Abelian symmetries in Multi-doublet models

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Introduction

• BSM $\rightarrow$ Multi-doublet models

• 2HDMs have been studied extensively, but very little work has been done for NHDM with $N \geq 2$.

• The huge number of free parameters makes it impossible to fully analyze NHDM potentials.

• The symmetries that can be imposed on the potential in NHDM are of interest.

• Here we introduce a strategy to find all realizable Abelian symmetry groups in the scalar sector NHDM.

• We explore; 1) which Abelian symmetry groups are realizable. 2) How to write a potential with this symmetry.

Multi-Scalar-Doublet Models

We introduce N complex scalar doublets:

$$\phi_a = \begin{pmatrix} \phi_a^+ \\ \phi_a^0 \end{pmatrix}, \quad a = 1, \ldots, N$$

The generic scalar potential can be written in a tensorial form:

$$V = Y_{\bar{a}b}(\phi_a^\dagger \phi_b) + Z_{\bar{a}b\bar{c}d}(\phi_a^\dagger \phi_b)(\phi_c^\dagger \phi_d)$$

This potential has $\frac{N^2(N^2+3)}{2}$ free parameters.

Unitary transformations

Reparametrization transformations form the group $U(N)$.

The overall phase factor multiplication is taken into account by the $U(1)_Y$.

This leaves us with $SU(N)$, which has a non-trivial center $Z(SU(N)) = Z_N$ generated by the diagonal matrix $\exp(2\pi i/N) \cdot 1_N$.

Therefore, the group of physically distinct unitary reparametrization transformations is

$$PSU(N) \simeq SU(N)/Z_N$$

Any symmetry group must be a subgroup of $PSU(N)$.

Strategy

• We find maximal Abelian subgroups of $PSU(N)$.

• Search for all the subgroups of each maximal Abelian subgroup whose potential is not invariant under a larger group. We call such subgroups realizable symmetry groups.

It can be proved that there are two types of maximal Abelian subgroups inside $PSU(N)$:

• The maximal tori, which will be constructed here.

• Certain finite Abelian groups, which are not subgroups of maximal tori and must be treated separately

Constructing maximal torus in $PSU(N)$

Starting from $SU(N)$; all maximal Abelian subgroups are maximal tori:

$$T_0 = [U(1)]^{N-1} = U(1) \times U(1) \times \cdots \times U(1)$$

and all such maximal tori are conjugate inside $SU(N)$.
Therefore without loss of generality one could pick up a specific maximal torus, for example, the one that is represented by phase rotations of individual doublets

$$\mathrm{diag}[e^{i\alpha_1}, e^{i\alpha_2}, \ldots, e^{i\alpha_{N-1}}, e^{-i\sum \alpha_i}]$$

and study its subgroups.
The diagonal transformation matrix could conveniently be written as a vector of phases:

$$\left(\alpha_1, \alpha_2, \ldots, \alpha_{N-1}, -\sum \alpha_i\right)$$

Maximal torus in $PSU(N)$

• T the maximal torus in $PSU(N)$, is the image of T_0 upon $SU(N) \to PSU(N)$.
• Now we check which subgroups of maximal torus T are realizable in the scalar sector of NHDM.
The most general T-symmetric potential:

$$V(T) = -\sum_a m_a^2(\phi_a^\dagger\phi_a) + \sum_{a,b}\lambda_{ab}(\phi_a^\dagger\phi_a)(\phi_b^\dagger\phi_b) + \sum_{a\neq b}\lambda'_{ab}(\phi_a^\dagger\phi_b)(\phi_b^\dagger\phi_a)$$

We add more terms to $V(T)$ and find under which realizable subgroups of T, the potential is invariant.

Identifying the symmetry groups

Each monomial gets a phase change under T:

$$\phi_a^\dagger \phi_b \to \exp[i(p_{ab}\alpha_1 + q_{ab}\alpha_2 + \cdots + t_{ab}\alpha_{N-1})] \cdot \phi_a^\dagger \phi_b$$

We write a potential with k terms with integer coefficients $p_i, q_i, \ldots t_i$, and construct the $X(V)$ matrix:

$$X(V) = \begin{pmatrix} p_1 & q_1 & \cdots & t_1 \\ p_2 & q_2 & \cdots & t_2 \\ \vdots & \vdots & & \vdots \\ p_k & q_k & \cdots & t_k \end{pmatrix}$$

The symmetry group of the potential is constructed from the set of solutions for α_i of the following equations:

$$X(V) \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_{N-1} \end{pmatrix} = \begin{pmatrix} 2\pi n_1 \\ \vdots \\ 2\pi n_{N-1} \end{pmatrix}$$

Identifying the symmetry groups

Diagonalizing the $X(V)$ matrix results in:

$$X(V) = \begin{pmatrix} d_1 & 0 & \cdots & 0 \\ 0 & d_2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & d_{N-1} \end{pmatrix}$$

Then we will have the symmetry group:

$$Z_{d_1} \times Z_{d_2} \times \cdots \times Z_{d_{N-1}}$$

Where $Z_1 \equiv$ no symmetry, and $Z_0 \equiv U(1)$ symmetry.

The 3HDM and 4HDM examples

Using this strategy, we have found the full list of realizable unitary Abelian symmetry groups in 3HDM and 4HDM.

• The finite groups in 3HDM:

$$Z_2, \quad Z_3, \quad Z_4, \quad Z_2 \times Z_2,$$

• The finite groups in 4HDM:

$$Z_k \text{ with } k = 2, \ldots, 8; \qquad Z_2 \times Z_k \text{ with } k = 2, 3, 4; \qquad Z_2 \times Z_2 \times Z_2$$

which interestingly all have order $\leq 2^{N-1}$.

• Conjecture: All Abelian groups with order $\leq 2^{N-1}$ are realizable in NHDM.

General NHDM

We have proved several statements for NHDM:

• Upper bound: For any given N the order of the realizable finite Abelian groups $\leq 2^{N-1}$.

• Cyclic groups: The cyclic group Z_p is realizable for any positive integer $p \leq 2^{N-1}$.

• Polycyclic groups: Let $(N-1) = \sum_{i=1}^{k} n_i$ be a partitioning of $(N-1)$ into a sum of non-negative integers n_i. Then, the finite group

$$G = Z_{p_1} \times Z_{p_2} \times \cdots \times Z_{p_k}$$

is realizable for any $0 < p_i \leq 2^{n_i}$.

We have almost proved our conjecture.

Conclusion

• For experimentalist:

We did some mathematical work which helps other people in their model building, hopefully of phenomenological interest.

• For theoreticians:

We characterized all Abelian symmetries realizable for any NHDM.