

# Abelian symmetries in Multi-Higgs-doublet models

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# Standard Model

- A gauge theory of Electroweak interactions
- Very well consistent with experiment
- There are aspects that SM doesn't explain: neutrino oscillations, dark matter, dark energy,...
- The last key ingredient of SM, the Higgs boson(s) is not discovered yet.
- Beyond Standard Model theories
- Some BSM require families of Higgs  $\Rightarrow$  Higgs doublets

- Several groups have studied 2HDM and their symmetries. Very few people have done this for 3HDM. And the task gets very complicated for NHDM.
- Here we introduce a strategy and a graphical understanding for 3HDM and 4HDM and intuitively derive results for NHDM. We focus on Abelian subgroups.
- We explore; 1) which Abelian symmetries are realizable. 2) How to write a potential with the symmetry. 3) How does the symmetry spontaneously break.

# General potential

- Higgs doublets:  $\phi_i = \begin{pmatrix} \phi_i^+ \\ \phi_i^0 \end{pmatrix}$ ,  $i = 1, \dots, N$
- The general potential has the form:

$$V = -Y_{ij}(\phi_i^\dagger \phi_j) + \frac{1}{2} Z_{ijkl}(\phi_i^\dagger \phi_j)(\phi_k^\dagger \phi_l)$$

- $N^2$  parameters in  $Y_{ij}$  and  $\frac{N^2(N^2 + 1)}{2}$  parameters in  $Z_{ijkl}$

# Field bilinears

- We introduce the field bilinears:

$$r_0 = \sqrt{\frac{N-1}{2N}} \sum_i \phi_i^\dagger \phi_i$$

$$r_a = \sum_{i,j} \phi_i^\dagger \lambda_{i,j}^a \phi_j$$

- The potential could be rewritten as:  $V = -M_\mu r^\mu + \frac{1}{2} \Lambda_{\mu\nu} r^\mu r^\nu$
- Geometrical properties make it easier to see the symmetries

# NHDM

- Symmetries in a NHDM: Subgroups of  $SU(N)$
- There is a reparametrization freedom by change of doublets
- One could also devise symmetries (subgroups of  $SU(N)$  which are realizable by the potential). The bigger the subgroup the less freedom there is.
- $SU(N)$  symmetric potential;  $V = -M_0(\Phi^\dagger\Phi) + \frac{1}{2}\Lambda(\Phi^\dagger\Phi)^2$  where  $\Phi^\dagger\Phi = \sum \phi_i^\dagger\phi_i$
- General classification of  $SU(N)$  subgroups (which are realizable) is still too difficult. We focus on the Abelian subgroups here.

# Theorems

- The largest Abelian subgroup of  $SU(3)$  is  $U(1) \times U(1)$
- All such subgroups are conjugates inside  $SU(3)$
- Any Abelian subgroup must lie inside a  $U(1) \times U(1)$  therefore we won't miss any symmetry

# $U(1) \times U(1)$

- One specific realization of  $U(1) \times U(1)$ ; Doublets transform according to  $\phi_i \rightarrow R_{ij}\phi_j$ , with

$$R = \text{diag}(e^{i\alpha}, e^{i\beta}, e^{-i\alpha-i\beta})$$

- Products transform as

$$\phi_1^\dagger \phi_2 \rightarrow e^{-i(\alpha-\beta)} \phi_1^\dagger \phi_2, \quad \phi_3^\dagger \phi_1 \rightarrow e^{-i(-2\alpha-\beta)} \phi_3^\dagger \phi_1, \quad \phi_2^\dagger \phi_3 \rightarrow e^{-i(\alpha+2\beta)} \phi_2^\dagger \phi_3$$

- Angles at which the doublets transform:

$$\psi_{12} = \alpha - \beta, \quad \psi_{31} = -2\alpha - \beta, \quad \psi_{23} = \alpha + 2\beta$$

# 3HDM

- In the case of a 3HDM the explicit expression for the field bilinears are:

$$r_3 = \frac{(\phi_1^\dagger \phi_1) - (\phi_2^\dagger \phi_2)}{2}, \quad r_8 = \frac{(\phi_1^\dagger \phi_1) + (\phi_2^\dagger \phi_2) - 2(\phi_3^\dagger \phi_3)}{2\sqrt{3}},$$

$$r_1 = \text{Re}(\phi_1^\dagger \phi_2), \quad r_2 = \text{Im}(\phi_1^\dagger \phi_2), \quad r_4 = \text{Re}(\phi_1^\dagger \phi_3),$$

$$r_5 = \text{Im}(\phi_1^\dagger \phi_3), \quad r_6 = \text{Re}(\phi_2^\dagger \phi_3), \quad r_7 = \text{Im}(\phi_2^\dagger \phi_3).$$

- The generic  $U(1) \times U(1)$  potential contains only terms

$$\text{any } r_3, r_8, \quad r_1^2 + r_2^2, \quad r_4^2 + r_5^2, \quad r_6^2 + r_7^2$$

# $U(1)$ and $U(1)'$

$$U(1) \times U(1) \rightarrow R = \text{diag}(e^{i\alpha}, e^{i\beta}, e^{-i\alpha-i\beta})$$

$$\beta = 0 \rightarrow R = \text{diag}(e^{i\alpha}, 1, e^{-i\alpha})$$

$$\alpha = 0 \rightarrow R = \text{diag}(1, e^{i\beta}, e^{-i\beta})$$

$$\alpha = \beta \rightarrow R = \text{diag}(e^{i\alpha}, e^{i\alpha}, e^{-2i\alpha})$$

- We define the  $U(1)$  and  $U(1)'$  groups by their transformation matrices:

$$U(1) : R = \text{diag}(e^{i\alpha}, e^{-i\alpha}, 1) \quad ; \quad [0, 2\pi]$$

$$U(1)' : R = \text{diag}(e^{i\alpha}, e^{i\alpha}, e^{-2i\alpha}) \quad ; \quad [0, \frac{2\pi}{3}]$$

$$\rightarrow U(1)' : R = \text{diag}(e^{\frac{i\alpha}{3}}, e^{\frac{i\alpha}{3}}, e^{-\frac{2i\alpha}{3}}) \quad ; \quad [0, 2\pi]$$

- we have developed terms that are invariant under these transformations

# Discrete subgroups

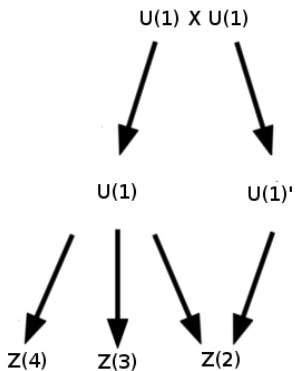
Consider first  $U(1)$  in the form  $R = (e^{i\alpha}, e^{-i\alpha}, 1)$  (i.e.  $\beta = -\alpha$ ). Then,

$$\psi_{12} = 2\alpha, \quad \psi_{31} = -\alpha, \quad \psi_{23} = -\alpha$$

Now we use discrete conditions.

- $Z_2$  **group** from  $\alpha + \alpha = 2\pi$ . Transformation is  $(-1, -1, 1)$ .
- $Z_3$  **group** from  $2\alpha + \alpha = 2\pi$ . Transformation is  $(e^{2\pi i/3}, e^{-2\pi i/3}, 1)$ .
- $Z_4$  **group** from  $2\alpha + 2\alpha = 2\pi$ . Transformation is  $(i, -i, 1)$ .

Discrete subgroups of  $U(1)'$ :  $Z_2$  **group** where the two transformations intersect



# 3HDM

- We identified two cases of  $U(1)$  subgroups:

$$U(1) : R = \text{diag}(e^{-i\alpha}, e^{i\alpha}, 1); \quad U(1)' : R = \text{diag}(e^{\frac{i\beta}{3}}, e^{\frac{i\beta}{3}}, e^{\frac{-2i\beta}{3}})$$

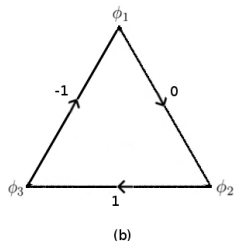
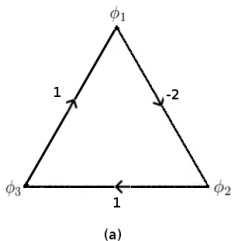
$$U(1) : R = \text{diag}(-\alpha, \alpha, 0); \quad U(1)' : R = \text{diag}\left(\frac{\beta}{3}, \frac{\beta}{3}, -\frac{2\beta}{3}\right)$$

$$U(1) : (-1, 1, 0); \quad U(1)' : \left(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}\right),$$

- 3D vectors lie in a plane orthogonal to  $(1, 1, 1)$ . Also orthogonal to each other. Therefore form a basis to represent any  $U(1) \times U(1)$  transformation in 3HDM.

## 3HDM

$$U(1) : (-1, 1, 0); \quad U(1)' : \left( \frac{1}{3}, \frac{1}{3}, -\frac{2}{3} \right),$$



- Space of graphs
- Scalar product  $\rightarrow$  Orthogonality:  $1 \cdot (-1) + (-2) \cdot 0 + 1 \cdot 1 = 0$

# 4HDM

- Maximal abelian subgroup of  $SU(4)$ :  $U(1) \times U(1) \times U(1)$  with  $R = \text{diag}(e^{i\alpha}, e^{i\beta}, e^{i\gamma}, e^{-i\alpha-i\beta-i\gamma})$
- The same type of graphs could be used for 4HDM. The basis transformations in this case are:

$$U(1) : \quad R = \text{diag}(e^{-i\alpha}, e^{i\alpha}, 1, 1),$$

$$U(1)' : \quad R = \text{diag}(e^{i\beta}, e^{i\beta}, e^{-2i\beta}, 1),$$

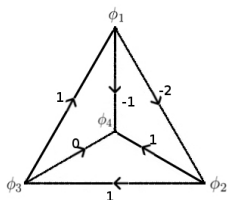
$$U(1)'' : \quad R = \text{diag}(e^{\frac{i\gamma}{4}}, e^{\frac{i\gamma}{4}}, e^{\frac{i\gamma}{4}}, e^{-\frac{3i\gamma}{4}})$$

- All angles change from 0 to  $2\pi$

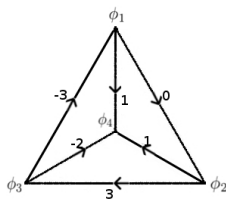
# 4HDM

- Orthogonal to each other and to  $(1, 1, 1, 1)$

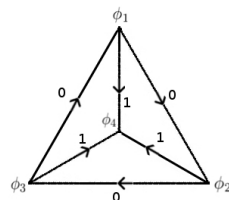
$$U(1) : (-1, 1, 0, 0), \quad U(1)' : (1, 1, -2, 0), \quad U(1)'' : \left( \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, -\frac{3}{4} \right)$$



(a)



(b)



(c)

- If sum/difference of two edges is zero in some graph, the term is symmetric under the full corresponding  $U(1)$ .
- If sum/difference of two edges is  $k$  in some graph, the term is symmetric under the  $Z_k$  subgroup of the corresponding  $U(1)$ .
- The discrete subgroups in this case are;

$$U(1) : Z_2, Z_3, Z_4$$

$$U(1)' : Z_2, Z_3, Z_4, Z_5, Z_6$$

$$U(1)'' : Z_2$$

## 4HDM

- Maximal discrete group  $Z_6 \Rightarrow Z_6 \times U(1)''$  which is continuous
- Next candidate  $Z_5$  from  $U(1)'$  :  $(\phi_1^\dagger \phi_3)(\phi_4^\dagger \phi_3) + (\phi_2^\dagger \phi_3)(\phi_4^\dagger \phi_3)$

$$U(1)' : R = \text{diag}(e^{i\beta}, e^{i\beta}, e^{-2i\beta}, 1) \quad \text{with} \quad \beta = \frac{2\pi}{5}$$

These terms are additionally invariant under  $Z_3$  coming from a permuted version of  $U(1)$ :

$$(1, 1, e^{-i\alpha}, e^{i\alpha}) \quad \text{with} \quad \alpha = 2\pi/3.$$

Also symmetric under  $Z_2$  subgroup coming from a permuted version of  $U(1)''$ :

$$(e^{\frac{i\gamma}{4}}, e^{\frac{i\gamma}{4}}, e^{-\frac{3i\gamma}{4}}, e^{\frac{i\gamma}{4}}) \quad \text{with} \quad \gamma = \pi,$$

Therefore  $(\phi_1^\dagger \phi_3)(\phi_4^\dagger \phi_3) + (\phi_2^\dagger \phi_3)(\phi_4^\dagger \phi_3)$  is symmetric under the discrete group

$$Z_5 \times Z_3 \times Z_2 = Z_{30}.$$

A generator of this group can be constructed as the product of the above transformations and is characterized by phases:

$$\left( \frac{39}{60}\pi, \frac{39}{60}\pi, -\frac{13}{60}\pi, \frac{55}{60}\pi \right).$$

This is an example of a surprisingly large discrete subgroup in a 4HDM

# NHDM

The basis  $U(1)$  transformations in NHDM are:

$$U(1) : (-1, 1, \dots, 0, 0),$$

$$U(1)' : (1, 1, -2, \dots, 0, 0),$$

$$U(1)'' : (1, 1, 1, -3, \dots, 0, 0),$$

...

$$U(1)^{(N-2)} : (1, 1, \dots, 1, -(N-2), 0),$$

$$U(1)^{(N-1)} : \left( \frac{1}{N}, \frac{1}{N}, \dots, \frac{1}{N}, -\frac{(N-1)}{N} \right).$$

- Discrete subgroups:  $Z_2, Z_{N-2}, Z_{N-1}, Z_N, Z_{2N-4}, Z_{2N-3}, Z_{2N-2}$ .
- Largest candidate for a cyclic group:  $Z_{2N-2} \Rightarrow Z_{2N-2} \times U(1)$
- Next candidate  $Z_{2N-3} : \sum_{i=1}^{N-2} (\phi_{N-1}^\dagger \phi_N)(\phi_{N-1}^\dagger \phi_i)$

# Conclusions

- We have developed a strategy to find Abelian subgroups in multi-Higgs-doublet models; where one can find discrete, polycyclic and continuous subgroups.
- This is a clear method to find invariant terms under certain symmetries and construct a potential accordingly
- What remains to study is how these symmetries spontaneously break