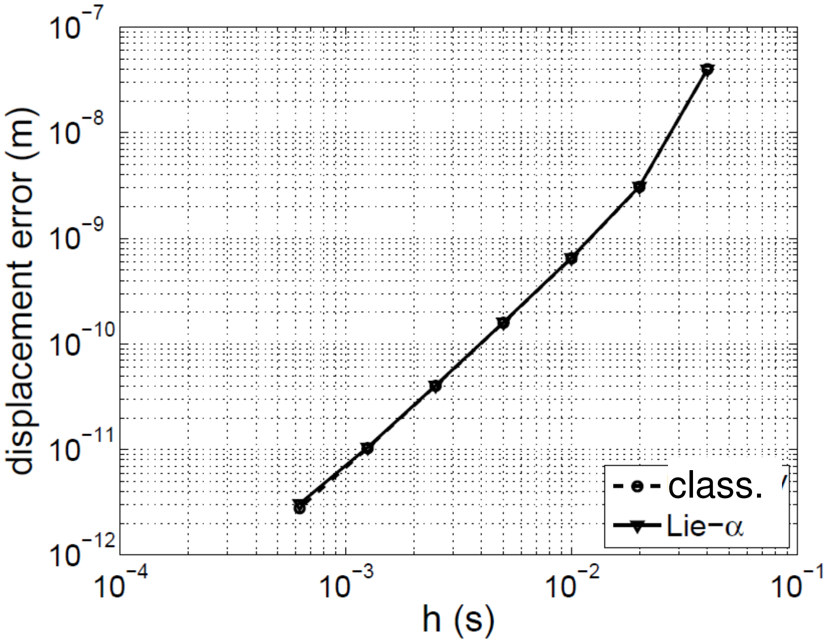


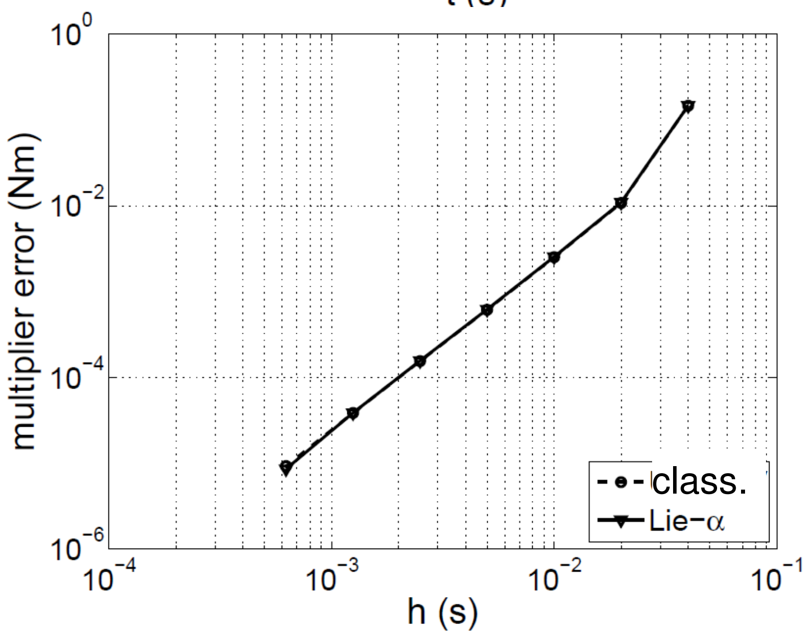


Flexible four bar mechanism









Sensitivity analysis on a Lie group

Let us consider a single design parameter p

$$\begin{aligned} \dot{q} &= q\tilde{\mathbf{v}} \\ \mathbf{M}(p)\dot{\mathbf{v}} - \hat{\mathbf{v}}^T\mathbf{M}(p)\mathbf{v} + \mathbf{g}(q,p,t) + \mathbf{B}^T(q,p)\boldsymbol{\lambda} &= \mathbf{0} \\ \boldsymbol{\Phi}(q,p) &= \mathbf{0} \end{aligned}$$

and a single criterion function

$$\Psi(p) = G(q(t_f), \mathbf{v}(t_f), p) + \int_{t_0}^{t_f} F(q, \mathbf{v}, \boldsymbol{\lambda}, p)\, dt$$

Sensitivity in the Lie algebra: $q' = q\tilde{\mathbf{w}}$

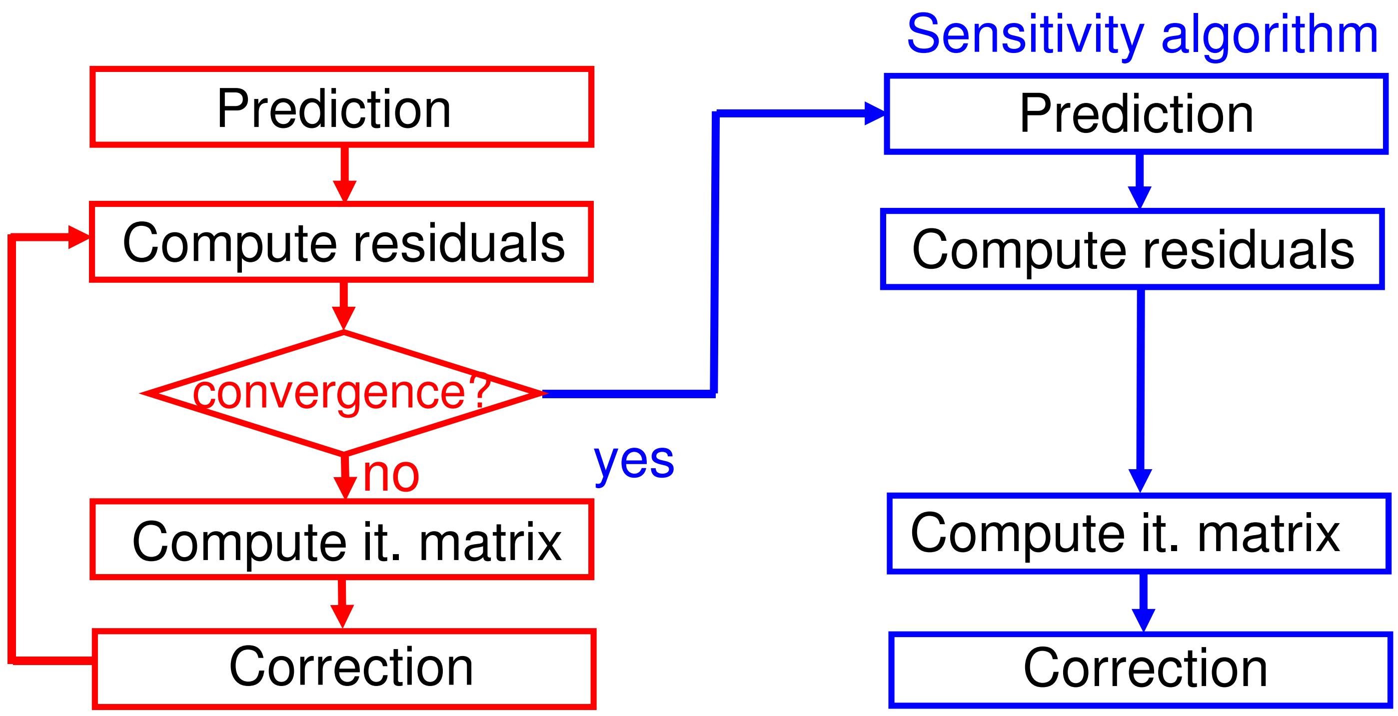
Extension to several parameters and criteria is straightforward

Direct differentiation on a Lie group

$$\begin{aligned} \dot{\mathbf{w}} &= \mathbf{v}' - \tilde{\mathbf{v}}\mathbf{w} \\ \mathbf{M}\dot{\mathbf{v}}' + \mathbf{C}_t\mathbf{v}' + \mathbf{K}_t\mathbf{w} + \mathbf{B}^T\boldsymbol{\lambda}' &= -\mathbf{res}' \\ \mathbf{B}\mathbf{w} &= -\boldsymbol{\Phi}' \end{aligned}$$

With :
$$\begin{aligned} \mathbf{res}' &= (\partial\mathbf{M}/\partial p)\dot{\mathbf{v}} - \hat{\mathbf{v}}(\partial\mathbf{M}/\partial p)\mathbf{v} + (\partial\mathbf{g}/\partial p) + (\partial\mathbf{B}/\partial p)^T\boldsymbol{\lambda} \\ \boldsymbol{\Phi}' &= \partial\boldsymbol{\Phi}/\partial p \end{aligned}$$

For each design variable, one linear DAE for $\mathbf{w}$, $\mathbf{v}'$ and λ'



Adjoint variable method

$$\delta\Psi = (G_p + \boldsymbol{\rho}^T\boldsymbol{\chi}_p + \boldsymbol{\pi}^T\boldsymbol{\zeta}_p)\delta p + \int_{t_0}^{t_f} (F_p + \boldsymbol{\mu}^T\mathbf{r}_p + \boldsymbol{\nu}^T\boldsymbol{\Phi}_p)\delta p \, dt$$

provided that the adjoint variables satisfy

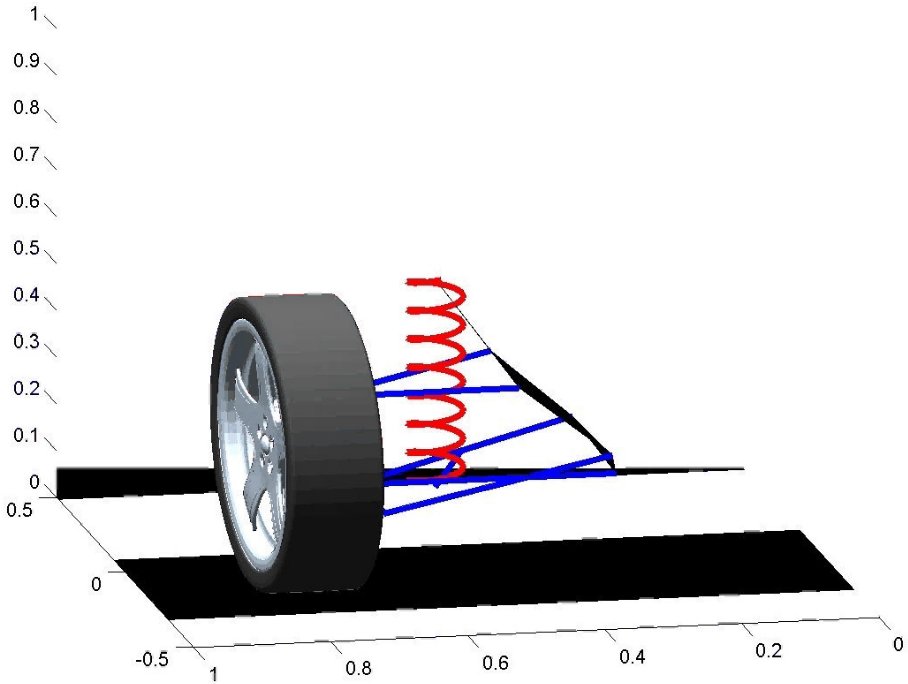
$$\begin{aligned} \mathbf{M}\ddot{\boldsymbol{\mu}} - (\mathbf{M}\hat{\mathbf{v}} + \mathbf{C}_t)^T\dot{\boldsymbol{\mu}} + (\mathbf{K}_t + \mathbf{C}_t\hat{\mathbf{v}} - \dot{\mathbf{C}}_t)^T\boldsymbol{\mu} + \mathbf{B}^T\boldsymbol{\nu} &= -F_{q*}^T \\ \mathbf{B}\boldsymbol{\mu} &= -F_{\boldsymbol{\lambda}}^T \end{aligned}$$

With :

$$\begin{aligned} \mathbf{r}_{\mathbf{M}}^T\boldsymbol{\mu}(t_f) &= -(G_{\mathbf{C}})_{t_f}^T \\ \mathbf{r}_{\mathbf{M}}^T\dot{\boldsymbol{\mu}}(t_f) &= (F_{\mathbf{C}} + \boldsymbol{\mu}^T\mathbf{r}_{\mathbf{C}} + G_{\mathbf{K}})_{t_f}^T \\ \boldsymbol{\chi}_{\mathbf{C}}^T\boldsymbol{\rho} &= (\boldsymbol{\mu}^T\mathbf{r}_{\mathbf{M}})_{t_0}^T \\ \boldsymbol{\zeta}_{\mathbf{K}}^T\boldsymbol{\pi} &= (F_{\mathbf{C}} + \boldsymbol{\mu}^T\mathbf{r}_{\mathbf{c}} - \dot{\boldsymbol{\mu}}^T\mathbf{r}_{\mathbf{M}} - \boldsymbol{\rho}^T\boldsymbol{\chi}_{\mathbf{K}})_{t_0}^T \end{aligned}$$

For each active criterion function, one linear DAE for μ and ν, which can be solved backward in time

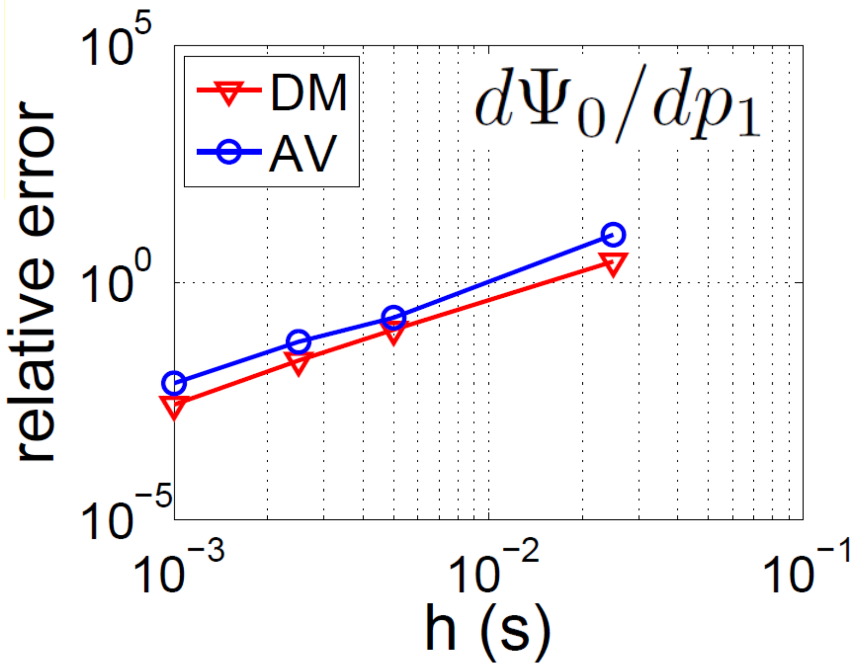
Numerical example



p_1 = damping coefficient

P_2 = stiffness coefficient

$$\Psi_0 = \int_{t_0}^{t_f} \dot{v}_{z,chassis}^2(t)\ dt$$



Conclusion

The FE method in flexible multibody dynamics has a high potential in mechanical applications for:

- simulation (virtual prototyping)
- dynamic response optimization

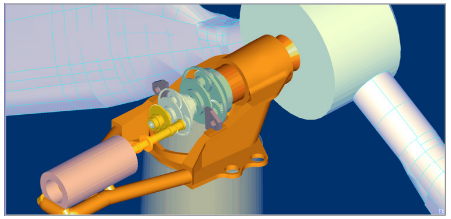
However, gradient-based methods require

- a careful formulation of the optimization problem
- efficient transient and sensitivity analysis

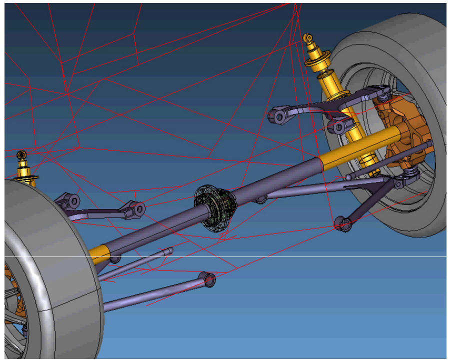
Lie group methods may improve the efficiency of 3D models

- parameterization-free formulations and time integration
- simplified algorithms but similar levels of accuracy
- well-suited for sensitivity analysis

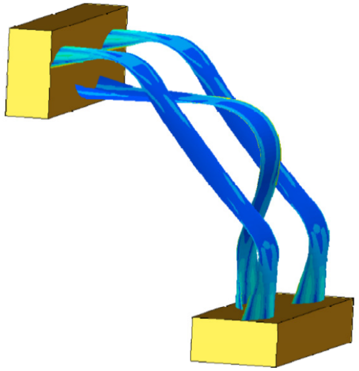
Thank you for your attention!



From direct to inverse analysis in flexible multibody dynamics



Olivier Brüls



Acknowledgements:
M. Arnold, G. Bastos, A. Cardona, Q.z. Chen, P. Duysinx,
S. Hoffait, G. Kerschen, R. Seifried, V. Sonneville, G. Virlez

Université de Liège

Annual GAMM Conference
Darmstadt, March 27, 2012

LTAS