Multi-scale modelling of fibre reinforced composite with non-local damage variable

L. Wu, L. Noels,

L. Adam and I. Doghri

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/Outline

- Introduction
- Mean Field Homogenization
- Nonlocal Approach and Implicit Gradient Formulation
- Ductile Damage in the Matrix of Composite
- Finite Element Implementation
- Validation and Simulation

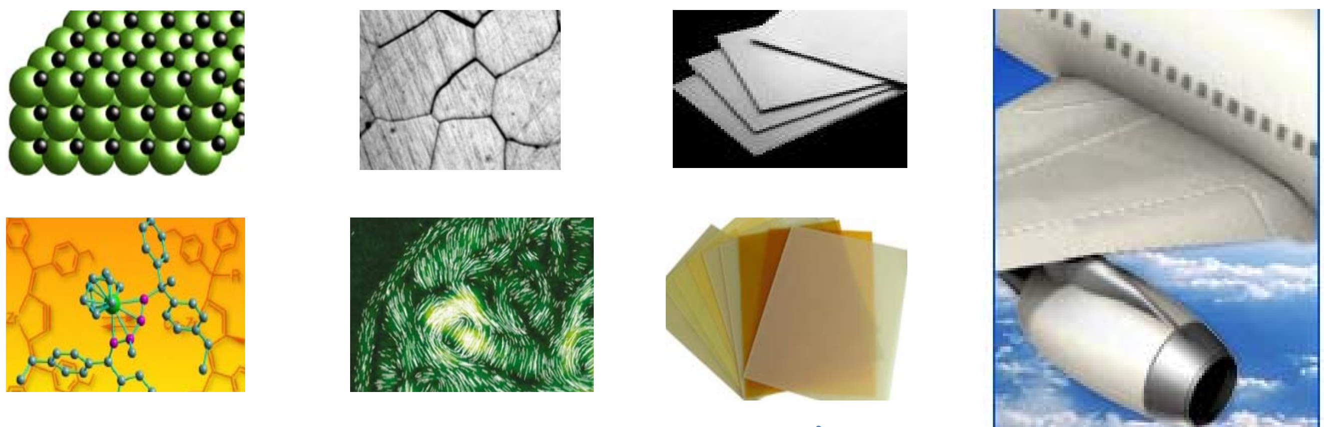
/Introduction

Why Multiscale?

- Materials are multiscale in nature:

Metals

Polymers



10^{-9} m — 1 m

Molecular Dynamics

Monte Carlo Method, Statistical Mechanics …

Continuum Mechanics

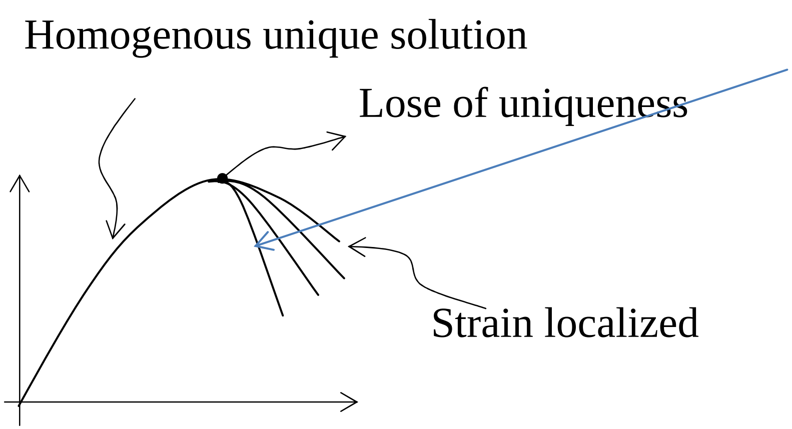
Why Multiscale?

- Multiscale Methods for Composites
  - ❖ For material design:
    These effective properties are difficult or expensive to measure.
  - ❖ For composite structures analysis:
    - Continuum mechanics analysis at Macroscale
      Accuracy!
    - Take into account the individual component properties and geometrical arrangements.
      Expensive, unreachable!
- Solution:
  - ❖ The engineering problems are solved at macroscopic scale with the homogenized properties.
  - ❖ The homogenized properties are obtained from the individual component properties and their microstructure.

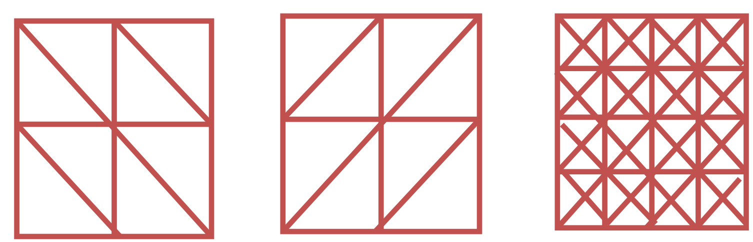
/Introduction

Problem in finite element simulations

- Finite element solutions for strain softening problems suffer from:
  - The loss the uniqueness and strain localization
  - Mesh dependence



The numerical results change with the size of mesh and direction of mesh



The numerical results change without convergence

/Introduction

Problem in finite element simulations

Multiscale Methods have this problem too!

- Solution:

  Introduce high order term in the continuum description

  Strain gradient model, nonlocal model…

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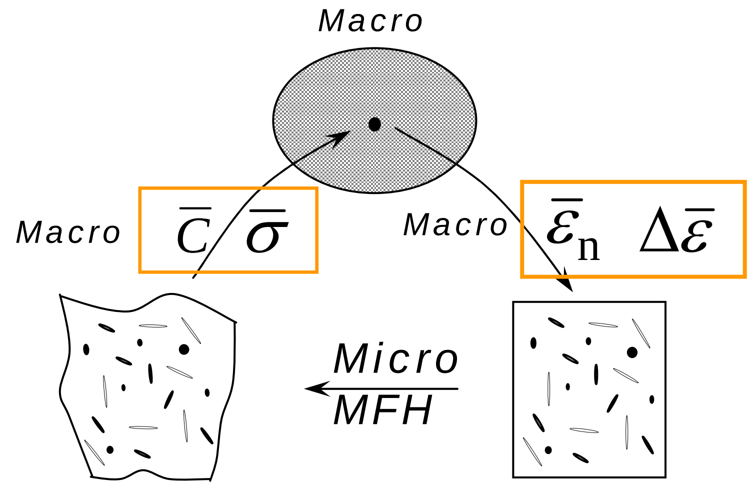
/Mean Field Homogenization

- At the macroscale, the problem is a classical continuum mechanics problem (Finite Element method).
- At a macroscopic material point the properties of the material correspond to a representative volume element (RVE) of the microstructure.

Basing on
The marco strain $\bar{\varepsilon}$ and stress $\bar{\sigma}$ equal the average strain $\langle \varepsilon \rangle$ and stress $\langle \boldsymbol{\sigma} \rangle$ over a RVE

$$\langle a \rangle = \frac{1}{V} \int_V a(\boldsymbol{X}) \mathrm{d}V$$



/Mean Field Homogenization

- How to get $\bar{C}$ in RVE? Such that $\langle \boldsymbol{\sigma} \rangle = \bar{\boldsymbol{C}} : \langle \boldsymbol{\varepsilon} \rangle$
  - Direct finite element simulation
  - Semi-analytical mean field homogenization models

(Voigt, Reuss, **Mori-Tanaka**, Double-Inclusion, Self-Consistent …)

- Two-phase composite
  - Volume fraction $v_0 + v_1 = 1$

$$\langle \boldsymbol{\sigma} \rangle = v_0 \langle \boldsymbol{\sigma} \rangle_{\omega_0} + v_1 \langle \boldsymbol{\sigma} \rangle_{\omega_1}$$

$$\langle \boldsymbol{\varepsilon} \rangle = v_0 \langle \boldsymbol{\varepsilon} \rangle_{\omega_0} + v_1 \langle \boldsymbol{\varepsilon} \rangle_{\omega_1}$$

$$\langle \boldsymbol{\sigma} \rangle_{\omega_1} = \bar{\boldsymbol{C}}_1 : \langle \boldsymbol{\varepsilon} \rangle_{\omega_1}$$

$$\langle \boldsymbol{\sigma} \rangle_{\omega_0} = \bar{\boldsymbol{C}}_0 : \langle \boldsymbol{\varepsilon} \rangle_{\omega_0}$$

???

$$\langle \varepsilon \rangle_{\omega_1} = B^{\varepsilon} : \langle \varepsilon \rangle_{\omega_0}$$

Subscription: 0(matrix) and 1(inclusion)

/Mean Field Homogenization

- Single inclusion problem

$$\langle \varepsilon \rangle_{\omega_1} = H^{\varepsilon}(I, \overline{C}_0, \overline{C}_1) : \varepsilon^{\infty}$$

H^{ε} is single inclusion strain concentration tensor (numerical, analytical)

$$H^{\varepsilon} = [I + S : \overline{C}_0^{-1} : (\overline{C}_1 - \overline{C}_0)]^{-1}$$

S is Eshelby's tensor

- Multiple inclusion problem

$$\langle \varepsilon \rangle_{\omega_1} = B^{\varepsilon} : \langle \varepsilon \rangle_{\omega_0} \qquad \langle \varepsilon \rangle_{\omega_1} = A^{\varepsilon} : \langle \varepsilon \rangle$$

Mori-Tanaka model:

$$\varepsilon^{\infty} = \langle \varepsilon \rangle_{\omega_0} \quad \text{and} \quad B^{\varepsilon} = H^{\varepsilon}$$

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/Nonlocal Approach

- Description

   Some variables (a) are replaced by their weighted (w) average over a characteristic volume (Vc) to reflect the interaction between neighboring material points.

$$\bar{a} = \frac{1}{V_c}\int_{V_c} a w dV$$

The state variable a can be strains, internal variables (eg. accumulated plastic strain, damage….)

Problem: Weight function w ?? Characteristic volume Vc ??

/Implicit Gradient Formulation

- Gradient model
  - Derived from non-local models by expanding the integration of $\bar{a}$ in Taylor series.

$$\bar{a} = a + c_1 \nabla^2 a + c_2 \nabla^4 a + \ldots$$

  - The coefficients *c*1, *c*2... depend on the weight function and the characteristic volume V_c.

❖ Explicit gradient formulation:

$$\bar{a} = a + c \nabla^2 a$$

c has the dimension of length squared.

/Implicit Gradient Formulation

- Implicit gradient formulation *:
  - ❖ Green's function $G(\boldsymbol{y};\boldsymbol{x})$ ⟶ weight function $w(\boldsymbol{y};\boldsymbol{x})$

$$\bar{a} = \frac{1}{V_c}\int_{V_c} awdV \longrightarrow \bar{a} - c\nabla^2\bar{a} = a$$

  - ❖ The natural boundary condition:

$$\frac{\partial\bar{a}}{\partial n} = n_i \frac{\partial\bar{a}}{\partial x_i} = 0$$

How can we use it in Mean Field Homogenization ?

* Peerlings et al., 1996

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/Ductile Damage in the Matrix of Composite

- Damage in matrix only (neglect the Damage in fiber)
- Lemaitre - Chaboche ductile damage model:

$$\dot{D} = (\frac{Y}{S_0})^n \dot{p}$$

where S_0 and n are the material parameters

Y is the strain energy release rate $Y = \frac{1}{2}\boldsymbol{\varepsilon}^{\mathrm{e}} : \boldsymbol{E}_0 : \boldsymbol{\varepsilon}^{\mathrm{e}}$

p is the accumulate plastic strain $\dot{p} = [\frac{2}{3}\dot{\boldsymbol{\varepsilon}}^{\mathrm{p}} : \dot{\boldsymbol{\varepsilon}}^{\mathrm{p}}]^{1/2}$, $p = \int \dot{p}\,\mathrm{d}t$

- Nonlocal damage:

$$\dot{D} = (\frac{Y}{S_0})^n (\dot{p} + c_1 \nabla^2 \dot{p} + c_2 \nabla^4 \dot{p} + \dots)$$

$$= (\frac{Y}{S_0})^n \dot{\bar{p}}$$

$$\bar{p} - c\nabla^2 \bar{p} = p$$

/Ductile Damage in the Matrix of Composite

- Considering the damage in matrix, the incremental form of stress in composite*:

$$\delta\boldsymbol{\sigma} = \upsilon_1 \delta\boldsymbol{\sigma}_1 + \upsilon_0 \delta\boldsymbol{\sigma}_0$$

$$\delta\boldsymbol{\sigma}_0 = (1-D)\boldsymbol{C}_0^{\mathrm{alg}} : \delta\boldsymbol{\varepsilon}_0 - \hat{\boldsymbol{\sigma}}_0 \delta D \qquad \hat{\boldsymbol{\sigma}}_0 = \boldsymbol{\sigma}_0/(1-D)$$

$$\delta\boldsymbol{\sigma} = \underbrace{\upsilon_1 \boldsymbol{C}_{\mathrm{I}}^{alg} \delta\boldsymbol{\varepsilon}_{\mathrm{I}} + \upsilon_0 (1-D)\boldsymbol{C}_0^{\mathrm{alg}} : \delta\boldsymbol{\varepsilon}_0}_{\text{Mori-Tanaka}} - \upsilon_0 \hat{\boldsymbol{\sigma}}_0 \delta D$$

$$\delta\boldsymbol{\sigma} = \overline{\boldsymbol{C}}^{\,algD} : \delta\boldsymbol{\varepsilon} + (\overline{\boldsymbol{C}}^{\,p} - \upsilon_0 \hat{\boldsymbol{\sigma}}_0 \frac{\partial D}{\partial \overline{p}}) \delta\overline{p}$$

Subscription: 0(matrix) and 1(inclusion)

* Doghri I. et al., 2003

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Governing equations

- For implicit gradient enhanced elastic-plasticity

$$\begin{cases} \nabla \boldsymbol{\sigma} + \boldsymbol{f} = \boldsymbol{0} & \text{for composite material} \\ \bar{p} - l^2 \nabla^2 \bar{p} = p & \text{for matrix material only} \end{cases}$$

where $\boldsymbol{f}$ - the body force vector;

l - the characteristic length of matrix material.

- Discretization (in each element)

$$\boldsymbol{U} = \boldsymbol{N}_u \boldsymbol{u} \qquad \bar{p} = \boldsymbol{N}_{\bar{p}} \bar{\boldsymbol{p}}$$

$$\boldsymbol{\varepsilon} = \boldsymbol{B}_u \boldsymbol{u} \qquad \nabla \bar{p} = \nabla \boldsymbol{N}_{\bar{p}} \bar{\boldsymbol{p}} = \boldsymbol{B}_{\bar{p}} \bar{\boldsymbol{p}}$$

$$\Rightarrow \begin{bmatrix} \boldsymbol{K}_{uu} & \boldsymbol{K}_{u\bar{p}} \\ \boldsymbol{K}_{\bar{p}u} & \boldsymbol{K}_{\bar{p}\bar{p}} \end{bmatrix} \begin{bmatrix} d\boldsymbol{u} \\ d\bar{\boldsymbol{p}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}_{\text{ext}} - \boldsymbol{F}_{\text{int}} \\ \boldsymbol{F}_p - \boldsymbol{F}_{\bar{p}} \end{bmatrix}$$

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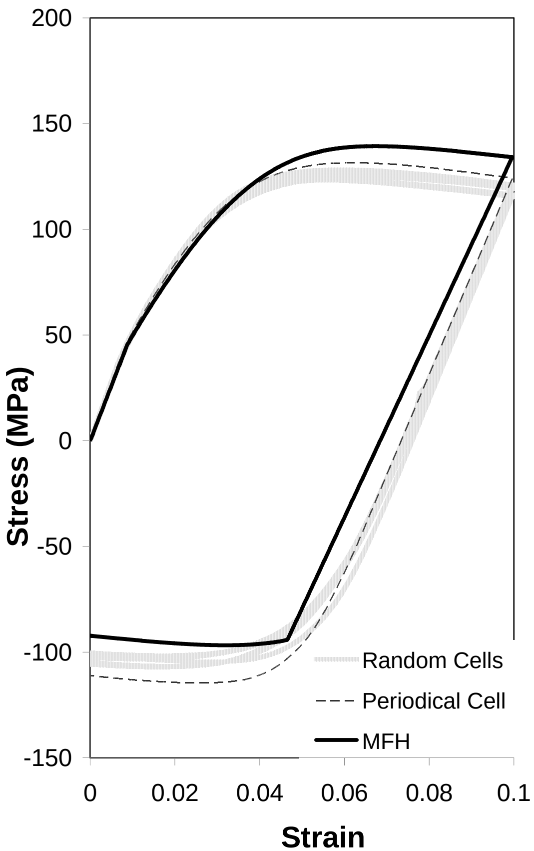
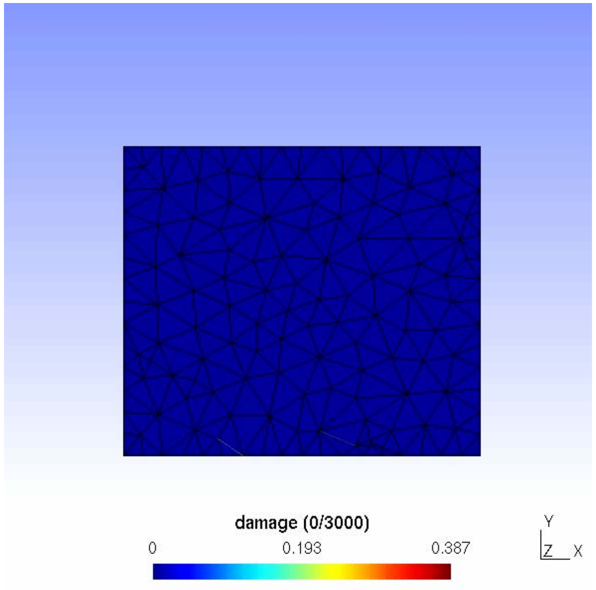
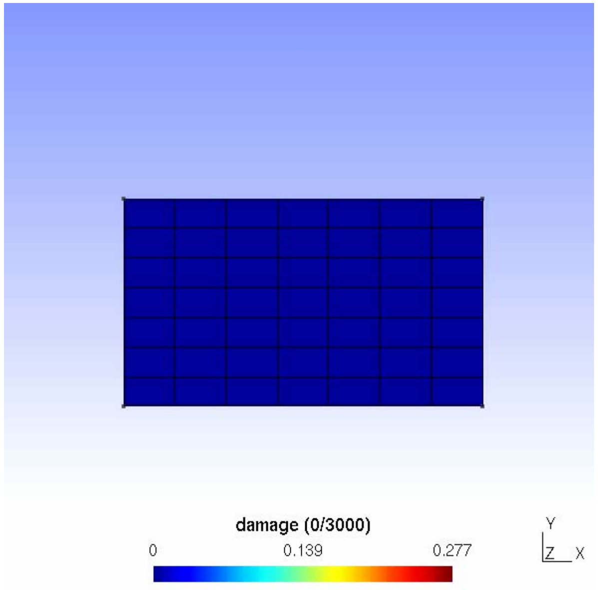
/Validation and Simulation

Validation: DNS vs. FE/MFH

- MFH with non-local damage (Epoxy-CF (30%))







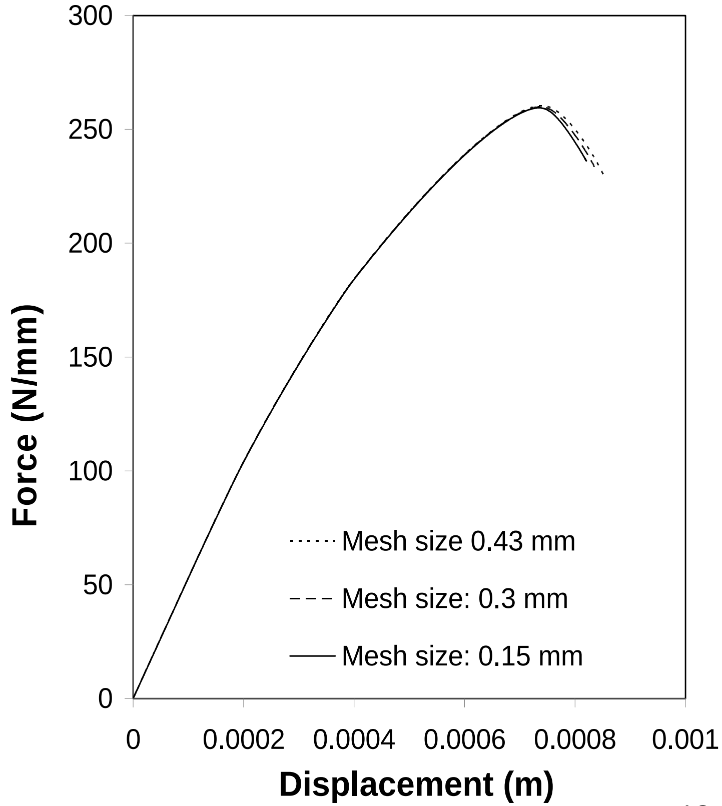
/Validation and Simulation

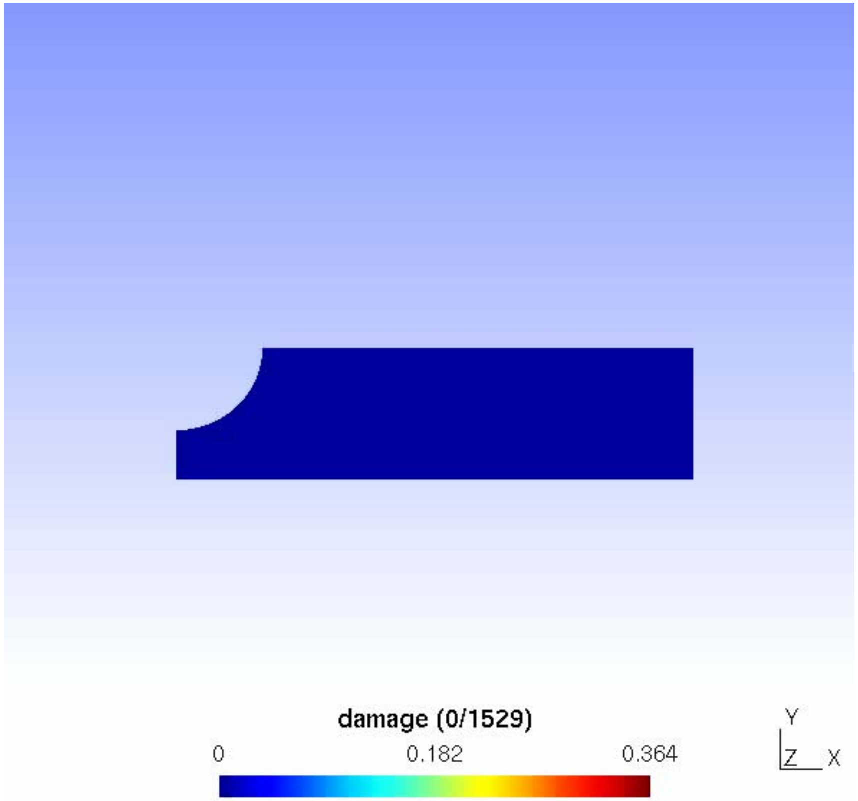
Simulation

- Notched ply

Epoxy-CF (30%), Transverse loading, single ply

Mesh independent



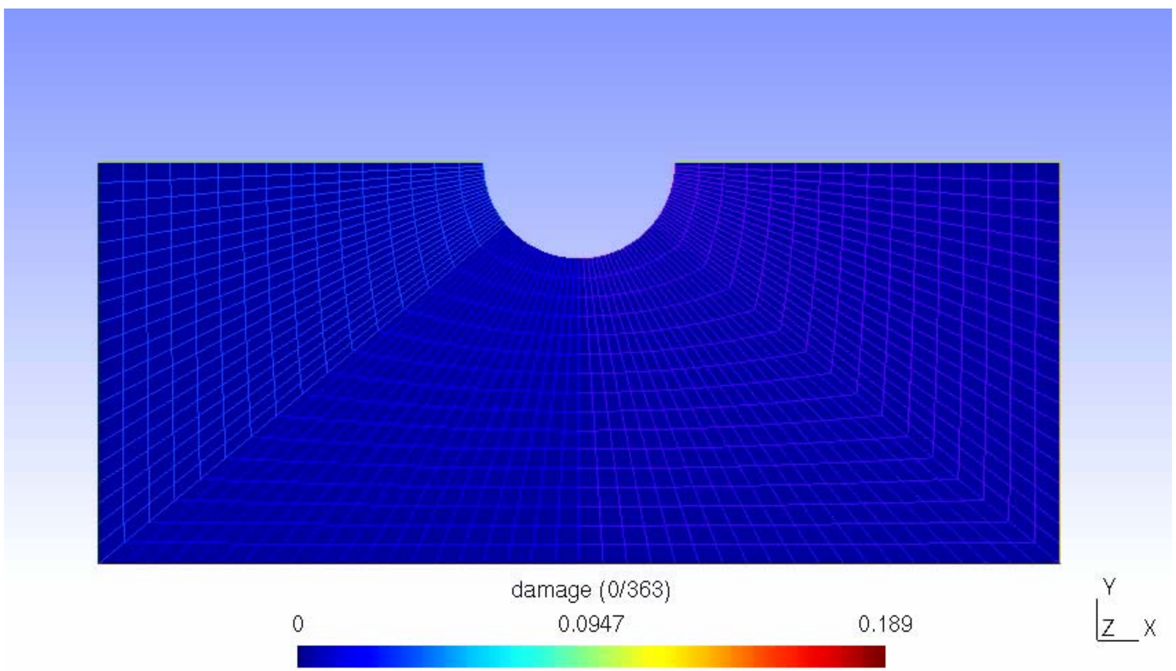


/Validation and Simulation

Simulation

- Laminates with hole

  Epoxy-CF (30%), in plane loading,
  laminates (-45/45/45/-45) → 4 layers of elements



Thank you!

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