Abelian symmetries in Multi-Higgs-doublet models

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Standard Model

- A gauge theory of Electroweak interactions
- Very well consistent with experiment
- There are aspects that SM doesn't explain: neutrino oscillations, dark matter, dark energy,...
- The last key ingredient of SM, the Higgs boson(s) is not discovered yet.
- Beyond Standard Model theories
- Some BSM require families of Higgs $\Rightarrow$ Higgs doublets

• Several groups have studied 2HDM and their symmetries. Very few people have done this for 3HDM. And the task gets very complicated for NHDM.

• Here we introduce a strategy and a graphical understanding for 3HDM and 4HDM and intuitively derive results for NHDM. We focus on Abelian subgroups.

• We explore; 1) which Abelian symmetries are realizable. 2) How to write a potential with the symmetry. 3) How does the symmetry spontaneously breaks.

General potential

- Higgs doublets: $\phi_i = \begin{pmatrix} \phi_i^+ \\ \phi_i^0 \end{pmatrix}$, $i = 1, ..., N$
- The general potential has the form:

$$V = -Y_{ij}(\phi_i^\dagger \phi_j) + \frac{1}{2} Z_{ijkl}(\phi_i^\dagger \phi_j)(\phi_k^\dagger \phi_l)$$

- N^2 parameters in Y_{ij} and $\dfrac{N^2(N^2+1)}{2}$ parameters in Z_{ijkl}

Field bilinears

- We introduce the field bilinears:

$$r_0 = \sqrt{\frac{N-1}{2N}} \sum_a \phi_i^\dagger \phi_i$$

$$r_a = \sum_{i,j} \phi_i^\dagger \lambda_{i,j}^a \phi_j$$

- The potential could be rewritten as: $V = -M_\mu r^\mu + \frac{1}{2}\Lambda_{\mu\nu} r^\mu r^\nu$

- Geometrical properties make it easier to see the symmetries

NHDM

- Symmetries in a NHDM: Subgroups of SU(N)

- There is a reparametrization freedom by change of doublets

- One could also devise symmetries (subgroups of SU(N) which are realizable by the potential). The bigger the subgroup the less freedom there is.

- Largest group; $V = -M_0(\Phi^\dagger\Phi) + \frac{1}{2}\Lambda(\Phi^\dagger\Phi)^2$ where $\Phi^\dagger\Phi = \sum \phi_i^\dagger\phi_i$

- General classification of SU(N) subgroups (which are realizable) is still too difficult. We focus on the Abelian subgroups here.

Abelian subgroups of SU(3)

• The largest Abelian subgroup of SU(3) is $U(1) \times U(1)$

• All such subgroups are conjugates inside SU(3)

• Any Abelian subgroup must lie inside a $U(1) \times U(1)$ therefore we won't miss any symmetry

• One specific realization; Doublets transform according to $\phi_i \to R_{ij}\phi_j$, with

$$R = \mathrm{diag}(e^{i\alpha},\, e^{i\beta},\, e^{-i\alpha-i\beta})$$

- A group has studied this:"Discrete and continuous symmetries in multi-Higgs-doublet models" by: P. M. Ferreira, Joao P. Silva Phys.Rev.D78:116007,2008:

$$S = \begin{pmatrix} 1 & & \\ & e^{i\alpha} & \\ & & e^{i\beta} \end{pmatrix} \qquad \begin{bmatrix} 0 & -\alpha & -\beta \\ \alpha & 0 & \alpha-\beta \\ \beta & \beta-\alpha & 0 \end{bmatrix}$$

$$\begin{bmatrix} \begin{bmatrix} 0 & -\alpha & -\beta \\ \alpha & 0 & \alpha-\beta \\ \beta & \beta-\alpha & 0 \end{bmatrix} & \begin{bmatrix} -\alpha & -2\alpha & -\alpha-\beta \\ 0 & -\alpha & -\beta \\ \beta-\alpha & \beta-2\alpha & -\alpha \end{bmatrix} & \begin{bmatrix} -\beta & -\alpha-\beta & -2\beta \\ \alpha-\beta & -\beta & \alpha-2\beta \\ 0 & -\alpha & -\beta \end{bmatrix} \\ \begin{bmatrix} \alpha & 0 & \alpha-\beta \\ 2\alpha & \alpha & 2\alpha-\beta \\ \alpha+\beta & \beta & \alpha \end{bmatrix} & \begin{bmatrix} 0 & -\alpha & -\beta \\ \alpha & 0 & \alpha-\beta \\ \beta & \beta-\alpha & 0 \end{bmatrix} & \begin{bmatrix} \alpha-\beta & -\beta & \alpha-2\beta \\ 2\alpha-\beta & \alpha-\beta & 2\alpha-2\beta \\ \alpha & 0 & \alpha-\beta \end{bmatrix} \\ \begin{bmatrix} \beta & \beta-\alpha & 0 \\ \alpha+\beta & \beta & \alpha \\ 2\beta & 2\beta-\alpha & \beta \end{bmatrix} & \begin{bmatrix} \beta-\alpha & \beta-2\alpha & -\alpha \\ \beta & \beta-\alpha & 0 \\ 2\beta-\alpha & 2\beta-2\alpha & \beta-\alpha \end{bmatrix} & \begin{bmatrix} 0 & -\alpha & -\beta \\ \alpha & 0 & \alpha-\beta \\ \beta & \beta-\alpha & 0 \end{bmatrix} \end{bmatrix}$$

$U(1) \times U(1)$

- Transformation matrix

$$R = \mathrm{diag}(e^{i\alpha},\, e^{i\beta},\, e^{-i\alpha-i\beta})$$

- Products transform as

$$\phi_1^\dagger\phi_2 \to e^{-i(\alpha-\beta)}\phi_1^\dagger\phi_2\,, \quad \phi_3^\dagger\phi_1 \to e^{-i(-2\alpha-\beta)}\phi_3^\dagger\phi_1\,, \quad \phi_2^\dagger\phi_3 \to e^{-i(\alpha+2\beta)}\phi_2^\dagger\phi_3$$

- Angles at which the doublets transform:

$$\psi_{12} = \alpha - \beta\,, \quad \psi_{31} = -2\alpha - \beta\,, \quad \psi_{23} = \alpha + 2\beta$$

3HDM

- In the case of a 3HDM the explicit expression for the field bilinears are:

$$r_3 = \frac{(\phi_1^\dagger \phi_1) - (\phi_2^\dagger \phi_2)}{2}\,, \quad r_8 = \frac{(\phi_1^\dagger \phi_1) + (\phi_2^\dagger \phi_2) - 2(\phi_3^\dagger \phi_3)}{2\sqrt{3}}\,,$$

$$r_1 = \mathrm{Re}(\phi_1^\dagger \phi_2)\,,\ r_2 = \mathrm{Im}(\phi_1^\dagger \phi_2)\,,\ r_4 = \mathrm{Re}(\phi_1^\dagger \phi_3)\,,$$

$$r_5 = \mathrm{Im}(\phi_1^\dagger \phi_3)\,,\ r_6 = \mathrm{Re}(\phi_2^\dagger \phi_3)\,,\ r_7 = \mathrm{Im}(\phi_2^\dagger \phi_3)\,.$$

- The generic $U(1) \times U(1)$ potential contains only terms

$$\text{any } r_3,\ r_8, \quad r_1^2 + r_2^2\,, \quad r_4^2 + r_5^2\,, \quad r_6^2 + r_7^2$$

$$r_1^2 + r_2^2 \rightarrow \mathrm{Re}(e^{-i\psi_{12}} \phi_1^\dagger \phi_2)^2 + \mathrm{Im}(e^{-i\psi_{12}} \phi_1^\dagger \phi_2)^2 = r_1^2 + r_2^2$$

$U(1)$ and $U(1)'$

$U(1) \times U(1) \to$ R= diag($e^{i\alpha}$, $e^{i\beta}$, $e^{-i\alpha-i\beta}$)

$$\beta = 0 \to \quad R = \mathrm{diag}(e^{i\alpha},\, 1,\, e^{-i\alpha})$$
$$\alpha = 0 \to \quad R = \mathrm{diag}(1,\, e^{i\beta},\, e^{-i\beta})$$
$$\alpha = \beta \to \quad R = \mathrm{diag}(e^{i\alpha},\, e^{i\alpha},\, e^{-2i\alpha})$$

- We define the $U(1)$ and $U(1)'$ groups by their transformation matrices:

$$U(1): \quad R = \mathrm{diag}(e^{i\alpha},\, e^{-i\alpha},\, 1) \quad ; \quad [0, 2\pi]$$
$$U(1)': \quad R = \mathrm{diag}(e^{i\alpha},\, e^{i\alpha},\, e^{-2i\alpha}) \quad ; \quad [0, \frac{2\pi}{3}]$$
$$\to U(1)': \quad R = \mathrm{diag}(e^{\frac{i\alpha}{3}},\, e^{\frac{i\alpha}{3}},\, e^{\frac{-2i\alpha}{3}}) \quad ; \quad [0, 2\pi]$$

$U(1)$ symmetric potential

$U(1)$s come from such conditions:

- $\psi_{31} - \psi_{23} = (-2\alpha - \beta) - (\alpha + 2\beta) = -3(\alpha + \beta) = 2\pi n$;

$$r_4 r_6 - r_5 r_7\,, \quad r_4 r_7 + r_5 r_6\,.$$

$$\begin{aligned}
& r_4 r_6 - r_5 r_7 \\
& = \mathrm{Re}(\phi_1^\dagger \phi_3)\mathrm{Re}(\phi_2^\dagger \phi_3) - \mathrm{Im}(\phi_1^\dagger \phi_3)\mathrm{Im}(\phi_2^\dagger \phi_3) \\
& \to \mathrm{Re}(e^{i\psi_{31}}\phi_1^\dagger \phi_3)\mathrm{Re}(e^{-i\psi_{23}}\phi_2^\dagger \phi_3) - \mathrm{Im}(e^{i\psi_{31}}\phi_1^\dagger \phi_3)\mathrm{Im}(e^{-i\psi_{23}}\phi_2^\dagger \phi_3) \\
& = \cos(\psi_{31} - \psi_{23})[r_4 r_6 - r_5 r_7] + \sin(\psi_{31} - \psi_{23})[r_4 r_7 + r_5 r_6] \\
& = r_4 r_6 - r_5 r_7
\end{aligned}$$

$U(1)'$ symmetric potential

$U(1)'$s come from the such conditions:

• $\psi_{31} + \psi_{23} = (-2\alpha - \beta) + (\alpha + 2\beta) = -\alpha + \beta = 2\pi n$;

$$
\begin{array}{l}
r_4 r_6 + r_5 r_7 \,, \quad r_4 r_7 - r_5 r_6 \\
r_4^2 - r_5^2 \,, \quad 2 r_4 r_5 \\
r_6^2 - r_7^2 \,, \quad 2 r_6 r_7
\end{array}
$$

and any dependence on r_1, r_2

• U(1)' appears in a wider class of potentials than U(1), because it's much less restrictive.

Discrete subgroups

Consider first $U(1)$ in the form $R = (e^{i\alpha}, e^{-i\alpha}, 1)$ (i.e. $\beta = -\alpha$). Then,

$$\psi_{12} = 2\alpha\,, \quad \psi_{31} = -\alpha\,, \quad \psi_{23} = -\alpha$$

$$\psi^*_{12} = -2\alpha\,, \quad \psi^*_{31} = \alpha\,, \quad \psi^*_{23} = \alpha$$

Now we use discrete conditions.

- Z_2 **group** from $\alpha + \alpha = 2\pi$. Transformation is $(-1, -1, 1)$. Extra terms are $r_4 r_6 + r_5 r_7$ and $r_4 r_7 - r_5 r_6$, and $r_4^2 - r_5^2$, $r_4 r_5$, $r_6^2 - r_7^2$, $r_6 r_7$
- Z_3 **group** from $2\alpha + \alpha = 2\pi$. Transformation is $(e^{2\pi i/3}, e^{-2\pi i/3}, 1)$. Extra terms:

$$r_1 r_4 + r_2 r_5\,, \quad r_1 r_5 - r_2 r_4\,, \quad r_1 r_6 - r_2 r_7\,, \quad r_1 r_7 + r_2 r_6\,.$$

- Z_4 **group** from $2\alpha + 2\alpha = 2\pi$. Transformation is $(i, -i, 1)$. Extra terms: $r_1^2 - r_2^2$, $r_1 r_2$, which implies any bilinear product of r_1, r_2.

Discrete subgroups

Now consider $U(1)'$, for example, in the realization $\alpha = \beta$, $R = (e^{\frac{i\alpha}{3}}, e^{\frac{i\alpha}{3}}, e^{\frac{-2i\alpha}{3}})$. Transformation angles for $\vec{r}$'s are

$$\psi_{12} = 0\,, \quad \psi_{31} = -\alpha\,, \quad \psi_{23} = \alpha$$

- $\psi_{23} - \psi_{31} = 2\pi \Rightarrow 2\alpha = 2\pi n$. But we can get from here only $\alpha = \pi$, which is again a Z_2 group as before. (This is where U(1) and U(1)' intersect)

- So, we do not generate any new discrete group from here.

3HDM

- We identified two cases of $U(1)$ subgroups:

$$U(1): \quad R = \mathrm{diag}(e^{-i\alpha},\, e^{i\alpha},\, 1)\,; \qquad U(1)': \quad R = \mathrm{diag}(e^{\frac{i\beta}{3}}, e^{\frac{i\beta}{3}}, e^{\frac{-2i\beta}{3}})$$

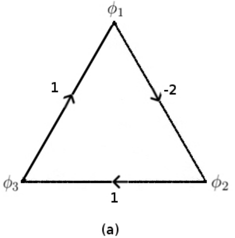
$$U(1): \quad R = \mathrm{diag}(-\alpha,\, \alpha,\, 0)\,; \qquad U(1)': \quad R = \mathrm{diag}\left(\frac{\beta}{3}, \frac{\beta}{3}, -\frac{2\beta}{3}\right)$$

$$U(1): \quad (-1,\, 1,\, 0\,); \qquad U(1)': \quad \left(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}\right),$$

- 3D vectors lie in a plane ortogonal to $(1,1,1)$. Also orthogonal to each other. Therefore form a basis to represent any $U(1) \times U(1)$ transformation in 3HDM.

3HDM

$$U(1): \quad (-1,\, 1,\, 0\,); \qquad U(1)': \quad \left(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}\right),$$



- Space of graphs
- Scalar product → Orthogonality: $1 \cdot (-1) + (-2) \cdot 0 + 1 \cdot 1 = 0$

3HDM

- Invariant terms $(\phi_3^\dagger\phi_2)(\phi_3^\dagger\phi_1)$ under U(1), and $(\phi_3^\dagger\phi_2)(\phi_1^\dagger\phi_3)$ under U(1)'
- Discrete symetries: Z_2, Z_3, Z_4 for U(1) and Z_2 for U(1)'
- $(\phi_1^\dagger\phi_2)^2 \Rightarrow Z_4 \times U(1)'$
- Polycyclic groups $Z_2 \times Z_2$: $(\phi_1^\dagger\phi_3)^2 + (\phi_3^\dagger\phi_2)^2$

3HDM

- Example Z_3 :

$$(\phi_1{}^\dagger\phi_3)(\phi_1{}^\dagger\phi_2)$$
$$\rightarrow [e^{i(-(-\alpha)+0)}(\phi_1{}^\dagger\phi_3)][e^{i(-(-\alpha)+\alpha)}(\phi_1{}^\dagger\phi_2)]$$
$$= e^{3i\alpha}(\phi_1{}^\dagger\phi_3)(\phi_1{}^\dagger\phi_2)$$
$$3\alpha = 2\pi n \Rightarrow Z_3 \quad \text{group}$$

4HDM

- Maximal abelian subgroup of SU(4): $U(1) \times U(1) \times U(1)$ with $R = \mathrm{diag}(e^{i\alpha}, e^{i\beta}, e^{i\gamma}, e^{-i\alpha - i\beta - i\gamma})$

- The same type of graphs could be used for 4HDM. The basis transfromations in this case are:

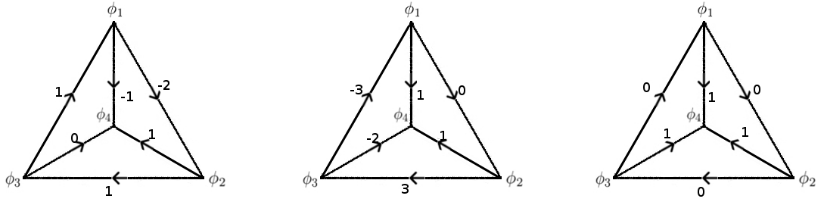
$$\begin{aligned} U(1): \quad & R = \mathrm{diag}(e^{-i\alpha}, e^{i\alpha}, 1, 1)\,, \\ U(1)': \quad & R = \mathrm{diag}(e^{i\beta}, e^{i\beta}, e^{-2i\beta}, 1)\,, \\ U(1)'': \quad & R = \mathrm{diag}(e^{\frac{i\gamma}{4}}, e^{\frac{i\gamma}{4}}, e^{\frac{i\gamma}{4}}, e^{-\frac{3i\gamma}{4}}) \end{aligned}$$

- All angles change from 0 to 2π

4HDM

- Orthogonal to each other and to $(1,1,1,1)$

$$U(1) : (-1,\ 1,\ 0,\ 0)\,, \quad U(1)' : (1,\ 1,\ -2,\ 0)\,, \quad U(1)'' : \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, -\frac{3}{4}\right)$$



(a) (b) (c)

• If sum/difference of two edges is zero in some graph, the term is symmetric under the full corresponding U(1).

• If sum/difference of two edges is k in some graph, the term is symmetric under the Z_k subgroup of the corresponding U(1).

• The discrete subgroups in this case are;

$$U(1): \; Z(2),\, Z(3),\, Z(4)$$
$$U(1)': \; Z(2),\, Z(3),\, Z(4),\, Z(5),\, Z(6)$$
$$U(1)'': \; Z(2)$$

4HDM

- Maximal discrete group $Z_6 \Rightarrow Z_6 \times U(1)''$ which is continuous
- Next candidate Z_5 from U(1)' : $(\phi_1^\dagger\phi_3)(\phi_4^\dagger\phi_3) + (\phi_2^\dagger\phi_3)(\phi_4^\dagger\phi_3)$

$$U(1)' : R = \mathrm{diag}(e^{i\beta},\, e^{i\beta},\, e^{-2i\beta},\, 1)$$
$$(-\beta - 2\beta) + (0 - 2\beta) = 5\beta$$
$$5\beta = 2\pi n \Rightarrow Z_5 \quad \text{group}$$

These terms are additionally invariant under Z_3 coming from a permuted version of $U(1)$:

$$(1, 1, e^{-i\alpha}, e^{i\alpha}) \quad \text{with} \quad \alpha = 2\pi/3\,.$$

Also symmetric under Z_2 subgroup coming from a permuted version of $U(1)''$:

$$(e^{\frac{i\gamma}{4}}, e^{\frac{i\gamma}{4}}, e^{-\frac{3i\gamma}{4}}, e^{\frac{i\gamma}{4}}) \quad \text{with} \quad \gamma = \pi\,,$$

Therefore $(\phi_1^\dagger\phi_3)(\phi_4^\dagger\phi_3) + (\phi_2^\dagger\phi_3)(\phi_4^\dagger\phi_3)$ is symmetric under the discrete group

$$Z_5 \times Z_3 \times Z_2 = Z_{30}\,.$$

A generator of this group can be constructed as the product of the above transformations and is characterized by phases:

$$\left(\frac{39}{60}\pi,\ \frac{39}{60}\pi,\ -\frac{13}{60}\pi,\ \frac{55}{60}\pi\right)\,.$$

NHDM

The basis $U(1)$ transformations in NHDM are:

$$
\begin{aligned}
U(1): &\qquad (-1,\, 1,\, ...,\, 0,\, 0)\,,\\
U(1)': &\qquad (1,\, 1,\, -2,\, ...,\, 0,\, 0)\,,\\
U(1)'': &\qquad (1,\, 1,\, 1,\, -3,\, ...,\, 0,\, 0)\,,\\
&\qquad \dots\\
U(1)^{(N-2)}: &\qquad (1,\, 1,\, ...\, 1,\, -(N-2),\, 0)\,,\\
U(1)^{(N-1)}: &\qquad \left(\frac{1}{N}, \frac{1}{N}, ..., \frac{1}{N}, -\frac{(N-1)}{N}\right)\,.
\end{aligned}
$$

- N-1 graphs each an N-vertex tetrahedron

NHDM

- Discrete subgroups: Z_2, Z_{N-2}, Z_{N-1}, Z_N, Z_{2N-4}, Z_{2N-3}, Z_{2N-2}.
- Largest candidate for a cyclic group: $Z_{2N-2} \Rightarrow Z_{2N-2} \times U(1)$
- Next candidate Z_{2N-3} :$\sum_{i=1}^{N-2}(\phi^\dagger_{N-1}\phi_N)(\phi^\dagger_{N-1}\phi_i)$

Conclusions

- We have developed a strategy to find Abelian subgroups in multi-Higgs-doublet models; where one can find discrete, polycyclic and continuous subgroups.

- This is a clear method to find invariant terms under certain symmetries and construct a potential accordingly

- What remains to study is how these symmetries spontaneously break