Frustrated symmetries in Multi-Higgs-doublet models

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Standard Model

- A gauge theory of Electroweak interactions
- Very well consistant with experiment
- There are aspects that SM doesn't explain: neutrino oscillations, dark matter, dark energy,...
- The last key ingredient of SM, the Higgs boson(s) is not discovered yet.
- Beyond Standard Model theories
- SUSY requires families of Higgs $\Rightarrow$ Higgs doublets

Higgs Mechanism

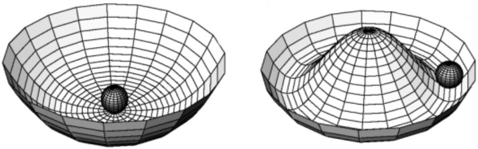
- How gauge bosons acquire their masses by coupling to the Higgs field:

- Consider the lagrangian: $L = \frac{-1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \phi|^2 - V(\phi)$

Where: $V(\phi) = \mu^2 |\phi|^2 + \lambda(|\phi|^2)^2 \qquad D_\mu = \partial_\mu - ieA_\mu$

- The potential is gauge invariant but minimizing it, shows that there is a non-zero vacuum expectation value, and the symmetry is broken:

Spontaneous symmetry breaking



- $\mu^2 > 0$ v.e.v is zero and symmetry is explicit
- $\mu^2 < 0$ v.e.v is non-zero and the symmetry is broken

Higgs doublets

- We start with minimal extension of the SM: a 2HDM
- Two Higgs doublets: $\phi_1 = \begin{pmatrix} \phi_a^+ \\ \phi_a^0 \end{pmatrix}$, $\phi_2 = \begin{pmatrix} \phi_b^+ \\ \phi_b^0 \end{pmatrix}$

- The general potential, a gauge-invariant combinations of ϕ_1 and ϕ_2 would have 14 free parameters:

$$V = V_2 + V_4$$

$$V = \frac{-1}{2}[m_{11}^2(\phi_1^\dagger\phi_1) + m_{22}^2(\phi_2^\dagger\phi_2) + m_{12}^2(\phi_1^\dagger\phi_2) + m_{12}^{2}{}^*(\phi_2^\dagger\phi_1)] + \frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \frac{1}{2}[\lambda_5(\phi_1^\dagger\phi_2)^2 + \lambda_5^*(\phi_2^\dagger\phi_1)^2] + \{[\lambda_6(\phi_1^\dagger\phi_1) + \lambda_7(\phi_2^\dagger\phi_2)](\phi_1^\dagger\phi_2) + h.c.\}$$

Higgs doublets

- At this level special symmetries can be imposed and it is possible to minimize the potential and solve the problem analitycally

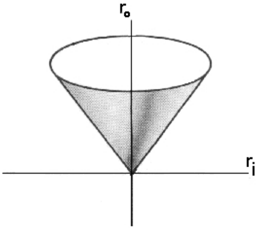
- Minimizing the potential gives: $\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}$, $\langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} u \\ v_2 e^{i\varepsilon} \end{pmatrix}$

$u = 0$: neutral vacuum , $u \neq 0$: charge breaking vacuum

- It's difficult to see the symmetries $\Rightarrow$ introduce the orbit space

Orbit space

• Introducing the orbit space $r^{\mu} : (r^0, r^i)$ would make it easier to see these symmetries, specially in multi-doublet-Higgs models.



$r^0 = \phi_a^{\dagger}\phi_a \qquad r^i = \phi_a^{\dagger}\sigma^i_{ab}\phi_b \qquad i = 1, 2, 3$

σ^i are Pauli matrices

Orbit space

- The potential could be written as $V = -M_\mu r^\mu + \frac{1}{2}\Lambda_{\mu\nu} r^\mu r^\nu$
- N^2 parameters in vector M_μ : (M_0, M_1, M_2, M_3)
- $\dfrac{N^2(N^2+1)}{2}$ parameters in tensor $\Lambda_{\mu\nu}$:

$$\begin{pmatrix} \Lambda_{00} & \Lambda_{01} & \Lambda_{02} & \Lambda_{03} \\ \Lambda_{10} & \Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\ \Lambda_{20} & \Lambda_{21} & \Lambda_{22} & \Lambda_{23} \\ \Lambda_{30} & \Lambda_{31} & \Lambda_{32} & \Lambda_{33} \end{pmatrix}$$

NHDM

- In NHDM we introduce N Higgs doublets ϕ_a, a=1,...,N.
- The general potential would be a gauge-invariant combination of $\phi_a^\dagger \phi_b$'s.
- The number of free parameters would be $\frac{N^2(N^2+3)}{2}$
- There is no hope to explicitly study and minimize the potential, and in particular to find the possible symmetries in such a model. However, we have found certain symmetries that are always broken aftre the EWSB.

Orbit space

- The orbit space would be represented as:

$$r_0 = \sqrt{\frac{N-1}{2N}} \sum_a \phi_a^\dagger \phi_a$$

$$r_i = \sum_{a,b} \phi_a^\dagger \lambda_{a,b}^i \phi_b$$

where λ_i are generators of $SU(N)$.

- The potential has the same form as before:

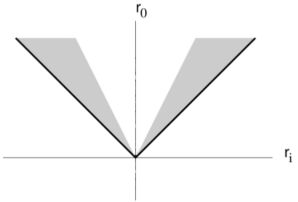
$$V = -M_\mu r^\mu + \frac{1}{2}\Lambda_{\mu\nu} r^\mu r^\nu$$

- The geometry of orbit space is more complicated.

Geometric properties

- It is located inside a conical shell lying between two coaxial cones; the forward light-cone and a certain inner cone:

$$\frac{N-2}{2(N-1)} \leq \vec{n}^2 \leq 1 \ , \ n_i \equiv \frac{r_i}{r_0}$$

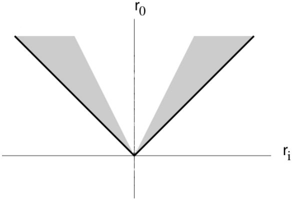


- $\langle \vec{n}^2 \rangle = 1$ on the surface of the light-cone; neutral vacuum
- $\langle \vec{n}^2 \rangle < 1$ charge-breaking vacuum
- for 2HDM, $N = 2$, the inner cone disappears. v.e.v's:

$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad \langle \phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$N > 2$

• We can devise symmetries where the only point that could conserve the symmetry would be: $\vec{r} = 0, r_0 \neq 0$



• Such point is not physically realizable in the case of $N > 2$

• Therefore these symmetries are always broken aftre EWSB in NHDM with $N > 2$

K matrix formalism

• Another look at the orbit space is offered by a complex hermitean NxN matrix: $K_{ab} \equiv (\phi_{b^\dagger}\phi_a)$

$$r_0 = \sqrt{\frac{N-1}{2N}} \sum_a \phi_a^\dagger \phi_a$$

$$r_i = \sum_{a,b} \phi_a^\dagger \lambda^i_{a,b} \phi_b$$

$$K \equiv r_0 \sqrt{\frac{2}{N(N-1)}} 1_N + r_i \lambda_i \qquad i = 1, ..., N^2 - 1$$

• Having $r_i = 0$ and $r_0 \neq 0 \quad \Longrightarrow \quad K \equiv (const) 1_N$
which gives: $(\phi_{b^\dagger}\phi_a) = 0 \quad , \quad (\phi_{a^\dagger}\phi_a) \neq 0$ and equal

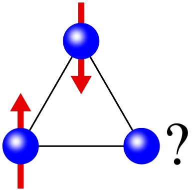
Geometrical frustration

- Having $\vec{r} = 0$, $r_0 \neq 0$ means that all products $(\phi_b^\dagger \phi_a)$ should be zero while keeping $(|\phi_a|)^2$ non-zero and equal.

- This not possible to achive with more than two doublets:

$$\begin{pmatrix} 0 \\ u \end{pmatrix} \quad \begin{pmatrix} u \\ 0 \end{pmatrix} \quad \begin{pmatrix} ? \\ ? \end{pmatrix}$$

- The situation is similar to the case of geometrical frustration in Condensed matter physics:

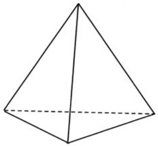


Examples in 3HDM

We have found two "frustrated" symmetries in three-Higgs-doublet models:

• The tetrahedral symmetry:

$$V = -M_0 r_0 + \Lambda_0 r_0^2 + \Lambda_1(r_1^2 + r_4^2 + r_6^2) + \Lambda_2(r_2^2 + r_5^2 + r_7^2) + \Lambda_3(r_3^2 + r_8^2)$$
$$+\Lambda_4(r_1 r_2 - r_4 r_5 + r_6 r_7)$$



$r_1 \to r_6 \to r_4 \to r_1$

$r_1 \to r_6 \to -r_4 \to r_1$

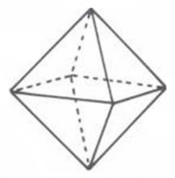
$r_1 \to -r_6 \to -r_4 \to r_1$

$r_1 \to -r_6 \to r_4 \to r_1$

Examples in 3HDM

- The octahedral symmetry:

$$V = -M_0 r_0 + \Lambda_0 r_0^2 + \Lambda_1 (r_1^2 + r_4^2 + r_6^2) + \Lambda_2 (r_2^2 + r_5^2 + r_7^2) + \Lambda_3 (r_3^2 + r_8^2)$$



- The properties of these models are being studied.

Conclusions

• In 2HDM any symmetry imposed on the potential can be conserved in the vacuum state.

• Whereas in NHDM there are certain symmetries that aren't conserved in the vacuum state.

• We call such symmetries "Frustrated symmetries", because of their symmilarity to "Geometrical frustration" in Condensed matter physics.

Current work

- Studying Abelian symmetries in 3HDM and 4HDM and trying to generalize it to NHDM

- Symmetries that can be imposed on NHDM of the form:
$U_{(1)} x U_{(1)} x \ldots$, $Z_{(i)} x Z_{(i)} x \ldots$, $Z_{(i)} x U_{(1)}$