UGR-Liège 2011

"Multiperiod vehicle loading optimization with stochastic supply"

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LIEGE-BELGIUM

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1. Main Message

From manual optimization over decomposed deterministic sub-problems to a multi-period stochastic policy

"Local optima over current data
vs
global policy including uncertainty"

Sénèque : *"It is not because things are difficult that we do not try, it is because we do not try that things are difficult"*

G. Santayana : *"What is difficult is what can be done straight away. What is impossible is what takes some more time"*

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1. Main Message : Litterature

New blend of wellknown OR problems and techniques

Closest problem Petrol Stations Replenishment (Laporte)
- for the **Problem** : bin-packing
- for the **Model** : set-covering
- for the **Optimization technique** CPLEX default setting
- for the **Stochastic optimization** terminology (Birge and Louveaux)
- for the heuristics **Consensus algorithms** (R. Bent and P. Van Hentenryck)

2. Industrial Motivation

Coils to be loaded on truck : **BIN-PACKING**
Objective function *min cost* :
Truck (fixed + tons) + Penalty for double un/loadings
Most expensive customer if 2
Constraints : Weight constraint
Usually 1-2, sometimes 3, exceptionally 4 coils per truck

Data :
1 production site Liège (B) with several warehouses
800 customers in Europe (Mostly Germany and France)
350 trucks per day

Manually Intractable (Feasible Y, Optimal N)

3. Rules and Manual Optimization

Consequence : Problem decomposed over

1. Time = period per period with the current stock
2. Space = ZIP code, lander or department
3. Customer = customer per customer

RULES : DIVIDE TIME AND SPACE TO GET SMALLER SUB-PROBLEMS

Results : Tractable instances manually optimized including up to 10 coils representing 7-8 trucks

4.1 Model : Set-covering

MIP approach to handle larger instances

Indices : i for M coils, j for N patterns

Parameters :

- A_{ij} pattern j contains coil i Truckload
- C_j cost of shipping pattern j

Variables : $x_j \in \{0,1\}\ \forall j = 1, ..., N$

Objective Function : $min\ Z = min \sum_{j=1}^{j=N} C_j x_j$

Constraints :

$\sum_{j=1}^{j=N} A_{ij} x_j \geq 1 \forall i = 1, ...M$ every coil is sent

Advantages : Pattern includes weight constraint, pattern costs penalties and complex truck cost function

Options : differents kinds of trucks and fleet size limits

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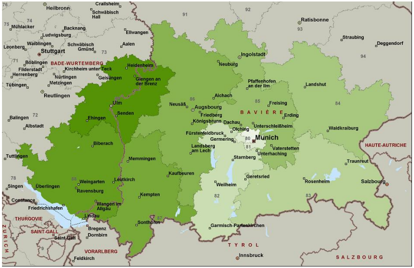
4.2 Results

Merge ZIP codes or departments (up to 10) to create large sizes instances up to 100 coils and act over

SPACE



Optimization technique : EXACT

Patterns Generation and Set Covering Problem

Generation of all feasible loaded trucks, their costs and selection of the cheapest composition (CPLEX)

4.2 Results

In Bavaria compare to individual optimization on ZIP Codes 80 to 89, over industrial instances,

1. the **number of trucks** is reduced by 16,9%
2. and the **cost** by 12,7 % (double unloadings)

These are well-known techniques.

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4.3 Extensions

Computational limits for a small items instance
Capacity : 23.5T
Weight : min 4.5 T (Max 4 coils), max 8-12-15-19T
Penalties : 2 un/loadings Time limit : 50 sec
Network : only 3 clients and 3 depots
Coils : 20, 40, 60, 80 Test : 20 instances

See Monoperiod Analysis.XLS (Data and Graphs)
Opportunities

- Many patterns => RAM Limit => Column generation
- New dimension **TIME** => Multi-period setting

NB : Multi-period is not Periodic ! (Bus, Train...)

Creation of a new model taking into account **production forecasts over a rolling horizon H**

Penalties related to Time Windows : INV, EAR, LATE

5.1 Theoretical cases

Extreme case : Improvement ratio = T

TW length T

Truck capacity C

Coil weight $\leq$ C/T

1 coil per period

One-period : T * 1 truck with 1 coil per period = T trucks

Multi-period : 1 truck with T coil of weight C/T = 1 truck

	Periods					
Coils Weight	**P1**	**P2**	**P...**	**Pi**	**P...**	**PT**
A *C/T*	1		...			
B *C/T*		1	...			
... *C/T*			...			
i *C/T*				1		
... *C/T*					...	
T *C/T*						1

e.g : T = 2 periods

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5.1 Theoretical cases

4 coils case with penalties and limits

	Periods				
Coils Weight	**P1**	**P2**	**P3**	**P4**	**P5**
A *0.6*	1	*LAT*			
B *0.8*		1	*EAR*	*TW*	*LAT*
C *0.2*	1	*TW*	*TW*	*TW*	*LAT*
D *0.4*	1	*EAR*	*TW*	*LAT*	

- P-INV < P-EAR < P-LAT < Truck cost
- late or early delivery TW +/- 1 period P-EAR or P-LAT
- not allowed before EAR and after LAT (semi-soft TW)
- one period delivery time

Decisions : WAIT or SEND available coils in P1
e.g. : AC(P1) + D(P2) + B(P3) vs AD(P1) + BC(P3)
P-LAT + 1 P-INV + 1 P-INV vs P-LAT + P-EAR + 3 P-INV

5.2 Biases H, Policy and "End of horizon"

Model formulation : set covering problem of patterns

A pattern is a truckload of coils
At any time *t*, a given pattern is available or not

Pattern cost indexed by *t* includes the truck cost based on the weight + Un/loads + INV, EAR, LATE
For any pattern there is a cheapest shipping period !
Consequence : model size is reduced (RAM)
Implementation lead to :

3 biases

1. **H rolling horizon** length or look-ahead periods
2. **Policy** not a solution
3. **End of horizon** (not same coils send)

5.2 Biases H, Policy and "End of horizon"

A Policy is an iterative process that generates a sequence of decisions and not a full-horizon planning :

1. Evaluate the best decision over P1 to P(1+H)
2. Implement decisions for P1 Always feasible ! !
3. Update extra period P(2+H) and remaining coils
4. Reevaluate the best decision from P2 to P(2+H)
5. Implement new P2 repeat...until... P(i+H)=End

Rolling horizon and End of horizon

e.g : (H=3P over 6P) 1-3,2-4,3-5,4-6 extra periods (5,6)
Remaing costs for P5 and P6 are not included
=> Average variable expedition costs $/T and 22P

Optimal rolling horizon length H

Look-ahead periods add information for decisions in P1
H large, more patterns => RAM problem and CPU time
Tests : H=5P (tractable) TW=4P [Ear, Inv1, Inv2, Late]
H must be evaluated according to instances classes
Results H 40-60.XLS

5.3 Fleet cut and Weighted Matching Problem

Aim : Reduce the CPU Time

Important for testing, in real life just optimization of P1

Fleet cut for the B&B

Set a constraint linked to the minimum of truck found

Results : Slower and worst => Cut effect.XLS

- 7-8 cuts, but checked at every step and still B&B
- Cut not strictly valid

Weighted matching problem

Polynomial time algorithm (Edmond's or Blossoms)

Drawback : only valid for 1-2 coils per truck, not practical

State of the art version (Kolgomorov) to be embedded

=> Kept as an option for future work

Minimum cost flow problem

If we remove some penalties...

6.1 Theoretical case : 2 coils

Forecasts contain uncertainty on production availability
Example : 2 coils cases

Weights	P1	P2	P3	P4
A *0.5*	1	**TW**	Late	X
B *0.5*		0.49	0.51	**TW**

Stochastic :
SEND A en (P1) : cost 2 trucks
WAIT A en (P1) : cost P-INV(A) +
If B is available in P2 : 1 truck (AB) + P-LATE(A) + P-EAR(B)
If B is available in P3 : 1 truck (A) + P-LATE(A) + 1 truck (B)
Average : cost 1.5 truck + penalties

Weights	P1	P2	P3	P4
A *0.5*	1	**TW**	Late	X
B *0.5*			1	**TW**

"Deterministic approach" : "Modal Period" 0,51 => 1
SEND A in (P1) : cost 2 trucks

6.1 Theoretical case : 4 coils

Independent and Identically Distributed I.I.D.

Weights	P1	P2	P3	P4
A *0.6*	1	**TW**	Late	X
B *0.6*	1	**TW**	Late	X
C *0.4*		0.49	0.51	**TW**
D *0.4*		0.49	0.51	**TW**

Stochastic I.I.D. (4 scenarios)
SEND A,B or WAIT A, SEND B in P1 : cost 3 trucks
WAIT A and B in P1 : cost P-INV + P-LAT(A and B)
If C and D unavailable(1) or if C (2) ou D (3) available in P2 :
2 trucks (A,B) + 1 truck(CD) + P-INV(C or D)
If C and D available in P2 (4) : 2 trucks
Average : $(3 * 3 + 1 * 2)/4 =$ **2.75** trucks + penalties
Stochastic Non Independant.I.D. (2 scenarios)
WAIT A and B in P1 : cost P-INV + P-LAT(A and B)
If C and D unavailable in P2 (1)
2 trucks (A,B) + 1 truck(CD) + P-INV(C or D)
If C and D available in P2 (2) : 2 trucks
Average : $(3 + 2)/2 =$ **2.5** trucks + penalties

DECISION : only I.I.D. cases => All scenarios

6.2 Optimal representation : Scenario Tree

	Periods				
Weights	**P1**	**P2**	**P3**	**P4**	**P5**
A *0.6*	1	*TW*			
B *0.8*		0.9	0.1	*TW*	
C *0.3*			0.2	0.8	
D *0.2*	1	*TW*	*TW*	*TW*	
E *0.4*	1		*TW*		

New objective function : *"Minimize expected cost"* **E***
Scenarios tree : Deterministic equivalent with scenarios and non-anticipativity constraints (IP Problem)

- e.g. : 4 scenarios
  1. B(P2) C(P3) Pr(0.18)
  2. B(P2) C(P4) Pr(0.72) Modal Periods EMod
  3. B(P3) C(P3) Pr(0.02)
  4. B(P3) C(P4) Pr(0.08)

Scenarios tree expected cost should be better than EMod
Aim : find a model or algorithm that approximates E*

6.2 Optimal representation : Scenario Tree

Drawback : huge number of scenarios
Limit for optimization 12 coils, H=3 periods
Distribution law over 2 periods, 2^{12} scenarios
=> Intractable (CPLEX)
"Heuristic" because of the model and/or the method
Basic ideas : Simplified solution in P1 is valid for the whole problem in P1

1. A representative scenario : EMod or EMean
2. A subtree of scenarios (few non-anticipativity links)
3. A subset of independant scenarios aggregated in a consensus solution

SCENARIOS SELECTION

1. Monte-carlo random generation of scenarios
2. Stratified generation of scenarios (no worst case)

SOLUTION VALIDATION

- Variance due to scenario sampling (N=30)
- Compare policies from a collection of results

6.3 Models O*, EMod, EMean, C1, LO

Heuristics comparaizon H = 5, TW = 4, N = 30

1. **O*** "Knowledge of God", full information revealed **LB**
2. **EMod** Deterministic equivalent modal period
3. **EMean** Deterministic equivalent expected period
4. **C1** Consensus : Send in P1 if Yes $\geq$ 6 /10 scenarios => Send only those coils in P1 (LO) Always feasible !
5. **LO** One-period only P1 (revealed info) ***UB***

The gap between **O* (LB)** and **LO (UB)** gives the **"Value of the multi-period and perfect information model"**

The gap between **O*** and **E*** gives the **"Value of the Perfect Information VPI"**

The min gap between **O*** and **EMod, EMean, C1** gives an **"Upper Bound on the VPI"**

6.4 Distribution laws and Results

No industrial data for a distribution law

Test 1 : Distribution law 2P Random [40,60], TW 4P
=> Value of a multi-period model
=> Simple heuristics are similar to perfect information O*
Results 2P O E C LO.XLS
When number of coils increases

- Average cost decreases
- Value of the multi-period model decreases
- Value of perfect information decreases
- Computing time increases

Test 2 : Distribution laws 4P over TW 4P in %

1. Uniform [25,25,25,25]
2. "Early" [40,30,20,10] Positive Skew
3. "Late" [10,20,30,40] Negative Skew
4. Binomial [12.5,37.5,37.5,12.5]

6.4 Distribution laws and Results

See RESULTS 10 C.XLS
=> High value of multi-period and perfect information model
=> Quality of solutions is better for early information
=> Quality of solutions is better with number of coils
=> Average cost reduces with number of coils
=> Average cost increases with a negative skew law
Consensus C1 is not better over all distributions laws

Improvement of our "consensus" ?

C2 : C1 send + coils available in P1 for free

Other idea : "local expectation" evaluation

C3 : cross-evalution of one decision over other scenarios
As any decision in P1 is valid for any other scenario, we evaluate the cost of applying solution 1 in scenario 2, 3...
The decision that generates the lowest cost over those scenarios is applied

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6.4 Distribution laws and Results

No heuristics outperforms the others
C2 seems better than C1
Yet, C3 never underperforms and often outperforms
Is C3 a "robust" algorithm over all distribution laws ?
Is C3 never statistically overclassed by an other algorithm ?
Is C3 the only algorithm that is never overclassed ?
Statistical comparison of means !
(Work in progress...)

7. Conclusions and Perspectives

Conclusions

- New model Transportation/Production
- Pattern generation seems an appropriate formulation
- Multi-period model is better than One-period model
- Rolling horizon H depends on TW, (H=5P,TW=4P)
- Distribution laws and number of coils seems to influence the quality of the solution obtained for the VPI

Perspectives

- Check statistical performances of heuristics
- Managerial advices linked to instances classes
- Introduce Non-Independent I.D. distribution laws
- Add an error on distribution law to test "Robustness" Real distribution law different from expected law
- Weighted Matching Problem (Theoretical approach)

7. Conclusions and Perspectives

Thank you for
Global opinion (:-)) (:-()
Questions ?
Advices !
Remarks ! ! !
Comments...