Abelian symmetries in Multi-Higgs-doublet models

Venus Ebrahimi-Keus

IFPA, University of Liège
In collaboration with I. P. Ivanov and E. Vdovin

Scalars 2011, Warsaw, 27/08/11

Multi-Higgs-Doublet Models

• We introduce N complex Higgs doublets with electroweak isospin $Y = 1/2$:

$$\phi_a = \begin{pmatrix} \phi_a^+ \\ \phi_a^0 \end{pmatrix}, \quad a = 1, \dots, N$$

• The generic Higgs potential can be written in a tensorial form:

$$V = Y_{\bar{a}b}(\phi_a^\dagger \phi_b) + Z_{\bar{a}b\bar{c}d}(\phi_a^\dagger \phi_b)(\phi_c^\dagger \phi_d)$$

where all indices run from 1 to N.

• There are N^2 independent components in Y and $N^2(N^2+1)/2$ independent components in Z.

• The explicit analysis of the most general case is impossible.

Symmetries

Several questions concerning symmetry properties of the scalar sector of NHDM arise;

• What groups G are realizable as symmetry groups of some potential V?

• How to write examples of the Higgs potential whose symmetry group is equal to a given realizable group G?

G is a realizable symmetry group if there exists a G-symmetric potential and there's no larger group which includes G and keeps this potential invariant.

For $N = 2$ the model has been studied extensively, but for $N > 2$, these questions have not been answered yet.

Here we introduce a strategy to find all Abelian subgroups in NHDM.

Reparametrization trasformations

• Reparametrization transformations: non-degenerate linear transformations which mix different doublets ϕ_a without changing the intradoublet structure and which conserve the norm $\phi_a^\dagger \phi_a$.

• All such transformations must be unitary or antiunitary:

$$
\begin{aligned}
U: &\quad \phi_a \to U_{ab}\phi_b \\
U_{CP} = U \cdot CP: &\quad \phi_a \to U_{ab}\phi_b^\dagger
\end{aligned}
$$

with unitary matrix U_{ab}.

In this talk I focus on the unitary transformations.

Unitary transformations

• Such transformations form the group $U(N)$. The overall phase factor multiplication is taken into account by the $U(1)_Y$.

• This leaves us with $SU(N)$, which has a non-trivial center $Z(SU(N)) = Z_N$ generated by the diagonal matrix $\exp(2\pi i/N) \cdot 1_N$.

• Therefore, the group of physically distinct reparametrization transformations is

$$PSU(N) \simeq SU(N)/Z_N$$

Strategy

- At first we write maximal Abelian subgroups of $PSU(N)$.
- Then we find all the subgroups of each maximal Abelian subgroup.
- At the end we check the potential is not symmetric under a larger group.

It can be proved that there are two sorts of maximal Abelian subgroups inside $PSU(N)$:

- The maximal tori, which will be constructed here.

- The image of the extraspecial N-groups, which is at most one additional group for each N.

Constructing maximal torus in $PSU(N)$

• Starting from $SU(N)$; all maximal Abelian subgroups are maximal tori:

$$[U(1)]^{N-1} = U(1) \times U(1) \times \cdots \times U(1)$$

and all such maximal tori are conjugate inside $SU(N)$.

• Therefore without loss of generality one could pick up a specific maximal torus, for example, the one that is represented by phase rotations of individual doublets

$$\mathrm{diag}[e^{i\alpha_1},\ e^{i\alpha_2},\ \ldots,\ e^{i\alpha_{N-1}},\ e^{-i\sum \alpha_i}]$$

and study its subgroups.

• We construct the representative maximal torus in $PSU(N)$ analogously, with the center $Z(SU(N))$ localized in only one of the $U(1)$'s.

• A diagonal transformation matrix which performs phase rotations of doublets will be written as a vector of phases:

$$\left(\alpha_1,\, \alpha_2,\, \ldots,\, \alpha_{N-1}, -\sum \alpha_i\right)$$

• Then we construct a maximal torus in $PSU(N)$ which has this form

$$T = U(1)_1 \times U(1)_2 \times \cdots \times \tilde{U}(1)_{N-1}$$

where

$$\begin{array}{rcl} U(1)_1 & = & \alpha_1(-1,\, 1,\, 0,\, 0,\, \ldots,\, 0)\,, \\ U(1)_2 & = & \alpha_2(-2,\, 1,\, 1,\, 0,\, \ldots,\, 0)\,, \\ U(1)_3 & = & \alpha_3(-3,\, 1,\, 1,\, 1,\, \ldots,\, 0)\,, \\ \vdots & & \vdots \\ \tilde{U}(1)_{N-1} & = & \alpha_{N-1}\left(-\dfrac{N-1}{N},\, \dfrac{1}{N},\, \ldots,\, \dfrac{1}{N}\right) \end{array}$$

with all $\alpha_i \in [0, 2\pi)$.

Identifying the symmetry groups

• Now, using the strategy, we check which subgroups of maximal torus are realizable:

• The Higgs potential is a sum of monomial terms of the form: $\phi_a^\dagger \phi_b$ or $(\phi_a^\dagger \phi_b)(\phi_c^\dagger \phi_d)$.

• Each monomial gets a phase factor under T:

$$\exp[i(p\alpha_1 + q\alpha_2 + \cdots + t\alpha_{N-1})]$$

• The coefficients $p, q, \ldots, t$ of such terms can be easily calculated for every monomial.

Identifying the symmetry groups

• Consider a Higgs potential V which is a sum of k terms, with coefficients $p_1, q_1, \ldots t_1$ to $p_k, q_k, \ldots t_k$. This potential defines the following $(N-1) \times k$ matrix of coefficients:

$$X(V) = \begin{pmatrix} p_1 & q_1 & \cdots & t_1 \\ p_2 & q_2 & \cdots & t_2 \\ \vdots & \vdots & & \vdots \\ p_k & q_k & \cdots & t_k \end{pmatrix}$$

• The symmetry group of this potential can be constructed from the set of solutions for α_i of the following equations:

$$X(V) \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_{N-1} \end{pmatrix} = \begin{pmatrix} 2\pi n_1 \\ \vdots \\ 2\pi n_{N-1} \end{pmatrix}$$

- There are two major possibilities depending on the rank of matrix X:

- If rank of this matrix is less than $N-1$, there exists a hyperplane in the space of angles α_i, which solves this equation for $n_i = 0$. The potential is symmetric under $[U(1)]^D$, where $D = N - 1 - rank(X)$.

- If rank$X(V) = N - 1$, there is no continuous symmetry. Instead, there exists a unique solution for any n_i.
All such solutions form the finite group of phase rotations of the given potential.

finite groups

- One could easily diagonalize matrix $X(V)$ with integer entries.
- Diagonalizing the $X(V)$ matrix results in:

$$X(V) = \begin{pmatrix} d_1 & 0 & \cdots & 0 \\ 0 & d_2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & d_{N-1} \end{pmatrix}$$

Then we will have the finite symmetry group:

$$Z_{d_1} \times Z_{d_2} \times \cdots \times Z_{d_{N-1}}$$

Now we're done with the strategy of the work.

It's time for some examples in 3HDM and 4HDM.

The 3HDM example

• In the 3HDM the representative maximal torus $T \subset PSU(3)$ is parametrized as

$$T = U(1)_1 \times U(1)_2\,, \quad U(1)_1 = \alpha(-1,\,1,\,0)\,, \quad U(1)_2 = \beta\left(-\frac{2}{3},\,\frac{1}{3},\,\frac{1}{3}\right)$$

• There are six bilinear combinations of doublets transforming non-trivialy under T

$$(\phi_a^\dagger\phi_b) \to \exp[i(p\alpha + q\beta)](\phi_a^\dagger\phi_b)$$

	p	q
$(\phi_2^\dagger\phi_1)$	-2	-1
$(\phi_3^\dagger\phi_2)$	1	0
$(\phi_1^\dagger\phi_3)$	1	1

and their conjugates with opposite coefficients p and q.

• Any monomial term is symmetric under a $U(1)$, because $rank X(V) = 1$. In order to have a finite group we need at least 2 terms.

• To find all realizable groups, one has to write the full list of possible terms and then calculate the symmetry group of all distinct pairs of terms. For example, if the two monomials are $v_1 = (\phi_1^\dagger \phi_2)(\phi_1^\dagger \phi_3)$ and $v_2 = (\phi_2^\dagger \phi_1)(\phi_2^\dagger \phi_3)$, then the matrix $X(v_1 + v_2)$ has form

$$X(v_1 + v_2) = \begin{pmatrix} 3 & 2 \\ -3 & -1 \end{pmatrix} \to \begin{pmatrix} 3 & 2 \\ 0 & 1 \end{pmatrix} \to \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$$

and it produces the symmetry group Z_3. The solution of the equation

$$X(v_1 + v_2) \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 2\pi n_1 \\ 2\pi n_2 \end{pmatrix}$$

yields $\alpha = 2\pi/3 \cdot k$, $\beta = 0$.

Full list of subgroups of 3HDM

• Checking all possible combination of monomials, we arrive at the full list of unitary Abelian subgroups of the maximal torus:

$$Z_2, \quad Z_3, \quad Z_4, \quad Z_2 \times Z_2,$$
$$U(1), \quad U(1) \times Z_2, \quad U(1) \times U(1)$$

The 4HDM example

• In the case of 4 Higgs doublets, the representative maximal torus in $PSU(4)$ is $T = U(1)_1 \times U(1)_2 \times U(1)_3$, where

$$U(1)_1 = \alpha(-1,\, 1,\, 0,\, 0)\,, \quad U(1)_2 = \beta(-2,\, 1,\, 1,\, 0)\,, \quad U(1)_3 = \gamma\left(-\frac{3}{4},\, \frac{1}{4},\, \frac{1}{4},\, \frac{1}{4}\right)$$

• The phase rotations of a generic bilinear of doublets under T is characterized by three integers p, q, r,

$$(\phi_a^\dagger \phi_b) \to \exp[i(p\alpha + q\beta + r\gamma)](\phi_a^\dagger \phi_b)$$

• Using the strategy we have found all finite unitary Abelian groups with order ≤ 8

$$Z_k \text{ with } k = 2, \ldots, 8; \qquad Z_2 \times Z_k \text{ with } k = 2,\ 3,\ 4; \qquad Z_2 \times Z_2 \times Z_2$$

• And all realizable continuous groups:

$$U(1) \times U(1) \times U(1)\,, \quad U(1) \times U(1) \times Z_2\,, \quad U(1) \times Z_k\,, \quad k = 2, 3, 4, 5, 6$$

General NHDM

• The algorithm described above can be used to find all Abelian groups realizable as the symmetry groups of the Higgs potential for any N.

Several statements:

• Upper bound on the order of finite Abelian groups:
For any given N there exists an upper bound on the order of finite Abelian groups realizable as symmetry groups of the NHDM potentials: The order of any such group must be $\leq 2^{\frac{3}{2}(N-1)}$. We suspect that this bound could be improved to 2^{N-1}.

• Cyclic groups:
The cyclic group Z_p is realizable for any positive integer $p \leq 2^{N-1}$.

• Polycyclic groups:
Let $(N-1) = \sum_{i=1}^{k} n_i$ be a partitioning of $(N-1)$ into a sum of non-negative integers n_i. Then, the finite group

$$G = Z_{p_1} \times Z_{p_2} \times \cdots \times Z_{p_k}$$

is realizable for any $0 < p_i \leq 2^{n_i}$.

Conclusion

To summarize:

• NHDM are interesting because one can introduce many non-trivial symmetries. Finding such symmetries is one the hot topics.

• In this work we have focused on Abelian symmetries and developed a strategy to find all Abelian groups realizable for any NHDM.
Specific examples of 3HDM and 4HDM have been shown.

• We have derived some general conclusion for NHDM.