# NONLINEAR MODAL ANALYSIS OF THE SMALLSAT SPACECRAFT

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## ABSTRACT

Non-linear elements are present in practically all spacecraft structures. When such non-linear effects are important linear modal analysis can no longer be applied. The development of a non-linear analogue of linear modal analysis is therefore an important endeavor. The objective of this paper is to show that nonlinear normal modes (NNMs) represent a useful and practical tool in this context. A full-scale spacecraft structure is considered and is modeled using the finite element method. Its NNMs are computed using advanced numerical algorithms, and the resulting dynamics is then carefully analyzed. Nonlinear phenomena with no linear counterpart including nonlinear modal interactions are also highlighted.

Key words: Nonlinear modal analysis; Nonlinear normal modes; Modal interaction; Localization; Spacecraft.

## 1. INTRODUCTION

Linear modal analysis (LMA) is a widely and extensively used tool in the structural dynamics community. The concept of linear normal modes (LNMs), with their obvious physical interpretation and their interesting mathematical properties (i.e. they decouple the equations of motion), is exploited for experimental modal analysis, model updating, substructuring, and health monitoring. Despite LMA is quite mature and sophisticated, it fails to capture the system dynamics in the presence of nonlinearities and cannot explain basic nonlinear phenomena such as localization and modal interactions. To address this issue, new rigorous mathematical tools need to be developed. In this context, this paper will show that nonlinear normal modes (NNMs) represent a useful framework.

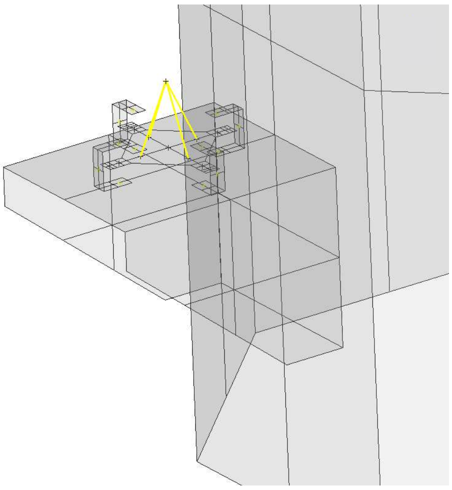
In particular, this paper presents the theoretical nonlinear modal analysis of a real-life structure, the SmallSat spacecraft. The NNMs are computed and the resulting dynamics is carefully analyzed. Nonlinear phenomena with no linear counterpart are observed on this example and their practical consequences are discussed.

The rest of the paper is organized in three sections. Section 2 introduces the spacecraft structure, its finite element model (FEM), and the nonlinearities considered. Section 3 presents the NNMs and analyzes the resulting dynamics. Finally, Section 4 draws conclusions on the nonlinear investigations. Note that a theoretical introduction to NNMs is beyond the scope of this paper and the reader is referred to [1] for further details.

## 2. SMALLSAT STRUCTURE AND MODELING

The SmallSat structure has been conceived as a low-cost structure for small low-earth orbit satellite [2]. It is a monocoque tube structure which is 1.2 $m$ long and 1 $m$ large. It incorporates eight flat faces for equipment mounting purposes, creating an octagon shape, as shown in Figure 1. The octagon is manufactured using carbon fibre reinforced plastic by means of a filament winding process. The structure thickness is 4.0 $mm$ with an additional 0.25 $mm$ thick skin of Kevlar applied to both the inside and outside surfaces to provide protection against debris. The interface between the spacecraft and launch vehicle is achieved through four aluminium brackets located around cut-outs at the base of the structure. The total mass including the interface brackets is around 64 $kg$. A finite element model (FEM) of the SmallSat was created in Samcef software and is used in the present study to conduct numerical experiments. It comprises about 65,000 degrees-of-freedom (DOFs) and the comparison with experimental measurements revealed its good predictive capability. The model idealizes the composite tube structure using shell elements whose behavior is supposed to be orthotropic and meets boundary conditions through 4 clamped nodes.

The SmallSat structure supports a telescope dummy



(a)

(b)

Figure 1: Global view of the SmallSat FEM (b) and zoom on the WEMS device (a). For confidentiality, the FEM is used to display the WEMS.

composed of two stages of base-plates and struts supporting various concentrated masses; its mass is around 140 $kg$. The telescope dummy plate is connected to the SmallSat top floor via three shock attenuators, termed SASSA (Shock Attenuation System for Spacecraft and Adaptator) [3], the behavior of which is considered as linear in the present study. The top floor is a 1 square meter sandwich aluminium panel, with 25 $mm$ core and 1 $mm$ skins, modeled using shell elements. Finally, as shown in Figure 1 (a), a support bracket connects to one of the eight walls the so-called Wheel Elastomer Mounting System (WEMS) device which is loaded with an 8 $kg$ inertia wheel dummy. The purpose of this device is to isolate the spacecraft structure from disturbances coming from the inertia wheel through the presence of a soft interface (made up of elastomer plots) between the fixed and mobile parts. In addition, mechanical stops limit the axial and lateral motion of the WEMS mobile part during launch, which gives rise to nonlinear dynamic phenomena. Shells model both the bracket and the wheel support. The four nonlinear connections between the WEMS mobile part (the inertia wheel and its cross-shaped support) and fixed part (the bracket and, by extension, the spacecraft itself) are considered as having piecewise-linear behaviors along two directions.

From a numerical standpoint, piecewise-linear behavior requires special numerical treatment. In order to decrease the computational burden and facilitate the convergence process, the piecewise-linear behaviors were regularized using Hermite polynomials. This approach considers a $2\Delta$ regularization area centered on the clearance and uses Hermite (cubic) polynomials to smooth the transition between both linear regimes. This approach has the advantage to keep the restoring forces as purely linear out of the regularization area.

## 3. SMALLSAT NONLINEAR NORMAL MODES

Prior to NNMs computation, a LMA is performed to underline the relevant modes for the future nonlinear investigations, i.e., the modes that will give rise to nonlinear phenomena. In this context, local WEMS modes are of particular interest. An interesting observation is that, due to the important flexibility of the WEMS, almost every LNM in the frequency range considered involves WEMS motion. The only exception is the $9^{th}$ mode for which the WEMS is quiescent. The interested reader can find additional information about the LMA in [4].

NNMs are computed using a two-step algorithm. The first step considers a shooting algorithm to compute isolated NNM motions (i.e. isolated periodic solutions). The second step involves a continuation algorithm (the pseudo-arclength continuation algorithm) in order to compute the evolution of the periodic solution with energy and to obtain the frequency-energy dependence of the NNMs. Further details about the algorithm are presented in [5].

Computing NNMs for the full-scale FEM was not feasible in a reasonable amount of time. Therefore, a reduced order model (ROM) of the structure was created using the Craig-Bampton technique [6]. This ROM is made of 8 nodes (24 translational DOFs) and 10 internal modes. Considering a maximum relative error on frequencies of 1% and modal assurance criterion (MAC) values above 0.9, this ROM is accurate with respect to the full model for the 15 first LNMs (i.e. in the $[0 - 80.4]$ Hz frequency range). The study of results sensitivity to ROM showed that ROM with few internal modes are relevant to capture the fundamental nonlinear dynamics [4].

The first NNM is pictured in a frequency-energy plot (FEP) in Figure 2. This plot highlights a basic property of NNMs, the frequency-energy dependence. At low energy, no mechanical stop is activated. The NNM frequency remains constant, an indication of a linear regime of motion. The frequency is equal to the first LNM frequency. Once one of the WEMS relative displacements enters into the regularization area of the piecewise-linear restoring forces, there is a very sudden increase in the NMM frequency, and nonlinear phenomena come into play. A hardening effect is observed with an overall frequency increase of about 0.4 Hz (3.8%). Specifically, the fundamental frequency of the periodic solution as well as its harmonics increase along the branch. Once a harmonic component has a frequency similar to another NNM frequency, a dynamic coupling between both NNMs can exist. For the first "tongue" in Figure 2, a $3 : 1$ modal interaction occurs between NNMs 1 and 7.

A second internal resonance appears further along the branch and involves modes 1 and 12. This $5 : 1$ interaction is described in Figure 3. Figures 3(a)-(f) show the evolution of the periodic solution on the tongue. The frequency content as well as the modal shapes are presented. From points (a) to (c), the amplitudes of the third and the fifth harmonics increase whereas the fundamental frequency amplitude decreases. The point (d) is a bifurcation point where the fifth harmonic component is the only frequency component in the periodic solution. The modal shapes evolve from an excellent matching with LNM 1 (point (a), MAC = 0.99) to an excellent matching with LNM 12 (point (d), MAC = 1). Finally, from points (e) to (f), harmonics amplitudes decrease, and a modal shape similar to the first LNM is recovered.

It turns out that the modal shapes computed on the backbone branch are close to the corresponding LNM. Indeed, SmallSat nonlinearities are localized and have a small influence on the overall structure deformation. In the particular case of local WEMS motions, this observation is even more true if we consider the position and orientation of the nonlinearities with respect to the LNMs modal shapes.

Another phenomenon observed on the seventh and eighth modes is the "localization" phenomenon. It highlights the existence of new periodic solutions that are localized to some structural parts and that do not resemble LNMs. Figure 4 presents the FEP of NNM 7 for which the frequency increase is about 0.9 Hz (2.8%). The frequency remains constant until high energies ($\approx 10^2$ $[J]$). However, the continuation of the backbone indicates a bifurcation where the curve starts to bend backwards. The evolution of the modal shape and the frequency content of the periodic solutions are presented in Figures 4 (a)-(d). Point (a) is a purely linear solution where harmonics are not visible and where the modal shape is similar to the corresponding LNM (MAC = 1). For the same energy level, point (b) is characterized by a different (but still mostly monochromatic) frequency content. In addition, the modal shape is modified and WEMS displacements become more important. The MAC with respect to the first LNM decreases to 0.74. Despite a very similar frequency content, the structural deformation at point (c) is dramatically different. The entire energy is localized to the WEMS. This new modal shape cannot be identified in the basis of the different LNMs and is a purely new nonlinear solution. We note that this phenomenon is different from modal interactions for which strong harmonic components are observed. As energy increases, point (d) shows that the modal shape still evolves but again with a similar frequency content.

For some modes, the energy required to have relative displacements large enough to activate the nonlinearities is very high so that they can be considered as linear. This is the case for the ninth mode where the frequency remains constant as energy increases (Figure 5).

The first, the seventh, and the ninth NNM were adequate to illustrate fundamental nonlinear phenomena. Similar observations were performed on other NNMs but, for conciseness, they are not discussed in the present paper. The reader is referred to [4] for further details.

## 4. CONCLUSIONS

The objective of the present paper was to compute the NNMs of a real-life structure in order to demonstrate that they form a useful and practical tool for the structural dynamicist. We demonstrated that NNMs can capture the basic frequency-energy dependence of the modes as well as important nonlinear phenomena with no linear counterparts (modal interactions, localization). Such phenomena that were already observed on academic examples are now observed on this real-life structure.

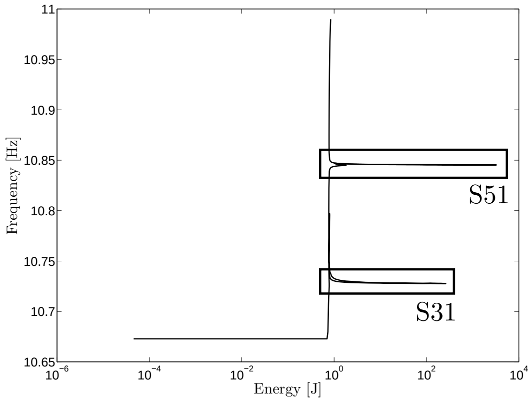



Figure 2: FEP of the first NNM with S31 and S51 modal interactions.

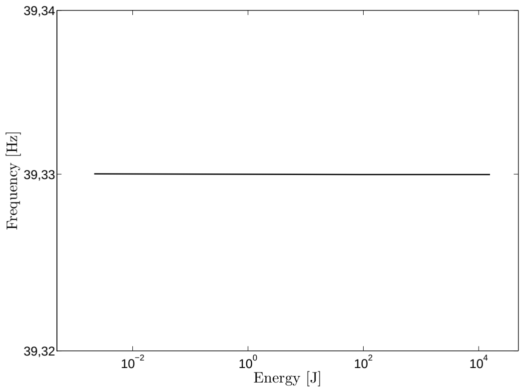



Figure 5: FEP of NNM 9. The mode remains linear and the frequency does not depend on the energy.

Due to the hardening characteristics of the WEMS, investigations presented in Section 3 showed that the fundamental frequency of vibration modes increases with the energy. We also observed that almost all modes investigated behave in a nonlinear fashion since they involve a WEMS device motion. Finally, one important conclusion of this study is that, due to nonlinear modal interactions, a mode of higher frequency can be excited through the excitation of a mode of lower frequency. In the particular case of the SmallSat spacecraft, we observed that the excitation of a local mode of the SmallSat spacecraft with a small modal effective mass can trigger the excitation of higher-frequency global modes of the structure (through, e.g, 3:1 and 5:1 modal interactions). This shows that the frequency-based criterion, which ensures that there is no dynamic coupling between the launch vehicle and the spacecraft, should be applied with great care in the presence of nonlinear phenomena. More precisely, when nonlinearities are present, low-frequency modes with a small effective modal mass should not be neglected on the basis that they do not represent a danger for the structural integrity of the spacecraft. We also stress that nonlinear modal interactions were encountered during experimental tests performed at Astrium-UK in Stevenage. Specifically, 2:1 and 3:1 modal interactions were evidenced. The NNM framework gives a rigorous theoretical interpretation of such experimental observations. In addition, since NNMs have a clear conceptual relation with LNMs, they constitute the natural nonlinear extension of LMA.

## ACKNOWLEDGMENTS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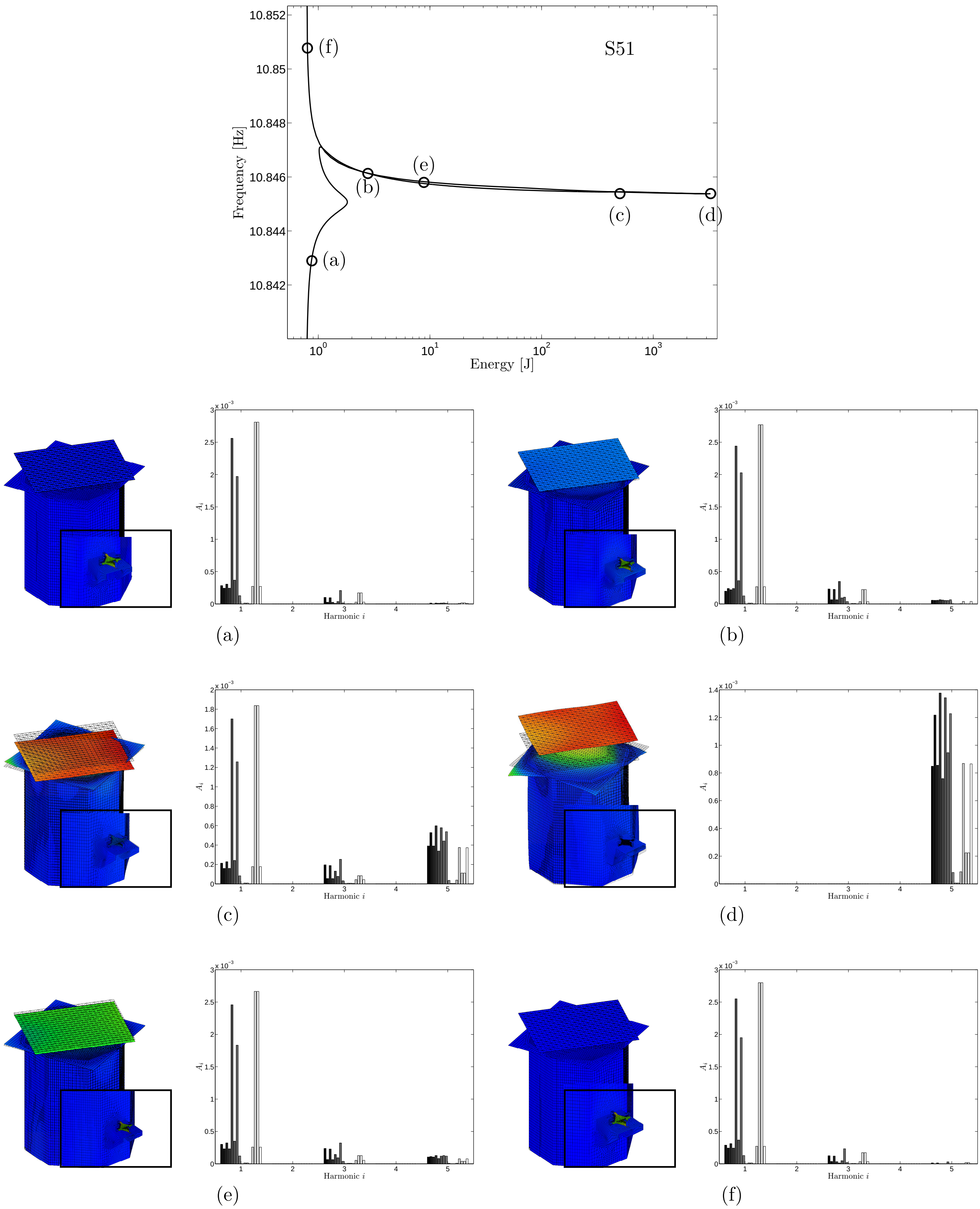


Figure 3: Close-up of the 5:1 internal resonance branch of the first NNM. (a)-(f) Modal shapes and frequency contents associated with the points defined in the FEP. The frequency content corresponds to Fourier coefficients computed using cosine terms only.

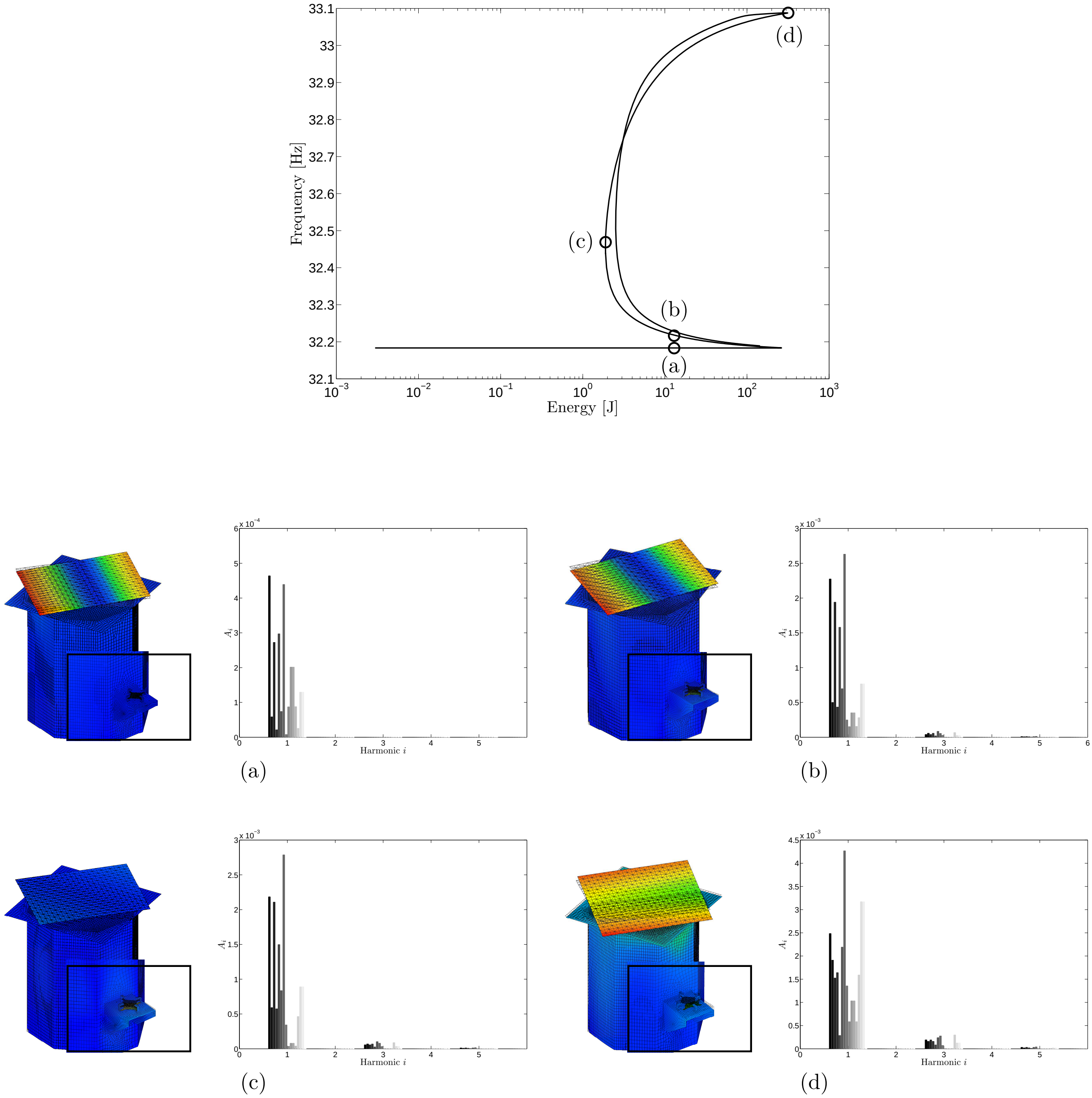


Figure 4: FEP of NNM 7. A localization phenomenon (on the WEMS) is observed. (a) - (d) represents the modal shapes and the frequency content at the different points of the backbone. The frequency content corresponds to Fourier coefficients computed using cosine terms only.