Frustrated symmetries in Multi-Higgs-doublet models

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1. Standard Model

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Standard Model

- A gauge theory of Electroweak interactions
- Very well consistant with experiment
- There are aspects that SM doesn’t explain: neutrino oscillations, dark matter, dark energy,...
- The last key ingredient of SM, the Higgs boson(s) is not discovered yet.
- Beyond Standard Model theories
- SUSY requires families of Higgs ⇒ Higgs doublets
Higgs Mechanism

• How gauge bosons acquire their masses by coupling to the Higgs field:

• Consider the lagrangian:  \[ L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \phi|^2 - V(\phi) \]

Where:  \[ V(\phi) = \mu^2 |\phi|^2 + \lambda (|\phi|^2)^2 \]

\[ D_\mu = \partial_\mu - ieA_\mu \]

• The potential is gauge invariant but minimizing it, shows that there is a non-zero vacuum expectation value, and the symmetry is broken:  Spontaneous symmetry breaking

•  \( \mu^2 > 0 \) v.e.v is zero and symmetry is explicit

•  \( \mu^2 < 0 \) v.e.v is non-zero and the symmetry is broken
Higgs doublets

- We start with minimal extension of the SM: a 2HDM
- Two Higgs doublets: \( \phi_1 = \begin{pmatrix} \phi_1^+ \\ a_1 \\ \phi_0 \\ a_1 \\ \end{pmatrix} \), \( \phi_2 = \begin{pmatrix} \phi_2^+ \\ b_2 \\ \phi_0 \\ b_2 \\ \end{pmatrix} \)
- The general potential, a gauge-invariant combinations of \( \phi_1 \) and \( \phi_2 \) would have 14 free parameters:

\[
V = V_2 + V_4
\]

\[
V = \frac{-1}{2} [m_{11}^2 (\phi_1^\dagger \phi_1) + m_{22}^2 (\phi_2^\dagger \phi_2) + m_{12}^2 (\phi_1^\dagger \phi_2) + m_{12}^2 \star (\phi_2^\dagger \phi_1)] + \frac{\lambda_1}{2} (\phi_1^\dagger \phi_1)^2 + \frac{\lambda_2}{2} (\phi_2^\dagger \phi_2)^2 + \lambda_3 (\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + \lambda_4 (\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) + \frac{1}{2} [\lambda_5 (\phi_1^\dagger \phi_2)^2 + \lambda_5^* (\phi_2^\dagger \phi_1)^2] + \\
\{[\lambda_6 (\phi_1^\dagger \phi_1) + \lambda_7 (\phi_2^\dagger \phi_2)](\phi_1^\dagger \phi_2) + h.c.\} 
\]
Higgs doublets

- At this level special symmetries can be imposed and it is possible to minimize the potential and solve the problem analytically.

- Minimizing the potential gives: $\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v_1 \end{array} \right)$, $\langle \phi_2 \rangle = \frac{1}{\sqrt{2}} \left( \begin{array}{c} u \\ v_2 e^{i\varepsilon} \end{array} \right)$

  $u = 0$: neutral vacuum, $u \neq 0$: charge breaking vacuum

- It's difficult to see the symmetries $\Rightarrow$ introduce the orbit space.
Orbit space

- Introducing the orbit space $r^\mu : (r^0, r^i)$ would make it easier to see these symmetries, specially in multi-doublet-Higgs models.

\[
\begin{align*}
  r^0 &= \phi_a^\dagger \phi_a \\
  r^i &= \phi_a^\dagger \sigma_i^{ab} \phi_b \\
  \sigma^i &\text{ are Pauli matrices}
\end{align*}
\]
Orbit space

• The potential could be written as $V = -M_\mu r^\mu + \frac{1}{2} \Lambda_{\mu\nu} r^\mu r^\nu$

• $N^2$ parameters in vector $M_\mu : (M_0, M_1, M_2, M_3)$

• $\frac{N^2(N^2 + 1)}{2}$ parameters in tensor $\Lambda_{\mu\nu}$:

$$
\begin{pmatrix}
\Lambda_{00} & \Lambda_{01} & \Lambda_{02} & \Lambda_{03} \\
\Lambda_{10} & \Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\
\Lambda_{20} & \Lambda_{21} & \Lambda_{22} & \Lambda_{23} \\
\Lambda_{30} & \Lambda_{31} & \Lambda_{32} & \Lambda_{33}
\end{pmatrix}
$$
• In NHDM we introduce \( N \) Higgs doublets \( \phi_a, \ a=1,...,N \).

• The general potential would be a gauge-invariant combination of \( \phi_a^\dagger \phi_b \)'s.

• The number of free parameters would be \( \frac{N^2(N^2+3)}{2} \)

• There is no hope to explicitly study and minimize the potential, and in particular to find the possible symmetries in such a model. However, we have found certain symmetries that are always broken after the EWSB.
Orbit space

- The orbit space would be represented as:

\[ r_0 = \sqrt{\frac{N-1}{2N}} \sum_a \phi^+_a \phi_a \]
\[ r_i = \sum_{a,b} \phi^+_a \lambda^i_{a,b} \phi_b \]

where \( \lambda_i \) are generators of \( SU(N) \).

- The potential has the same form as before:

\[ V = -M_\mu r^\mu + \frac{1}{2} \Lambda_{\mu\nu} r^\mu r^\nu \]

- The geometry of orbit space is more complicated.
Geometric properties

- It is located inside a conical shell lying between two coaxial cones; the forward light-cone and a certain inner cone:
  \[
  \frac{N - 2}{2(N - 1)} \leq \bar{n}^2 \leq 1 , \quad n_i \equiv \frac{r_i}{r_0}
  \]

- \( \langle \bar{n}^2 \rangle = 1 \) on the surface of the light-cone; neutral vacuum
- \( \langle \bar{n}^2 \rangle < 1 \) charge-breaking vacuum
- for 2HDM, \( N = 2 \), the inner cone disappears. v.e.v’s:
  \[
  \langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu \end{pmatrix} \quad \langle \phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \nu \\ 0 \end{pmatrix}
  \]
• We can devise symmetries where the only point that could conserve the symmetry would be: \( \vec{r}' = 0, r_0 \neq 0 \)

• Such point is not physically realizable in the case of \( N > 2 \)
• Therefore these symmetries are always broken after EWSB in NHDM with \( N > 2 \)
Another look at the orbit space is offered by a complex hermitean $N \times N$ matrix: $K_{ab} \equiv (\phi_b^\dagger \phi_a)$

$$r_0 = \sqrt{\frac{N-1}{2N}} \sum_a \phi_a^\dagger \phi_a$$

$$r_i = \sum_{a,b} \phi_a^\dagger \lambda_{a,b} \phi_b$$

$$K \equiv r_0 \sqrt{\frac{2}{N(N-1)}} 1_N + r_i \lambda_i \quad i = 1, \ldots, N^2 - 1$$

Having $r_i = 0$ and $r_0 \neq 0 \implies K \equiv (\text{const}) 1_N$

which gives: $(\phi_b^\dagger \phi_a) = 0$, $(\phi_a^\dagger \phi_a) \neq 0$ and equal
Geometrical frustration

• Having \( \vec{r} = 0 \), \( r_0 \neq 0 \) means that all products \( (\phi_b^\dagger \phi_a) \) should be zero while keeping \( (|\phi_a|^2) \) non-zero and equal.

• This not possible to achieve with more than two doublets:

\[
\begin{pmatrix}
0 \\
u
\end{pmatrix}
\begin{pmatrix}
u \\
0
\end{pmatrix}
\begin{pmatrix}
? \\
?
\end{pmatrix}
\]

• The situation is similar to the case of geometrical frustration in Condensed matter physics:
Examples in 3HDM

We have found two ”frustrated” symmetries in three-Higgs-doublet models:

- The tetrahedral symmetry:
  \[ V = -M_0 r_0 + \Lambda_0 r_0^2 + \Lambda_1 (r_1^2 + r_4^2 + r_6^2) + \Lambda_2 (r_2^2 + r_5^2 + r_7^2) + \Lambda_3 (r_3^2 + r_8^2) + \Lambda_4 (r_1 r_2 - r_4 r_5 + r_6 r_7) \]

\[ r_1 \rightarrow r_6 \rightarrow r_4 \rightarrow r_1 \]
\[ r_1 \rightarrow r_6 \rightarrow -r_4 \rightarrow r_1 \]
\[ r_1 \rightarrow -r_6 \rightarrow -r_4 \rightarrow r_1 \]
\[ r_1 \rightarrow -r_6 \rightarrow r_4 \rightarrow r_1 \]
Examples in 3HDM

- The octahedral symmetry:

\[ V = -M_0 r_0 + \Lambda_0 r_0^2 + \Lambda_1 (r_1^2 + r_4^2 + r_6^2) + \Lambda_2 (r_2^2 + r_5^2 + r_7^2) + \Lambda_3 (r_3^2 + r_8^2) \]

- The properties of these models are being studied.
Conclusions

• In 2HDM any symmetry imposed on the potential can be conserved in the vacuum state.

• Whereas in NHDM there are certain symmetries that aren’t conserved in the vacuum state.

• We call such symmetries ”Frustrated symmetries”, because of their symmilarity to ”Geometrical frustration” in Condensed matter physics.
Current work

• Studying Abelian symmetries in 3HDM and 4HDM and trying to generalize it to NHDM

• Symmetries that can be imposed on NHDM of the form:
  \[ U(1) \times U(1) \times \ldots, Z(i) \times Z(i) \times \ldots, Z(i) \times U(1) \]