From direct to inverse analysis in flexible multibody dynamics

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1. Overview and added-value in mechanical applications

2. Potential and challenges in design optimization

3. Emerging formalisms for inverse analysis (Lie group approach)
High-fidelity system-level simulation

1. Overview and added-value in mechanical applications
   - The FE method in flexible multibody dynamics
   - Simulation of vehicle driveline
   - Simulation of compliant deployable structures

2. Potential and challenges in design optimization

3. Emerging formalisms for inverse analysis
   (Lie group approach)
Examples of application fields

Mecano models (Courtesy: LMS Samtech)
Integrated simulation approach

- Rigid bodies
- Joints
- Flexible bodies
- Hydraulics
- Electronics
- Control

Flexible mechanism (FE formalism)

Control system (block diagram)

Numerical assembly

Strongly coupled eqns

Time integration
FE method in flexible multibody dynamics

Technical system

Model

Equations of motion

Time integration

Results

Dynamic performance?
Mechanical loads & stresses?
FE method in flexible multibody dynamics

Technical system

Model

Equations of motion

Time integration

Results
FE method in flexible multibody dynamics

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Results

If rotations are parameterized:

\[
M(q)\ddot{q} = g(q, \dot{q}, t) - \Phi_q^T \lambda + Ly
\]

\[
0 = \Phi(q, t)
\]

\[
\dot{x} = f(q, \dot{q}, \ddot{q}, \lambda, x, y, t)
\]

\[
y = h(q, \dot{q}, \ddot{q}, \lambda, x, y, t)
\]

(Géradin & Cardona 2001)
FE method in flexible multibody dynamics

Technical system

Model

Numerical solution of coupled 1st and 2nd order DAEs with index 3

Equations of motion

Generalized-α method
(Chung & Hulbert, 1993)

Time integration

Results
FE method in flexible multibody dynamics

Technical system

Model

Equations of motion

Time integration

Results
Stiff & large ODEs in structural dynamics (Chung & Hulbert 1993)
- Second-order accuracy
- Unconditional stability (A-stability)
- Controllable numerical damping (tuning parameter $\rho_\infty$)
- Newmark and HHT are special cases
- Equivalent to a multistep method (Erlicher et al 2002)

Direct integration of index-3 DAEs
- Linear stability (Cardona & Géradin 1989)
- 2nd order accuracy (Arnold & B. 2007)
- Mechatronics (B. & Golinval 2008)

Index reduction methods
Generalized-\(\alpha\) time integrator

Newmark implicit formulae:

\[
\begin{align*}
q_{n+1} &= q_n + h \ddot{q}_n + h^2 (0.5 - \beta) a_n + h^2 \beta a_{n+1} \\
\dot{q}_{n+1} &= \dot{q}_n + h (1 - \gamma) a_n + h \gamma a_{n+1} \\
x_{n+1} &= x_n + h (1 - \theta) w_n + h \theta w_{n+1}
\end{align*}
\]

Generalized-\(\alpha\) method (Chung & Hulbert, 1993)

\[
\begin{align*}
(1 - \alpha_m) a_{n+1} + \alpha_m a_n &= (1 - \alpha_f) \ddot{q}_{n+1} + \alpha_f \ddot{q}_n \\
(1 - \delta_m) w_{n+1} + \delta_m w_n &= (1 - \delta_f) \dot{x}_{n+1} + \delta_f \dot{x}_n
\end{align*}
\]

Equations of motion at time \(t_{n+1}\)

\[
\begin{align*}
M(q) \ddot{q} &= g(q, \dot{q}, t) - \Phi_q^T \lambda + Ly \\
0 &= \Phi(q, t) \\
\dot{x} &= f(q, \dot{q}, \ddot{q}, \lambda, x, y, t) \\
y &= h(q, \dot{q}, \ddot{q}, \lambda, x, y, t)
\end{align*}
\]
Comparison with other DAE solvers

Test equation:
\[ \ddot{q} + \omega^2 q = 0 \]

Numerical solution:
\[ x_n = (A(\omega h))^n x_0 \]

- Order of unconditionally stable BDF \( \leq 2 \)
- Less numerical dissipation with generalized-\( \alpha \) method
Modelling of vehicle drivelines

Modelling the components in their environment
Torsen limited slip differential

- Variable torque distribution between the output shafts
- Locking by friction between gear pairs & thrust washers
- 4 working modes
Gear pair element

- Connection between two wheels modelled as rigid bodies
- Local flexibility: spring (KR) and damper (C)
- Time fluctuation of mesh stiffness (ISO 6336)
- Backlash (GAP), load transmission error (ERR), misalignment
Case 1: flexible washers

- Rigid/flexible contact conditions
- 8000 dofs

TDR1 numerical : 3.90
TDR1 experimental (Torsen) : 4.02
Case 2: rigid washers

Rigid/rigid contact condition: continuous impact model with a coefficient of restitution (Lankarani & Nikravesh 1994)

Alternative: nonsmooth description and time-stepping algorithm
Tape-spring hinge

MAEVA hinge (METRAVIB & CNES)
- Guiding, driving and locking functions
- No contact between sliding surfaces

First model: ideal hinge
- No 3D behaviour
- No self-locking
Static analysis of a single tape-spring

- Fine mesh with second order Mindlin shell elements
- Symmetry is exploited
- Continuation vs. pseudo-dynamic methods

[Graph showing bending moment vs. opening angle with labels for holding and driving torque]
Static behaviour of a full hinge

Numerical results
Driving torque : 0.152 Nm
Holding torque : 6.67 Nm

Experimental tests (Metravib):
Driving torque > 0.15 Nm
Holding torque > 4.5 Nm
Dynamic behaviour of a full hinge

- Inertia of the rigid appendix (solar panel)
- No structural damping but numerical damping
Full hinge - Torsional mode blocked

\[ \Delta E_{\text{tot}} = -0.0457 \, \text{J} \]

\[ \Delta E_{\text{hyst}} = -0.0414 \, \text{J} \]}
Full hinge - Torsional mode free

(Hoffait et al 2010)

- Self-locking is caused by the hysteresis phenomenon
- The global dynamic response is acceptable even though the physical dissipation is not modelled!
Summary

Fully integrated approach in flexible multibody dynamics
  - Nonlinear finite element method
  - Block diagram language
  - Monolithic generalized-\(\alpha\) time integration

Added-value in applications:
  - Motion, vibration & control analysis
  - Stress computation with accurate dynamic loadings
  - Analysis of compliant systems

Can we use these simulation tools for inverse analysis?
The unknowns may be
  - the externally applied loads
  - the mechanical design
1. Overview and added-value in mechanical applications

2. Potential and challenges in design optimization
   - Inverse dynamics
   - Structural optimization
   - Sensitivity analysis

3. Emerging formalisms for inverse analysis
   (Lie group approach)
Inverse dynamics of flexible MBS

 Outputs $y$

 Inputs $u$

 Flexibility $\Rightarrow$ underactuated system

 Forward integration for differentially flat or minimum-phase systems (Blajer & Kolodziejczyk 2004, Seifried 2010)

 Stable inversion for systems in nonlinear I/O normal form (Seifried & Eberhard 2009)

\[
M(q)\ddot{q} + g(q, \dot{q}, t) + \Phi^T q \lambda = Au
\]
\[
\Phi(q) = 0
\]
\[
y(q) = y_d(t)
\]

Optimal control for flexible MBS in DAE form
Inverse dynamics of flexible MBS

\[
\begin{align*}
\min_{q(t), \lambda(t), u(t)} & \quad G(q(t_f), \dot{q}(t_f)) + \int_{t_0}^{t_f} F(q, \dot{q}, \lambda, u) \, d\tau \\
\text{s.t.} & \quad M(q)\ddot{q} + g(q, \dot{q}, t) + \Phi_q^T \lambda = A u \\
& \quad \Phi(q) = 0 \\
& \quad y(q) = y_d(t) + \text{other constraints}
\end{align*}
\]

Direct collocation method \Rightarrow large but sparse NLP problem

\[
x = (q_1, \dot{q}_1, \ddot{q}_1, a_1, \lambda_1, u_1, \ldots, q_N, \dot{q}_N, \ddot{q}_N, a_N, \lambda_N, u_N)
\]

\[
\begin{align*}
\min_x & \quad G(q_N, \dot{q}_N) + \sum_{n=2}^{N} h F(q_n, \dot{q}_n, \lambda_n, u_n) \\
\text{s.t.} & \quad \left\{ \begin{array}{l}
equations of motion integration formulae \end{array} \right\} \text{ at each step}
\end{align*}
\]
Inverse dynamics of flexible MBS

Manipulator with one passive joint (non-minimum phase)
(Bastos, Seifried & B. 2011)
Structural optimization

Topology optimization
(Bendsøe & Kikuchi 1988, Sigmund, 2001)

Complex structures can be optimized w.r.t. static loads:

Optimization of articulated systems with dynamic load cases?
Topology optimization of a planar robot arm
(B. et al 2007)

Point-to-point joint trajectory

One topology variable per beam (SIMP penalization)

\[
\begin{align*}
\min & \quad \frac{1}{t_f} \int_0^{t_f} \| \mathbf{r} - \mathbf{r}_{\text{rigid}} \|^2 \, dt \\
\text{subject to} & \quad V_{(i)} \leq 0.4 V_{\text{full},(i)}
\end{align*}
\]
Topology optimization of a planar robot arm

Beam densities

Flexible MBS simulation

CONLIN Optimizer

Objective function, design constraints + sensitivities

Objective function

Mean tip deflection vs iteration
Optimization of full-scale 3D MBS?

Parameters:
- Control
- Properties
- Shape
- Topology

Criteria:
- Dyn. performance
- Deformations
- Vibration levels
- Stresses

Optimization algorithm

Gradient-based & sparse methods (SQP, IP, CONLIN, MMA, etc)

- The problem should be carefully formulated
- An efficient evaluation of the sensitivities is needed
Methods for sensitivity analysis

High cost of finite differences for large scale problems
  ➢ \( n_p \) additional simulations for fwd/bwd differences (order 1)
  ➢ 2 \( n_p \) additional simulations for central differences (order 2)

Automatic differentiation
  ➢ High reliability but suboptimal code, so that a manual
    post-processing of the code is often required

Semi-analytical methods (direct differentiation / adjoint variable)
  ➢ Optimized but manual implementation
  ➢ Tend to amplify the intricacy of a simulation code
  ➢ Feasible for flexible MBS?
Rotational equilibrium of a free body: \( \mathbf{M}(\alpha) \ddot{\alpha} + \mathbf{g}(\alpha, \dot{\alpha}) = 0 \)

\[
\mathbf{M}(\alpha) = \mathbf{T}^T(\alpha) \mathbf{J} \mathbf{T}(\alpha)
\]

\[
\mathbf{g}(\alpha, \dot{\alpha}) = \mathbf{T}^T(\alpha)(\mathbf{J} \dot{\mathbf{T}}(\alpha, \dot{\alpha}) + (\mathbf{T}(\alpha) \dot{\alpha}) \times \mathbf{J} \mathbf{T}(\alpha) \dot{\alpha})
\]

Updated Lagrangian strategy (Cardona & Géradin 1989)

\[
\mathbf{R}(t_{n+1}) = \mathbf{R}(t_n) \mathbf{R}_{inc}(t_{n+1})
\]

- Only the incremental rotation is parameterized
- Geometrically exact and singularity-free approach
- Equivalent to a reparameterization at each time step

Successful for simulation codes but challenging for SA!

(B. & Eberhard 2008)
Dynamic response optimization

- The FEM in flexible multibody dynamics can be exploited for inverse dynamics & structural optimization
- This leads to large scale optimization problems involving transient analyses
- More efficient transient/sensitivity analyses are needed for the optimization of full-scale 3D systems
High-fidelity system-level simulation

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3. Emerging formalisms for inverse analysis
   - Lie group approach
   - Sensitivity analysis
The configuration of a MBS is described as an element of a matrix Lie group.
The equations of motion are formulated on the Lie group.
Numerical solution is computed on the Lie group.

Properties:
- parameterization-free (geometric) approach
- simpler formulations and numerical procedures
Lie group description of a MBS

Example: \( \mathbf{R}(t) \in SO(3) \)

\[ SO(3) = \{ \mathbf{R} \in \mathbb{R}^{3 \times 3} | \mathbf{R}^T \mathbf{R} = I_3, \text{det} \mathbf{R} = +1 \} \]

\[ \dot{\mathbf{R}} = \mathbf{R} \tilde{\Omega} \]

\[ \tilde{\Omega} = \begin{bmatrix} 0 & -\Omega_3 & \Omega_2 \\ \Omega_3 & 0 & -\Omega_1 \\ -\Omega_2 & \Omega_1 & 0 \end{bmatrix} \]

\( \tilde{\Omega} \in so(3) = \{ \tilde{\Omega} : \tilde{\Omega} + \tilde{\Omega}^T = 0 \} \)

A Lie group is not a linear space!
Kinematic compatibility equation (left translation map): $\dot{q} = q\tilde{V}$
Lie group description of a nodal variable

\( \mathbb{R}^3 \times \text{SO}(3) : \quad q = (x, R) \)

- Composition: \((x_1, R_1) \circ (x_2, R_2) = (x_1 + x_2, R_1 R_2)\)

- Velocity vector: \( v = \begin{bmatrix} u \\ \Omega \end{bmatrix} \) with \( \dot{x} = u \)

\( \text{SE}(3) : \quad q = \begin{bmatrix} R & x \\ 0_{1 \times 3} & 1 \end{bmatrix} \)

- Composition: product of 4x4 homogenous transf. matrices

- Velocity vector: \( v = \begin{bmatrix} \mathbf{U} \\ \Omega \end{bmatrix} \) with \( \dot{x} = RU \)
Configuration of a multibody system

- \( q \in G \) is a collection of nodal variables, so that,
  \[
  G = \mathbb{R}^3 \times SO(3) \times \ldots \times \mathbb{R}^3 \times SO(3)
  
  \text{or} \quad G = SE(3) \times \ldots \times SE(3)
  
- \( m \) kinematic constraints \( \Phi(q) \)
The configuration is described by the matrix $q$

The velocity is described by a vector $\mathbf{v}$, related with the matrix $\tilde{\mathbf{V}}$

The mass matrix is constant, inertia forces are quadratic

Parameterization-free formulation!
Overview of Lie group integration methods

Local (incremental) parameterization of the equations of motion

- Cardona & Géradin (1989): HHT method for flexible MBS

Integration formulae on a Lie group using the exponential map

- Crouch & Grossman (1993): RK and multistep methods for ODEs
Lie group generalized-$\alpha$ method

Solution of DAEs on a Lie group (B. & Cardona 2010)

$$M\dot{q}_{n+1} - \dot{v}_{n+1}^T M v_{n+1} = -g(q_{n+1}, t_{n+1}) - B(q_{n+1})^T \lambda_{n+1}$$

$$\Phi(q_{n+1}) = 0$$

$$q_{n+1} = q_n \exp(\Delta x_{n+1})$$

$$\Delta x_{n+1} = h v_n + (0.5 - \beta) h^2 a_n + \beta h^2 a_{n+1}$$

$$v_{n+1} = v_n + (1 - \gamma) h a_n + \gamma h a_{n+1}$$

$$(1 - \alpha_m)a_{n+1} + \alpha_m a_n = (1 - \alpha_f)\dot{v}_{n+1} + \alpha_f \dot{v}_n$$

- Inspired by Newmark / generalized-$\alpha$ methods
- Analytical form of the exponential map
- Newton iterations for vector unknowns (not matrices)
- Second-order convergence (B., Arnold, Cardona 2011)
- Reduced-index formulation (Arnold et al 2011)
Rightangle flexible beam

10 elements
HHT method
\( \alpha = 0.05 \)
\( h = 0.125 \) s
Flexible four bar mechanism

![Diagram of a flexible four bar mechanism with labels and a graph showing z-displacement at joint C over time.](image)

![Graphs showing displacement error and multiplier error over h (s) for class and Lie-α methods.](image)
Sensitivity analysis on a Lie group

Let us consider a single design parameter $p$

\[ \dot{q} = q\tilde{v} \]
\[ M(p)\dot{v} - \dot{\hat{v}}^T M(p)v + g(q, p, t) + B^T(q, p)\lambda = 0 \]
\[ \Phi(q, p) = 0 \]

and a single criterion function

\[ \Psi(p) = G(q(t_f), v(t_f), p) + \int_{t_0}^{t_f} F(q, v, \lambda, p) \, dt \]

Sensitivity in the Lie algebra: \[ q' = q\tilde{w} \]

Extension to several parameters and criteria is straightforward
For each design variable, one linear DAE for $w$, $v'$ and $\lambda'$

$$\dot{w} = v' - \tilde{v}w$$

$$M\dot{v}' + C_t v' + K_t w + B^T \lambda' = -\text{res}'$$

$$Bw = -\Phi'$$

With:

$$\text{res}' = (\partial M/\partial \lambda)v - \dot{v}(\partial M/\partial \lambda)v + (\partial g/\partial \lambda) + (\partial B/\partial \lambda)^T \lambda$$

$$\Phi' = \partial \Phi / \partial \lambda$$

Sensitivity algorithm

Prediction

Compute residuals

convergence?

no

Compute it. matrix

Correction

yes

Prediction

Compute residuals

Compute it. matrix

Correction
Adjoint variable method

\[ \delta \Psi = (G_p + \rho^T \chi_p + \pi^T \zeta_p) \delta p + \int_{t_0}^{t_f} (F_p + \mu^T r_p + \nu^T \Phi_p) \delta p \, dt \]

provided that the adjoint variables satisfy

\[ M \ddot{\mu} - (M \dot{\nu} + C_t)^T \dot{\mu} + (K_t + C_t \dot{\nu} - \dot{C}_t)^T \mu + B^T \nu = -F_{q*}^T \]

\[ B \mu = -F_{\chi}^T \]

With:

\[ r_M^T \mu(t_f) = -(G_C)^T_{t_f} \]
\[ r_M^T \dot{\mu}(t_f) = (F_C + \mu^T r_C + G_K)^T_{t_f} \]
\[ \chi_C^T \rho = (\mu^T r_M)^T_{t_0} \]
\[ \zeta_K^T \pi = (F_C + \mu^T r_c - \dot{\mu}^T r_M - \rho^T \chi_K)^T_{t_0} \]

For each active criterion function, one linear DAE for \( \mu \) and \( \nu \), which can be solved backward in time.
Numerical example

$p_1 = \text{damping coefficient}$

$P_2 = \text{stiffness coefficient}$

\[
\Psi_0 = \int_{t_0}^{t_f} \dot{v}_{z,\text{chassis}}^2(t) \, dt
\]
Conclusion

The FE method in flexible multibody dynamics has a high potential in mechanical applications for:

- simulation (virtual prototyping)
- dynamic response optimization

However, gradient-based methods require

- a careful formulation of the optimization problem
- efficient transient and sensitivity analysis

Lie group methods may improve the efficiency of 3D models

- parameterization-free formulations and time integration
- simplified algorithms but similar levels of accuracy
- well-suited for sensitivity analysis
Thank you for your attention!

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