

ORBEL-24 HEC-ULg 2010

"Vehicle loading optimization with stochastic supply"

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LIEGE-BELGIUM

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Main Message

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1. Main Message

From manual optimization over decomposed deterministic sub-problems to a multi-period stochastic policy.

**"Local optima over available data
vs
global policy including uncertainty".**

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Main Message

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2. Industrial Case

Coils to be loaded on truck : **BIN-PACKING**

Objective function *min cost* :

Truck (fixed + tons) + Penalty for double un/loadings

Constraints :

Weight constraint usually 1-2, sometimes 3 coils per truck



Data :

1 production site Liège (B)

800 customers in Europe (Germany and France)

350 trucks per day

MANUALLY UNTRACTABLE

3. Rules and Manual Optimization

Consequence : Problem decomposed over

1. Time = period per period with the actual stock
2. Space = ZIP code, lander or department
3. Customer = customer per customer

RULES : DIVIDE TIME AND SPACE TO GET SMALLER SUB-PROBLEMS

Results : Tractable instances manually optimized including up to 10 coils representing 7-8 trucks

4. One-period Deterministic : METHOD

- ▶ MIP approach to handle large instances
- ▶ Merge ZIP codes or departments (up to 10) to create large sizes instances up to 100 coils and act over

SPACE



Optimization technique : EXACT

Patterns Generation and Set Covering Problem

Generation of all feasible loaded trucks, their costs and selection of the cheapest composition (Cplex)

4. One-period Deterministic : MODEL

Indices : i for M coils, j for N patterns

Parameters :

- ▶ A_{ij} pattern j contains coil i
- ▶ C_j cost of shipping pattern j

Variables : $x_j \in \{0, 1\} \forall j = 1, \dots, N$

Objective Function : $\min Z = \min \sum_{j=1}^{j=N} C_j x_j$

Constraints :

$\sum_{j=1}^{j=N} A_{ij} x_j \geq 1 \forall i = 1, \dots, M$ every coil is sent

Advantages : Pattern includes weight constraint, pattern costs penalties and complex truck cost function

Option : different kinds of trucks and fleet size limits

4. One-period Deterministic : RESULTS

In Bavaria compare to individual optimization on ZIP Codes 80 to 89, over industrial instances,

1. the **number of trucks** is reduced by 16,9%
2. and the **cost** by 12,7 %

These are well-known techniques.

New dimension **TIME** : multi-period setting

Creation of a new model that takes into account **production forecasts** over a rolling horizon.

Penalties related to time : Inventory and Time-Windows

5. Multi-period Deterministic

Coils Weight	Periods				
	P1	P2	P3	P4	P5
A <i>0.6</i>	1	TW	Late	X	X
B <i>0.8</i>		1	Early	TW	Late
C <i>0.2</i>	1	TW	TW	TW	Late
D <i>0.4</i>	1	Early	TW	Late	X

Costs, penalties and limits

- ▶ inventory cost P-INV
- ▶ late or early delivery TW +/- 1 period P-EAR or P-LAT
- ▶ not allowed TW \geq +/- 2 periods
- ▶ one period delivery time
- ▶ P-INV < P-EAR < P-LAT < Truck cost

DECISIONS : WAIT or SEND for coils in PERIOD 1

e.g. : AC(P1) + D(P2) + B(P3) vs AD(P1) + BC(P3)

TW (OK) + 1 P-INV + 1 P-INV vs P-EAR + 2 P-INV

5. Multi-period Deterministic

Model formulation

Pattern generation and set covering problem

More efficient than alternative formulations

A pattern is a truckload of coils which are available at a given time t

It means from availability date to last $TW + 1 P$

Pattern cost indexed by t includes as a basis the truck cost based on the load + the penalties : inventory, earliness or lateness

The **set covering problem** is similar to the previous one with a new index t

Current implementation : patterns are generated off-line, no delayed column generation technique or dedicated set-covering algorithm

Tractable 100 coils over 3 periods

6. Multi-period Stochastic

Forecasts contain uncertainty on production availability

Coils Weight	Periods				
	P1	P2	P3	P4	P5
A 0.6	1	TW	Late	X	X
B 0.8		0.9	0.1	TW	Late
C 0.3			0.2	0.8	TW
D 0.2	1	TW	TW	TW	Late
E 0.4	1	Early	TW	Late	X

New objective function : *"Minimize expected cost"*

Scenarios tree : Deterministic equivalent with scenarios and non-anticipativity constraints

- ▶ e.g. : 4 scenarios
 1. B(P2) C(P3) Pr(0.18)
 2. B(P2) C(P4) Pr(0.72)* Most likely available
 3. B(P3) C(P3) Pr(0.02)
 4. B(P3) C(P4) Pr(0.08)

Policy : find solution for P1, implement, update data (remove P1, add a P) and repeat over the rolling horizon

6. Multi-period Stochastic : Worst case

Stochastic vs Deterministic "Most likely available"

Coils Weight	P1	P2	P3	P4
A 0.5	1	TW	Late	X
B 0.5		0.49	0.51	TW

Solutions for stochastic case : SEND A in (P1) : cost 2 trucks

WAIT A in (P1) : cost : P-INV(A) +

If B is available in P2(0.49) : 1 truck (AB) + P-LATE(A) + P-EAR(B)

If B is available in P3(0.51) : 1 truck (A) + P-LATE(A) + 1 truck (B)

Average cost : 1.5 truck + penalties

Coils Weight	P1	P2	P3	P4
A 0.5	1	TW	Late	X
B 0.5			1	TW

Solution for "deterministic" SEND A in (P1) : cost 2 trucks

GAP for optimization : 25% and it can be generalized for big sets

Remarks :

- depends on the **distribution function profile**
- if coils fit well together in P1, optimization gap is reduced
- with un/loading constraints, the gap increased up to 27,5% (WIP)

7. Scenario Sampling

Drawback : huge number of scenarios

Limit for optimization over all scenarios 22 coils 3 periods

Sample to solve over a subset of scenarios

SCENARIOS SELECTION

1. Monte-carlo random generation of scenarios
2. Stratified generation of scenarios

SAMPLE SIZE

1. use the largest sample size that we can handle
1 x [N] e.g. : N = 1-20-50-100 scenarios

SOLUTION VALIDATION

- ▶ Variance due to scenario sampling
- ▶ Compare policies from a collection of results

8. Results

5 solutions techniques or policies

1. One-period Deterministic repeated on every period
2. Multi-period Deterministic with most likely availability
3. Stochastic with all scenarios
4. Stochastic with sampling (Monte-Carlo generation)
5. Stochastic with stratified sampling

Cost function : coils sent and expected cost for not sent

Instances : 22 coils, 6 periods, rolling horizon P1, P2, P3

Sample size : 10 - 50 compare to thousands

Computation time : including scenario generation

Policies	Sample size	Value	Computation Time
One-period Deter.		6750	0.1 sec
Multi-period Deter.	(1)*	6202	0.1 sec
Stoch. All Scenarios	All	6192	417 sec
Stoch. Samp. Rand.	10	6192	2.5 sec
Stoch. Samp. Rand.	50	6197	3.1 sec
Stoch. Samp. Strat.	10	6200	1 sec
Stoch. Samp. strat.	50	6189	4.2 sec

8. Results

Theoretical case, based on the worst case
5 coils in P1, 5 coils in P2(40%) or P3(60%)

Policies	Sample size	Value
One-period Deter.		5726
Multi-period Deter.	(1)*	5726
Stoch. All Scenarios	All	4901
Stoch. Samp. Rand.	10 (32)	4966
Stoch. Samp. Strat.	10 (32)	4896

Deterministic "Most likely availability" = singlestage
Theoretical gap (without penalty) : 15.6%
Practical result (with low penalty cost) : 14.4%

Policies decisions :

- "All scenarios" Send 1 Wait 4 (due to penalties)
- Scenario samplings methods send from 1 to 3

9. Conclusions and Perspectives

Conclusions

- ▶ New model Transportation/Production
- ▶ Pattern generation seems an appropriate formulation to include penalties
- ▶ Determine the value added by successive enrichments of the model from deterministic to multi-period to stochastic

Perspectives

- ▶ Perform more computational tests to handle realistic instances
- ▶ Compare the policies using larger instances
- ▶ Add a delayed column generation technique and a dedicated algorithm for the set-covering
- ▶ Use consensus functions which aggregate several solutions obtained from different scenario samples