Non-local damage-enhanced MFH for multiscale simulations of composites

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ABSTRACT

In this work, a gradient-enhanced mean-field homogenization (MFH) procedure is proposed for fiber reinforced materials. In this approach, the fibers are assumed to remain linear elastic while the matrix material obeys an elasto-plastic behavior enhanced by a damage model. As classical finite element simulations face the problems of losing uniqueness and strain localization when strain softening of materials is involved, we develop the mean-field homogenization in a non-local way. Toward this end we use the so-called non-local implicit approach, reformulated in an anisotropic way to describe the damage in the matrix. As a result we have a multi-scale model that can be used to study the damage process at the meso-scale, and in particular the damaging of plies in a composite stack, in an efficient computational way. As a demonstration a stack with a hole is studied and it is shown that the model predicts the damaging process in bands oriented with the fiber directions.

1. INTRODUCTION

As direct numerical simulations of composite structures explicitly accounting for the individual phases are too complex to handle and as the computation expenses are unaffordable, homogenized properties are usually sought to perform structural analyzes. Homogenization techniques are known to be efficient tools to derive those homogenized material properties analytically and/or numerically, see [1,2] for exhaustive overviews.

Among those different methods, the mean-field homogenization (MFH) approach [3] provides predictions for the macroscopic behavior of heterogeneous materials and is thus of particular interest for the modeling of particle or fiber reinforced composites. MFH methods were first developed for linear elastic structures by extending the Eshelby single inclusion solution [4] to multiple inclusions interacting in an average way in the composite phase, as the Mori-Tanaka scheme [5, 6] and the self-consistent scheme [7, 8]. When extending MFH methods to the non-linear regime, the so-called incremental formulation rewrites the constitutive equations in a pseudo-linear form relating the stress and strain rates [9-12], allowing the use of the linear techniques.

Although homogenization schemes have achieved a high level of accuracy to capture the non-linear behavior of composites, accounting for material degradation remains highly challenging [2]. Besides the complexity of formulating the multiscale method, the governing partial differential equations lose ellipticity and finite element (FE) solution uniqueness at strain-softening onset. Recently, the authors have proposed an incremental non-local MFH accounting for damage happening in the matrix-material at the micro-scale [13]. In order to avoid the strain/damage localization caused by matrix material softening, a gradient-enhanced formulation [14] is adopted during the homogenization process. In this formulation, the non-local accumulated plastic strain of matrix is defined and depends on the local accumulated plastic strain and on its derivatives through the resolution of a new boundary value problem [14, 15]. This formulation avoids the need to develop higher-order elements, although the elements have now one additional degree of freedom per node. As a result this new formulation allows simulating the ply-level response under quasi-static loading conditions resulting from the coupled plasticity-damage model considered at the micro-scale.

This paper extends the non-local formulation of MFH with damage [13] to the analysis of composite laminates, which are meshed by considering in each ply finite elements whose constitutive behaviors are predicted by the MFH. The elements in each ply are integrated using a material constitutive behavior depending on the orientation of the ply fibers. This extension requires reformulating the non-local approach in an anisotropic way in order to introduce different characteristic sizes in the directions parallel and perpendicular to the fibers. Also, as the non-local formulation implies new nodal degrees of freedom and the resolution of new equations, the finite element approach presented introduces discontinuities in the non-local accumulated plastic strain field at plies interfaces to satisfy the new boundary conditions at material interfaces which are associated to the non-local variables. To illustrate the efficiency of this framework, a laminate with a hole is studied, and it is shown that the model predicts the damaging process in bands oriented with the fiber directions in the different plies.

2. ANISOTROPIC NON-LOCAL GRADIENT MODEL

A non-local formulation results from replacing an internal variable, as the accumulated plastic strain p, by a weighted average on a characteristic volume V_C:

$$\tilde{p}(\boldsymbol{X}) = \frac{1}{V_C} \int_{V_C} p(\boldsymbol{y}) \phi(\boldsymbol{y}; \boldsymbol{X}) dV \tag{1}$$

where $\phi(\boldsymbol{y}; \boldsymbol{X})$ are normalized weight functions. An elegant way to avoid the explicit integral evaluation during a finite element implementation is to reformulate this equation in an implicit way [14]

$$\tilde{p}(\boldsymbol{X}) - c\nabla \cdot \nabla \tilde{p}(\boldsymbol{X}) = p(\boldsymbol{X}) \tag{2}$$

where c comes from the integration of equation (1), and has the unit of the squared length. This new Helmholtz-type equation is completed by an appropriate boundary condition

$$\nabla \tilde{p} \cdot \boldsymbol{n} = 0 \tag{3}$$

on the whole body surface, where $\boldsymbol{n}$ is the outward unit normal. This model allows defining a non-local form of the accumulated plastic strain, which can be used to evaluate a damage evolution law without inducing loss of solution uniqueness at strain softening onset, see next section.

However, the set of equations (2)-(3) is characterized by a unique length scale of the material in all the directions. As we intend to combine this model with a MFH framework for which characteristic dimensions are advantageously linked to the fibers orientation, this non-local implicit model is reformulated in an anisotropic framework following our developments in [16]. Thus the governing equation reads

$$\widetilde{p} - \nabla \cdot \left(\boldsymbol{c}_{\mathrm{g}} \cdot \nabla \widetilde{p} \right) = p \tag{4}$$

with the new boundary condition

$$\left(\boldsymbol{c}_{\mathrm{g}} \cdot \nabla \widetilde{p} \right) \cdot \boldsymbol{n} = 0 \tag{5}$$

In these last two equations, the symmetric second order tensor $\boldsymbol{c}_{\mathrm{g}}$ represents the distribution of characteristic length sizes in the global axes where the finite element problem is formulated. If in the axes linked to the composite ply, for example with one axis along the fibers direction, the characteristic lengths are l^1 along the fibers, l^2 and l^3 in the two directions perpendicular to the fibers, and if $\boldsymbol{R}$ is the rotation matrix representing the change of orthonormal coordinates from global to local, then this tensor reads

$$\boldsymbol{c}_{\mathrm{g}} = \boldsymbol{R}^T \cdot \mathrm{diag}\left(\left(l^1\right)^2, \left(l^2\right)^2, \left(l^3\right)^2 \right) \cdot \boldsymbol{R} \tag{6}$$

Boundary condition (5) states that the non-local accumulated plastic strain gradient should be aligned with the fiber directions.

3. MFH WITH GRADIENT-ENHANCED DAMAGE MODEL

This section summarizes the extension of the non-local formulation of MFH with damage [13] to the finite-element analyzes of laminated composite.

3.1. Incremental MFH

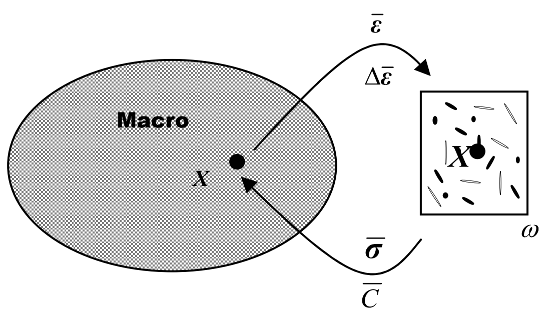


Fig. 1 Multiscale method

In the multiscale framework developed, at each macro-point $\boldsymbol{X}$ of the structure, the macro-strain tensor $\bar{\boldsymbol{\varepsilon}}$ is known, and the macro-stress tensor $\bar{\boldsymbol{\sigma}}$ is sought from the resolution of a micro-scale boundary value problem (BVP) as illustrated on Fig. 1.

The Hill-Mandell condition, expressing the equality between energies at both scales, transforms the relation between macro-strains and stresses into the relation between average strains and stresses over the RVE. For a two-phase isothermal composite with the volume fractions $v_0 + v_\mathrm{I} = 1$ (subscript 0 refers to the matrix and I to the inclusions), the average quantities are expressed in terms of the phase averages as

$$\bar{\boldsymbol{\varepsilon}} = v_0 \langle \boldsymbol{\varepsilon} \rangle_0 + v_\mathrm{I} \langle \boldsymbol{\varepsilon} \rangle_\mathrm{I} \tag{7}$$

$$\bar{\boldsymbol{\sigma}} = v_0 \langle \boldsymbol{\sigma} \rangle_0 + v_\mathrm{I} \langle \boldsymbol{\sigma} \rangle_\mathrm{I} \tag{8}$$

In the non-linear range no direct relation between the macro-strain tensor $\bar{\boldsymbol{\varepsilon}}$ and the macro-stress tensor $\bar{\boldsymbol{\sigma}}$ can be derived, and a linear comparison composite (LCC) is introduced by linearizing the behaviors of the two phases around the current strain state. Thus, for a given time step $[t_\mathrm{n}, t_\mathrm{n+1}]$, relations (7) and (8) are rewritten in an incremental form

$$\Delta\bar{\boldsymbol{\varepsilon}} = v_0 \langle \Delta\boldsymbol{\varepsilon} \rangle_0 + v_\mathrm{I} \langle \Delta\boldsymbol{\varepsilon} \rangle_\mathrm{I} \tag{9}$$

$$\Delta\bar{\boldsymbol{\sigma}} = v_0 \langle \Delta\boldsymbol{\sigma} \rangle_0 + v_\mathrm{I} \langle \Delta\boldsymbol{\sigma} \rangle_\mathrm{I} \tag{10}$$

Mori-Tanaka (M-T) assumption allows relating the strain increments in the different phases from

$$\langle \Delta\boldsymbol{\varepsilon} \rangle_\mathrm{I} = \boldsymbol{B}^{\varepsilon}\left(I, \overline{C}_0^{\mathrm{alg}}, \overline{C}_\mathrm{I}^{\mathrm{alg}}\right) : \langle \Delta\boldsymbol{\varepsilon} \rangle_0 \tag{11}$$

with the average values of the consistent algorithmic tangent modulii of the inclusions $\overline{C}_\mathrm{I}^{\mathrm{alg}}$ and of the matrix phase $\overline{C}_0^{\mathrm{alg}}$,

and with $\boldsymbol{B}^{\varepsilon}\left(I, \overline{C}_0^{\mathrm{alg}}, \overline{C}_\mathrm{I}^{\mathrm{alg}}\right)$ the Eshelby tensor [4]. Thus, the system of equations (9)-(11) simplifies into

$$\Delta\bar{\sigma} = \underbrace{\left[v_\mathrm{I} \overline{C}_\mathrm{I}^{\mathrm{alg}} : \boldsymbol{B}^{\varepsilon} + v_0 \overline{C}_0^{\mathrm{alg}}\right] : \left[v_\mathrm{I} \boldsymbol{B}^{\varepsilon} + v_0 \boldsymbol{I}\right]^{-1}}_{\breve{C}} \Delta\bar{\boldsymbol{\varepsilon}} \tag{12}$$

Finally, if the resolution scheme involves Newton-Raphson iterations, the Jacobian form of (12) is developed under the form

$$\delta\bar{\sigma} = \overline{C}^{\mathrm{alg}} : \delta\bar{\boldsymbol{\varepsilon}} \tag{13}$$

3.2. MFH with gradient-enhanced damage

In this section, the < > symbols are omitted for clarity, but the values are used in the mean sense. The damage model of Lemaître-Chaboche [17] is considered for the matrix phase. This allows defining an effective stress

$$\hat{\sigma}_0 = \frac{\sigma_0}{(1-D)} \tag{14}$$

where D is a representation of the damage in the matrix. The damage evolution reformulated in the non-local way obeys the law

$$\dot{D} = \begin{cases} 0 & \text{if} \quad \tilde{p} \le p_C \\ \left(\dfrac{\boldsymbol{\varepsilon}_0^{\mathrm{el}} : \boldsymbol{C}_0^{\mathrm{el}} : \boldsymbol{\varepsilon}_0^{\mathrm{el}}}{2S_0} \right)^s \dot{\tilde{p}} & \text{if} \quad \tilde{p} > p_C \end{cases} \tag{15}$$

where s, S_0 and p_C are model parameters. The damage evolution depends on the non-local accumulated plastic strain in the matrix, which is governed by relations (4)-(5).

From relations (14)-(15), it appears that the linearization of $\boldsymbol{\sigma}_0$ depends on $\boldsymbol{\varepsilon}_0$ and on $\tilde{p}$. Thus, the linearization of (10) reads

$$\delta\overline{\boldsymbol{\sigma}} = v_{\mathrm{I}}\overline{C}_{\mathrm{I}}^{\mathrm{alg}} : \delta\overline{\boldsymbol{\varepsilon}}_1 + v_0 \underbrace{(1-D)\overline{C}_0^{\mathrm{alg}}}_{\overline{C}_0^{\mathrm{algD}}} : \delta\boldsymbol{\varepsilon}_0 - v_0\left(\hat{\boldsymbol{\sigma}}_0 \otimes \frac{\partial D}{\partial \boldsymbol{\varepsilon}_0}\right) : \delta\boldsymbol{\varepsilon}_0 - v_0\hat{\boldsymbol{\sigma}}_0 \frac{\partial D}{\partial \tilde{p}} : \delta\tilde{p} \tag{16}$$

The first two terms at the right hand side of this linearization represent the stress variation of a composite with a fictitious matrix material of constant non-local damage. In this work we assume that the incremental Mori-Tanaka process can be applied on these first two terms. The last two terms are related to the softening of the matrix due to the damage in the matrix, and are not considered in the Mori-Tanaka process. Indeed M-T is only defined when the two tangent moduli are definite positive, which would not be the case in the softening part, see [13].

Thus, the MFH process with gradient enhanced damage during a time step $[t_n, t_{n+1}]$ is defined as follows. Knowing the internal variables, macro-strain tensor $\overline{\boldsymbol{\varepsilon}}_n$, macro-stress tensor $\overline{\boldsymbol{\sigma}}_n$ and non-local accumulated plastic strain $\tilde{p}_n$ at time t_n, for given increments in the macro-strain tensor $\Delta\overline{\boldsymbol{\varepsilon}}_{n+1}$, and in the non-local accumulated plastic strain $\Delta\tilde{p}_{n+1}$, the system to be solved reads

$$\Delta\overline{\boldsymbol{\varepsilon}}_{n+1} = v_0\Delta\boldsymbol{\varepsilon}_{0\,n+1} + v_{\mathrm{I}}\Delta\boldsymbol{\varepsilon}_{\mathrm{I}\,n+1} \tag{17}$$

$$\overline{\boldsymbol{\sigma}}_{n+1} = v_0\boldsymbol{\sigma}_{0\,n+1} + v_{\mathrm{I}}\boldsymbol{\sigma}_{\mathrm{I}\,n+1} \tag{18}$$

$$\Delta\boldsymbol{\varepsilon}_{\mathrm{I}\,n+1} = \boldsymbol{B}^{\varepsilon}\left(I, \overline{C}_0^{\mathrm{algD}}, \overline{C}_{\mathrm{I}}^{\mathrm{alg}}\right) : \Delta\boldsymbol{\varepsilon}_{0\,n+1} \tag{19}$$

The resolution of this system is detailed in [13].

4. FINITE ELEMENT IMPLEMENTATION

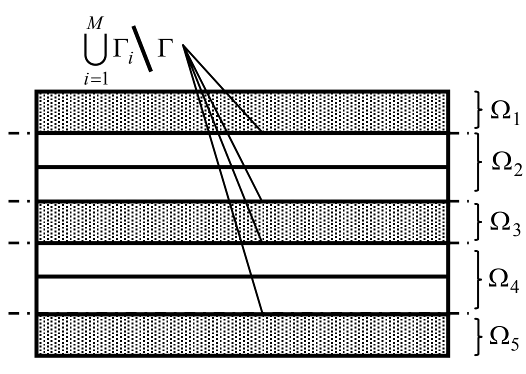


Fig. 2 Description of a laminate

The strong form of the problem is stated by the governing equations in the body Ω

$$\nabla \cdot \boldsymbol{\sigma}^T + \rho\boldsymbol{f} = 0 \tag{20}$$

$$\tilde{p} - \nabla \cdot \left(\boldsymbol{c}_{\mathrm{g}} \cdot \nabla\tilde{p}\right) = p \tag{21}$$

with the boundary conditions

$$\boldsymbol{U} = \overline{\boldsymbol{U}} \quad \text{on} \quad \Gamma_D \tag{22}$$

$$\boldsymbol{\sigma} \cdot \boldsymbol{n} = \overline{\boldsymbol{T}} \quad \text{on} \quad \Gamma_T \tag{23}$$

$$\left(\boldsymbol{c}_{\mathrm{g}} \cdot \nabla \widetilde{p}\right) \cdot \boldsymbol{n} = 0 \quad \text{on} \quad \Gamma_i \tag{24}$$

In these last equations Γ_D is the Dirichlet part of the whole body boundary and Γ_T its Neumann part. However, the boundary condition (24) should be enforced on the body surface Γ, but also on each interface between different material behaviors. In the case of a laminated structure, the different domains with different material behaviors correspond to the different plies with different fibers orientations see Fig. 2.

The finite element weak formulation of the problem is detailed in [16] and results to a system with four degrees of freedom per node: 3 displacements and one non-local accumulated plastic strain. However, condition (24) has to be satisfied on each interface between different material behaviors. This is accomplished by considering the field corresponding to $\widetilde{p}$ discontinuous at domain interfaces. This is achieved simply by defining for each node belonging to the interface, besides the three displacement degrees of freedom, 2 degrees of freedom corresponding to $\widetilde{p}$, one for the domain 1, one for the domain 2 (*n* in case of an interface between *n* domains).

5. NUMERICAL APPLICATIONS

Table 1 Properties of the composite constituents

Expoxy		Carbon Fibers	
E_0 [GPa]	2.89	E_{L} [GPa]	230
ν [-]	0.3	E_{T} [GPa]	40
σ_Y [MPa]	35	ν_{TT} [-]	0.2
h_0 [MPa]	73	ν_{LT} [GPa]	0.256
m [-]	60	G_{TT} [GPa]	16.7
S_0 [MPa]	2	G_{LT} [GPa]	24
s [-]	0.5		
p_C [-]	0		

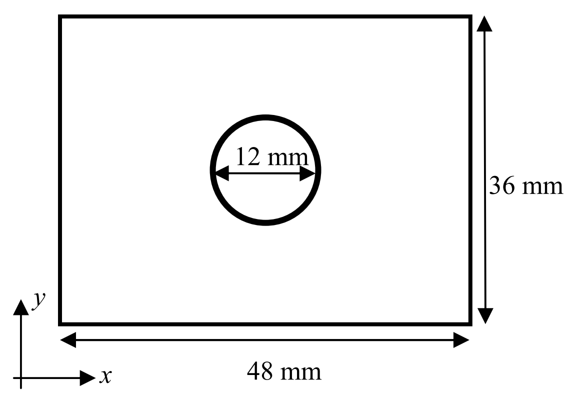
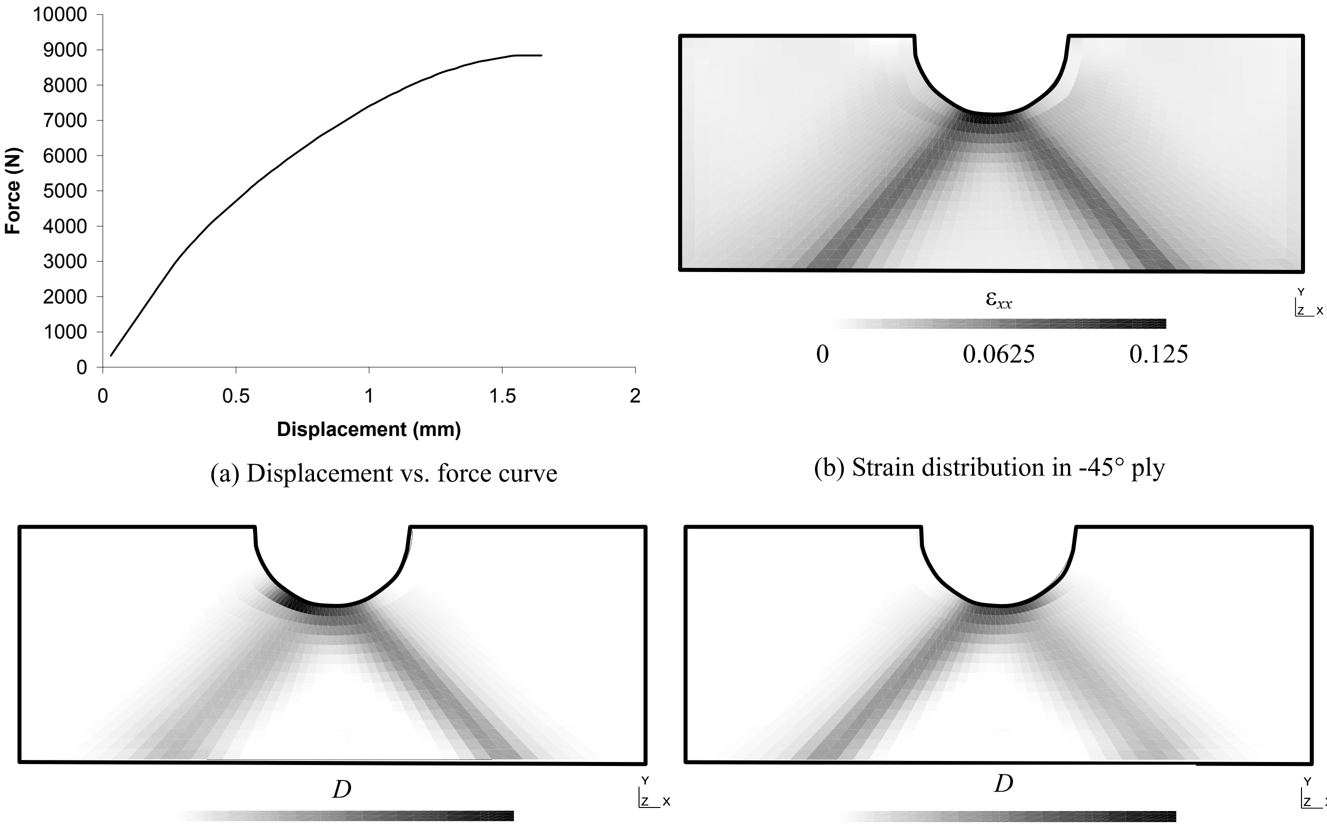


Fig. 3: Geometry of the laminate

In this section we consider composite structures made of continuous carbon fibers reinforced epoxy. The material is made of prepreg Hexply M10.1/38% / UD300/ HS (R), which results in a fiber volume fraction of 52.5%. Each ply has a thickness of 0.3 mm. The properties of the epoxy matrix are reported in Table 1, with a hardening law following an exponential law

$$R(p) = h_0\left(1 - e^{-mp}\right) \tag{25}$$

and a damage law following (15). The carbon fibers are linear elastic and transversely isotropic with the mechanical properties given in Table 1. A laminate plate with the stacking sequence $(-45)_2/(45)_4/(-45)_2$ has a hole in its center, see Fig. 3.



(a) Displacement vs. force curve

(b) Strain distribution in -45° ply

(c) Damage distribution in -45° ply

(d) Damage distribution in 45° ply

Fig. 4: Numerical results for the tension of a laminate with hole

Numerical results of a tensile test on the laminate with a hole are presented in Fig. 4. Although the plate is not symmetric due to the fiber orientations, the balanced distribution of the plies induces a quasi-symmetric stress/strain distribution at the macro-scale and only one half of the plate is represented. The deformation distribution at maximal force, in Fig. 4a, is presented in Fig. 4b. Strain concentration occurs in bands parallel to the fiber orientations. Similarly, the damage distributions in the -45°, Fig. 4c and in the +45°, Fig. 4d, plies exhibit bands, which initiate at the hole, and which propagate with the fiber orientations, respectively -45° and 45°. Due to the balanced stacking sequence, there is a second band in each ply, of lower intensity, which is parallel to fiber orientation of the other plies, respectively along 45° and -45°. It also appears that the location of the band initiation at the hole differs in the -45°, Fig. 4c and in the +45°, Fig. 4d, plies.

6. CONCLUSIONS

In order to analyze failure of laminated composite structures, a gradient-enhanced mean-field homogenization (MFH) procedure has been developed. In this approach, the fibers are assumed to remain linear elastic while the matrix material obeys an elasto-plastic behavior enhanced by a damage model. Loss of solution uniqueness at onset of strain softening has been avoided by using a non-local implicit approach, reformulated in an anisotropic way, to describe the damage in the matrix. As a result we have a multi-scale model that can model the damage evolution in each ply of a composite stack. As a demonstration a stack with a hole is studied, and it is shown that the model predicts the damaging process in bands oriented with the fibers directions.

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