A Capacity Game in Transportation Management

Guillaume Amand and Yasemin Arda

Abstract

Emerging concerns about competitiveness induce a growing number of firms to outsource their outbound transportation operations to 3PL service providers. The resulting increase in the number of actors often leads to sub-optimal supply chain actions due to the antagonistic nature of the economic objectives of the partners. With the aim of determining possible deviations from the optimal system performance in such supply chains, this study analyzes the contractual relation between a retailer and a 3PL service provider using game theoretical approaches.

The partners of the studied supply chain play a Stackelberg game in which the retailer is the leader and the carrier is the follower. The retailer faces an uncertain demand and needs to supply his store from his warehouse. He has the option of not meeting all the demand but must satisfy at least a minimum percentage of the final demand. On the other hand, the carrier has to determine the number of trucks needed to satisfy this demand before uncertainty is resolved. Once demand is realized, if the reserved transportation capacity is insufficient, the carrier also has the possibility to requisition trucks at a higher price.

The contract that is proposed with the aim of increaseing the efficiency of the supply chain has two parameters : the quantity of transported items and the number of trucks used. In our settings, the retailer is the one that submits the contract and the carrier decides if he accepts it or not. We compare this situation with a centralized model where a single decision maker takes all the decisions.

1. Introduction

In today's competitive markets, firms tend to outsource their non-core operations in order to improve their responsiveness to market changes and to maximise their profits. This behavior increases the number of decision-makers in the supply chain and often leads to sub-optimal supply chain actions. Therefore, one would like to design contracts that commits each member of the supply chain to behave with the aim to increase the efficiency of the whole supply chain while preserving the interests of each party. The aim of this paper is to analyze the contractual relation between a retailer and its supplier.

To study this situation, we analyze a general outsourcing system in which a retailer facing an uncertain demand needs to supply his store. He has the option of not meeting all the demand but must satisfy at least a minimum proportion of the final demand. The retailer outsources his production activities to a supplier by submitting a contract. The proposed contract is composed of two terms (a, p) where a is the amount paid by the distributor per unit of capacity installed at the carrier and p is the price paid per delivered item. The aim here is to find the optimal operating policies of each player that maximizes his outcome and analyze if the supply chain is coordinated with this arrengement.

The remainder of this paper is organized as follows. A review of the relevant litterature is presented in Section 2. We develop the model and hypothesis in Section 3. In Section 4, we analyze the ideal case of a centralized system. The Stackelberg game is considered in Section 5. In Section 6, we provide an algorithm to find the optimal solution of the Stackelberg game and conduct numerical analysis. Section 7 presents some concluding remarks.

2. Literature review

This paper is related in varying degrees to several research stream in the operations management literature. The situation analyzed combines a newsvendor problem, a capacity game and a contract negotiation. Each of these streams is well studied in the literature and we consider them as equally important in this study.

This paper study a system related to the newsvendor problem. Various extensions of this problem have been studied. For a classification of these extensions, see the literature reviews by Khouja [Kho99] and Qin [QWV$^+$11]. In our situation, the carrier faces a newsvendor problem with recourse : the transportation capacity can be increased after the real demand is known. Fisher and Raman [FR96] study a related problem where a seller may submit a second order at a given time during the selling period. This second order is limited in its quantity and is used to satisfy the backordered demand of the first sub-period and some part of the unknown demand of the second sub-period. A similar environnement is considered in Lau and Lau ([LL97b] and [LL97a]). They consider the same model as Fisher and Raman whithout the capacity constraint on the second order but with lead-times between the order and the reception of the goods. To our knowledge, the system studied by Khouja [Kho96] is the closest from the one considered in this study. In this system, the newsvendor can order additional items from an emergency supply at a higher cost once the demand is known. Still, all these problems differs from the one faced by our carrier. In our case, even if the emergency option is costly, the carrier may have to increase its capacity after the demand is known. In the paper cited precedingly, if the emergency option costs more than to backorder or to loose the sales, the problem is reduced to a classical newsvendor problem. Other types of newsvendor problem with recourse are also studied in Kouvelis and Milner [KM02] and Gallego and Moon [GM93].

Newsvendor problems concerns situation with one decision maker. There are two in the system studied in this paper. As a consequence, the newsvendor problem has to be widened to a capacity game. A capacity game modelizes the arrengement between two level of the same supply chain to assure supply to one of the party. Closely related to our research is the work by Cachon and Larivière [CL01]. They study a system where a manufacturer proposes a contract to an upstream supplier to install some capacity. They consider two settings : forced and voluntary compliance. Under forced compliance, the supplier is obliged to satisfy the manufacturer's maximum order and has to set its capacity accordingly. While, under voluntary compliance, the supplier may decide of its capacity freely. We consider a hybrid system where our carrier may choose its capacity freely at first but finaly is obligated to satisfy a minimum proportion of the retailer's demand. The major difference between the system of Cachon and Larivière and ours is in the abilities given to each player. In Cachon and Larivière, the leader may retain some information. As a consequence, the leader gains more power and is able to use it to increase its profits. While in our case, the follower is able to adjusts its decisions once the uncertainty is resolved. This makes the bargain decisions harder for the leader and induce greater expected profits for the follower. Other types of capacity games are described in Cachon and Larivière [CL99], for distribution chains, Wang and Gerchak [WG03], for assembly chains, and in Serel [Ser07], who considers uncertainty in the supply too.

This paper is also closely related to the literature on supply chain coordination with contracts and supply chain contract negotiation. Different types of contracts have been studied in the literature. The one that we propose is common in today's firm interactions but has received poor interest in the scientific research. Cachon [Cac03] gives a good survey on supply chain coordination with contracts. Cachon and Larivière [CL05] demonstrate that revenue sharing contract can coordinate a two-echelon supply chain with a price-setting newsvendor. Zhao et al. [ZWC+10] consider a two-level supply chain with a stochastic demand and show that coordination may be achieved using option contracts. Taylor [Tay05] studies sales-rebate contract with sales effort effects. He shows that when demand is affected by the retailer's sales effort, a target rebate contract and a returns contract can achieve channel coordination. Other contracts types that have been studied in the literature include quantity discounts (Weng [Wen95]) and quantity-fexibility contracts (Tsay [Tsa99]).

3.Model Description

This article consider an outsourcing problem in a two-level supply chain with a retailer and a supplier. The retailer, R, sells a certain item on the market and faces a stochastic demand for this item. The supplier, S, has to decide of the capacity dedicated to the retailer's demand. The problem is to determine the total capacity that will be reserved before the uncertainty is resolved.

The demand is denoted by a random variable X where $f(x)$ is the distribution function and $F(x)$ is the cumulative distribution function. It is assumed that $F(x) = 0$ for all $x \leq 0$. Shortages are allowed but the retailer is obligated to satisfy a minimum proportion of the final demand, u, which is determined exogeneously.

Different costs are incured during the process. First, the supplier pays a fixed quantity c_1 for each unit of capacity reserved before uncertainty is resolved. Once demand is realized, if the actual capacity is insufficient, the supplier has the opportunity to requisition additional units of capacity from an emergency option at a higher price $c_2 > c_1$. Finally, each item produced incurs a processing cost r and is sold at the price of s. It is assumed that the selling price is greater than the minimum distribution costs, i.e $s > r + c_1$, otherwhise it is not profitable to sell this item.

In order to ensure a minimum service level, the retailer submits a contract to the supplier. This contract is composed of two terms (a, p) where a is the amount paid by the retailer per unit of capacity reserved or requisitionned by the supplier and p is the price paid per item transported. Note that this contract is a buy-back contract : the retailer buys each unit of capacity at a price of $a + p$ and receives a partial credit equal to p for each unused unit of capacity. This contract has two advantages. First, it is a simple contract, easy to implement and based on the performance of the supplier. Second, it include different types of contracts. If $a = 0$, then it is a wholesale-price contract. While if $p = 0$, then it is a capacity contract. These two simple contracts has strenght and weakness. For example, in the case of a capacity contract, the supplier will generally build a very high capacity which reduces the shortages. This behavior is the optimal for the supplier but is clearly not for the retailer. This is also why we design the contract such that a is a compensation per unit of capacity installed, $a < c_1$. Consequently, p has to be greater than r. We suppose that the supplier accepts a contract only if his expected profit is nonnegative under the application of this contract.

During the game, the following sequence of events occurs : before demand is realised, the retailer proposes the contract to the supplier ; if the supplier accepts this contract, he decides how many trucks he reserves. Once the demand is known, the supplier decides to requisition additional trucks or not and satisfy the totality or a fraction of demand following the contract specifications.

Finally, the problem faced by each member of the supply chain is the following. The supplier has a two-stage problem. In the first stage, when demand is still uncertain, he has to decide the capacity reserved, Y. In the second stage, when the demand x is known, he decides the additional capacity requisitionned, Z, and the quantity that he delivers, Q. He also has some constraints. He must at least satisfy a minimum proportion of the demand, $u \leq 1$, but he cannot deliver more than the demanded amount. We suppose that all the decision variables Y, Z, and Q are continuous. The retailer's profit depends on the quantity delivered, Q, and on the total capacity installed, $Y + Z$. All the cited variables are decision variables of the supplier. The only decision variables of the retailer are the contract parameters, a and p.

The supplier's profit, Π_S is a function of the contract parameters a and p and is modelized as follows :

$$\begin{aligned} \Pi_S(a,p) &= \max E_x\left[\pi(x)\right] - c_1.Y \\ s.t &\quad Y \geq 0 \end{aligned}$$

where

$$\begin{aligned} \pi(x) &= \max a.(Y+Z) - c_2.Z + (p-r).Q \\ s.t &\quad Q \leq Y + Z && (1) \\ &\quad Q \geq u.x && (2) \\ &\quad Q \leq x && (3) \\ &\quad Q, Z, \geq 0 \end{aligned}$$

The quantity $\Pi_S(a,p)$ is the supplier's gain under the contract (a,p). Constraint (1) ensures that the quantity delivered does not exceed the total capacity build. Constraint (2) defines the minimum service level. Finally, constraint (3) limits the quantity delivered to be at most equal to the demand.

The optimization problem of the retailer is the following :

$$\begin{aligned} \Pi_R &= \max E_x\left[(s-p).Q^*(a,p,x) - a.(Y^*(a,p) + Z^*(a,p,x))\right] \\ s.t &\quad \Pi_S(a,p) \geq 0 && (4) \\ &\quad a, p \geq 0 \end{aligned}$$

where the functions $Y^*(a,p)$, $Z^*(a,p,x)$, and $Q^*(a,p,x)$ are the optimal values of the variables Y, Z, and Q in the supplier's problem under a contract with (a,p) as parameters and facing a demand x. In problem Π_R, the decision variables are a and p and the retailer's profit is a function of the demand satisfied minus the contract costs. Constraint (4) ensures that the supplier will accept the contract only if he obtains a nonnegative profit.

4. Centralized System

In this section, we develop the optimal solution when the supply chain is centralized, i.e, when a centralized decision maker takes all the decisions and incures all the costs. We will use the solution obtained as a benchmark for our developments. The problem considered is a two-stage problem. In the first stage, while demand is still uncertain, the decision maker builds a capacity Y. In the second stage, when demand is known, he requisitions additional capacity, Z, and satisfies a part of the demand, Q. Therefore, the model is the following :

$$\begin{aligned} \Pi_{cent} &= \max \ E_x\left[\pi_{cent}(x)\right] - c_1.Y \\ s.t &\quad Y \geq 0 \end{aligned}$$

where

$$\begin{aligned} \pi_{cent}(x) &= \max(s-r)Q - c_2.Z \\ s.t &\quad Q \leq Y + Z && (5) \\ &\quad Q \geq u.x && (6) \\ &\quad Q \leq x && (7) \\ &\quad Q, Z \geq 0 \end{aligned}$$

We need a closed form result for the expectation of π_{cent} to be able to calculate the objective function of the first stage. The following theorem gives the optimal value of the second stage variables Q, and Z.

Proposition 1

For fixed values of Y and x, we have the following :

1. *if $s \geq c_2 + r$ then*
   - *if $x \leq Y$ then $Q^* = x$, and $Z^* = 0$.*
   - *if $x > Y$ then $Q^* = x$, and $Z^* = x - Y$.*
2. *if $s < c_2 + r$ then*
   - *if $x \leq Y$ then $Q^* = x$, and $Z^* = 0$.*
   - *if $u.x \leq Y < x$ then $Q^* = Y$, and $Z^* = 0$.*
   - *if $u.x > Y$ then $Q^* = u.x$, and $Z^* = u.x - Y$.*

where Q^*, and Z^* are the optimal second stage decisions.

The proof of this proposition is given in Appendices. From this proposition, we are able to derive the following theorem.

Theorem 1

In the centralized system, the optimal first stage decision Y^* is :

1. *if $s \geq c_2 + r$,*

$$Y^* = F^{-1}(\frac{c_2 - c_1}{c_2})$$

2. *if $s < c_2 + r$, the solution of the following equation :*

$$c_2 - c_1 - (s - r).F(Y^*) + (s - r - c_2).F(\frac{Y^*}{u}) = 0$$

Using Theorem 1, we are able to compute the optimal first stage decision Y and the total profit of the centralized system.

5. Stackelberg Game

In this section, we consider that each member of the supply chain aims to maximize his own profit and do not consider the other party's objectives. This gives rises to a Stackelberg game where the retailer is the leader. As the leader, he can anticipate the supplier's decisions. If he is able to compute the supplier's optimal response to a contract (a, p), then he can use this knowledge to find the optimal parameters a^* and p^* that will maximize his profit. To compute the supplier's response function, we need to calculate the expectation of the second stage objective function of the model Π_S for fixed values of a, p and Y.

Proposition 2

For fixed values of a, p and Y, the supplier's optimal solution of the second stage is :

1. *if $c_2 + r \leq a + p$:*
   - *if $x \leq Y$, then $Q^*(a,p,Y) = x$, and $Z = 0$.*
   - *if $x > Y$, then $Q^*(a,p,Y) = x$, and $Z^*(a,p,Y) = x - Y$.*
2. *if $c_2 + r > a + p$:*
   - *if $x \leq Y$, then $Q^*(a,p,Y) = x$, and $Z = 0$.*
   - *if $u.x \leq Y < x$, then $Q^*(a,p,Y) = Y$, and $Z^*(a,p,Y) = 0$.*
   - *if $u.x > Y$, then $Q^*(a,p,Y) = u.x$, and $Z^*(a,p,Y) = u.x - Y$.*

Again, we can compute the expectation in the first stage objective function of the supplier's model using Proposition 2. We obtain the following theorem :

Theorem 2

In the Stackelberg game, the supplier's optimal first stage decision Y^* is equal to :

1. *if* $c_2 + r \leq a + p$,

$$Y^* = F^{-1}(\frac{c_2 - c_1}{c_2 - a})$$

2. *if* $c_2 + r > a + p$, *the solution of the following equation :*

$$c_2 - c_1 - (p - r).F(Y^*) + (p - r + a - c_2).F(\frac{Y^*}{u}) = 0$$

To conclude this section, we express the expectation of the optimal values of the supplier's second stage decision variables. We need these expressions to compute the optimal contract for the retailer. In fact, Theorem 2 will be used to compute the response $Y^*(a, p)$ of the supplier to a contract (a, p) and this response will be used to compute the different costs (capacity installed, quantity of items delivered) of the retailer.

Proposition 3

1. *If* $c_2 + r \leq a + p$, *then we have :*

$$\begin{aligned} Z^* &= E[x] + \int_0^{Y^*} F(x)dx - Y^* \\ Q^* &= E[x] \end{aligned}$$

2. *If* $c_2 + r > a + p$, *then we have :*

$$\begin{aligned} Z^* &= u.(E[x] + \int_0^{\frac{Y^*}{u}} F(x)dx) - Y^* \\ Q^* &= u.(E[x] + \int_0^{\frac{Y^*}{u}} F(x)dx) - \int_0^{Y^*} F(x)dx \end{aligned}$$

Using Proposition 3, we are now able to rewrite the retailer's model as two different non-linear programs :

$$\begin{aligned} \Pi_R^1 \quad = \quad & \max(s - p).E[x] - a.(E[x] + \int_0^{Y^*} F(x)dx) \\ s.t \quad & (p - r).E[x] + a.(E[x] + \int_0^{Y^*} F(x)dx) - c_2.(E[x] + \int_0^{Y^*} F(x)dx - Y^*) - c_1.Y^* \geq 0 && (8) \\ & Y^* = F^{-1}(\frac{c_2 - c_1}{c_2 - a}) && (9) \\ & a + p \geq c_2 + r && (10) \\ & a, p \geq 0 \end{aligned}$$

and

$$\begin{aligned} \Pi_R^2 \quad = \quad & \max(s - p).\left[u.(E[x] + \int_0^{\frac{Y^*}{u}} F(x)dx) - \int_0^{Y^*} F(x)dx\right] - a.(u.(E[x] + \int_0^{\frac{Y^*}{u}} F(x)dx) - Y^*) \\ s.t \quad & (p - r).\left[u.(E[x] + \int_0^{\frac{Y^*}{u}} F(x)dx) - \int_0^{Y^*} F(x)dx\right] + a.\left[u.(E[x] + \int_0^{\frac{Y^*}{u}} F(x)dx) - Y^*\right] \\ & -c_2.\left[u.(E[x] + \int_0^{\frac{Y^*}{u}} F(x)dx)\right] - c_1.Y^* \geq 0 && (11) \\ & c_2 - c_1 - (p - r).F(Y^*) + (p - r + a - c_2).F(\frac{Y^*}{u}) = 0 && (12) \\ & a + p < c_2 + r && (13) \\ & a, p \geq 0 \end{aligned}$$

Where constraints (8) and **??**11) represent the supplier's reservation profit. Constraints (9) and (11) are used to define the optimal value of Y^*, and constraints (13) and (12) are used to determine which case is currently considered.

Both these model can be solved with a general NLP solver. Once it is done, compare the two optimal solution, the one with the highest value is the optimal solution of the retailer's problem.

6. Numerical study

In this section, we provide numerical examples to compare the performance of different system structures, to attain qualitative insights into their sensitivity with respect to several parameters, and to investigate the applicability of the proposed contract.

We consider the following basic system parameters : $\mu = 5000$, $\sigma = 500$, $m = 40$, $c_1 = 200$, $c_2 = 400$, $s = 30$, $b = 10$, $r = 5$ and $u = 0.9$. All the tests were performed using MATLAB (R2007b) on a machine with an intel(R) Core(TM) i7 CPU (3.70 GHz) with 8,00 GB of installed memory. For each scenario tested, we made a numerical search to find the optimal values of a^*, p^* and $Y^*(a^*, p^*)$. Due the large number of parameters, we restrict our analyzis to tests on parameters of the same gender. The tests performed are on variations of the requisition cost (c_2), of the shortage cost (b) and on variations of the variability of the demand and on the capacity of trucks (σ and m). We do not analyze the reaction of our algorithm to a change in the mean μ of the demand because this is done in the tests analyzing the variation of m (a system where $\mu = 5000$ and $m = 10$ is the same as a system where $\mu = 10000$ and $m = 20$).

The effects of demand variability and of the truck capacity

In the first battery of tests, we investigate how the system reacts to modifications of σ and m. The range of the tested parameters, σ and m, are respectively $\{100, 500, 1000, 1500\}$ and $\{20, 40, 60, 80, 100\}$. Table 1 shows the obtained results. Here are some basic observations from Table 1 :

OBS 1. As expected, the contract proposed does not coordinate the supply chain but obtain good results (the ratio between the profit of the decentralized system over the centralized system is generally over 0.9).

OBS 2. The retailer captures nearly all the profits of the supply chain. The retailer only propose contracts of the second type, i.e such that $a + p.m < c_2 + r.m$.

A remark on the contract : the retailer propose a fee per truck used as close as possible of c_1 and a very small price per unit transported. This indicates that, under the chosen settings, the retailer prefers to avoid shortages than over capacity.

OBS 3. In both cases, the total profit decreases with the variability but increases with the trucks capacity.

This observation can be described as obvious. With a higher truck capacity, the cost per unit transported, $\frac{c_1}{m} + r$ in first period and $\frac{c_2}{m} + r$ in second, decreased so the total profit increases. Due to the contract, if the variability of the demand is high, the supplier will request a high number of trucks in first period so as to avoid shortages.

OBS 4. The contract proposed by the retailer is less attractive for the supplier when the truck capacity is high and when the variability of demand is low.

The cases with a high truck capacity or a low variability of demand can be described as more profitable for the supplier. Due to the hypothesis on the contract : $a < c_1$ and $p \geq r$, the supplier is able to make more profit when the truck capacity is high. Indeed, as the supplier gains money on the unit that he transports and loses money on the trucks that he uses, if he is able to sent more unit with the same amount of trucks, he obtains better profits. As for the variability, if it is low, the supplier takes less risks of shortages and over capacity, wich increases its profits. As a consequence, the supplier will accept less profitable contracts in these cases.

The effects of shortage and requisition costs

The second battery of tests investigates the effects of the costs c_2 and b on both systems. The tested range of c_2 and of b are $\{300, 400, 800, 1200\}$ and $\{0, 2.5, 5, 7.5, 10\}$ respectively. The results are shown in Table 2. Here are some basic observations from Table 2 :

OBS 5. Variations of both b and c_2 have small impact on the supplier's decision and on the retailer's profit.

As explained before, the retailer proposes a contract such that the risks of shortages are minimized. As a consequence, modification of costs parameters that relates to possible shortages : b if it is too costly for the

Table 1 – Optimal solutions for a range of values of σ and m

	Profit comparisons			Optimal decisions comparisons			
m	Π_{cent}	Π_R	Π_C	Y^*_{cent}	$Y^*(a^*, p^*)$	a^*	p^*
$\sigma = 100$							
20	74200	66490	5800	250	245	199	6.25
40	99600	94380	3580	125	124	199	5.75
60	108070	103950	2880	83.3	83	199	5.6
80	112300	108830	2440	62.5	65	199	5.5
100	114800	111820	1950	50	51	199	5.4
$\sigma = 500$							
20	71000	69000	20	250	254	199	5.1
40	98000	94630	250	125	127	199	5.1
60	107000	103170	340	83.3	85	199	5.1
80	111500	107440	100	62.5	63	199	5.05
100	114200	110060	900	50	52	199	5.2
$\sigma = 1000$							
20	67000	64340	280	250	283	199	5.2
40	96000	93090	130	125	141	199	5.1
60	105670	102960	250	83.3	94	199	5.1
80	110500	107490	80	62.5	71	199	5.05
100	113400	110370	110	50	57	199	5.05
$\sigma = 1500$							
20	63000	58530	560	250	312	199	5.3
40	94000	93090	530	125	156	199	5.2
60	105350	101200	190	83.3	104	199	5.1
80	109500	106550	260	62.5	78	199	5.1
100	112600	109750	50	50	62	199	5.05

supplier to requisition additional trucks and c_2 in the other case, have a small impact on the decentralized system.

7. Conclusion and future research

This paper studies the problem of coordination in a two-level supply chain with stochastic demand. We investigate the system under both centralized and decentralized decision-making situations. We propose a contract based on two simple observations : the number of trucks used and the number of transported items. At first glance, it seems that the proposed model is simple to solve, but after careful examination of the structure of the developed game, the optimal values of the contract parameters are obtained using nonlinear optimization problems.

Numerical analysis were conducted to study the effects of capacity, variability of demand, shortages and requisition costs. The results show that even if the proposed contract does not coordinate the supply chain, it performs reasonnably well. It is worthwhile to indicate that the retailer generally captures all the profits because he is the leader and has opportunity to act the first.

An interesting direction for future research is to study the problem under a cooperative game theory approach, where all the parties first try to coordinate the suply chain and then decide of how to allocate the total profits. An possible extension to the system studied is to consider the case where the retailer has full information about the demand distribution function and not the supplier and vice-versa. Consequently, the retailer may lie to the supplier to obtain a greater capacity at least cost. Moreover, the game could also imply different suppliers with different cost structures which compete for the contract proposed by the retailer. Considering a minimum load constraint per truck is another extension.

Table 2 – Optimal solutions for a range of values of c_2 and b

	Profit comparisons			Optimal decisions comparisons			
b	Π_{cent}	Π_R	Π_C	Y^*_{cent}	$Y^*(a^*,p^*)$	a^*	p^*
$c_2 = 300$							
0	98630	95860	60	119	127	199	5.05
2.5	98630	95550	60	119	127	199	5.05
5	98630	95250	60	119	127	199	5.05
7.5	98630	94950	60	119	127	199	5.05
10	98630	94650	60	119	127	199	5.05
c_2 =400							
0	98000	95820	60	125	127	199	5.06
2.5	98000	95520	60	125	127	199	5.06
5	98000	95220	60	125	127	199	5.06
7.5	98000	94930	60	125	127	199	5.06
10	98000	94430	60	125	127	199	5.06
c_2 =800							
0	96800	95600	45	133	127	199	5.1
2.5	96800	95310	45	133	127	199	5.1
5	96800	95000	45	133	127	199	5.1
7.5	96800	94730	45	133	127	199	5.1
10	96800	94430	45	133	127	199	5.1
c_2 =1200							
0	96680	95370	80	136	127	199	5.15
2.5	96360	95080	80	136	127	199	5.15
5	96200	94790	80	137	127	199	5.15
7.5	96200	94500	80	137	127	199	5.15
10	96200	94200	80	137	127	199	5.15

8. Appendices

Proof of Proposition 1

We analyze each case separately. In the first case, requisitioning additional capacity is profitable whenever the initial capacity is insufficient. As a consequence, the demand will always be satisfied and the quantity delivered, Q, will always be equal to x. If the capacity reserved is sufficient, i.e $Y \geq x$, then there is no need for additional trucks : $Z = 0$. Otherwhise, the additional capacity requested is equal to the difference between the demand and the initial capacity : $Z = x - Y$. It is not profitable to request more because the system would pay for useless capacity.

In the second case, the decision maker will not iuse the emergency option unless obligated. As a consequence, when the initial capacity is sufficient to satisfy at least the minimum requirements, i.e $m.Y \geq u/x$, then $Z = 0$. Otherwhise, the additional capacity requested is equal to the difference between the minimum requirement and the initial capacity : $Z = u.x - Y$.$\square$

Proof of Theorem 1

Using Proposition 1, we analyse the profit function, $\Pi_{cent}(Y)$, for each case :

– If $s \geq c_2 + r$, then :

$$\begin{aligned}\Pi_{cent}(Y) &= -c_1.Y + \int_0^Y (s-r).x.f(x)dx + \int_Y^{+\infty} \left[(s-r).x - c_2.(x-Y)\right].f(x)dx \\ &= -c_1.Y + (s-r).E\left[x\right] + c_2.Y.(1-F(Y)) - c_2.\int_Y^{+\infty} x.f(x)dx \\ &= (c_2 - c_1).Y + (s-r).E\left[x\right] - c_2.(E\left[x\right] + \int_0^Y F(x)dx)\end{aligned}$$

This function is a sum of concave functions so it is concave. As a consequence, we can use the first order condition, and obtain :

$$Y^* = F^{-1}(\tfrac{c_2-c_1}{c_2}).$$

– If $s < c_2 + r$, then :

$$\begin{aligned}
\Pi_{cent}(Y) &= -c_1.Y + \int_0^Y (s-r).x.f(x)dx + \int_Y^{\frac{Y}{u}} (s-r).Y.fxdx \\
&\quad + \int_{\frac{Y}{u}}^{+\infty} [(s-r).u.x - c_2.(u.x - Y)] \\
&= -c_1.Y + (s-r).\left[Y.F(Y) - \int_0^Y F(x)dx\right] + (s-r).Y.\left[F(\frac{Y}{u}) - F(Y)\right] \\
&\quad + c_2.Y.(1 - F(\frac{Y}{u})) + [(s-r-c_2).u].\left[E[x] - \frac{Y}{u}.F(\frac{Y}{u}) + \int_0^{\frac{Y}{u}} F(x)dx\right] \\
&= (c_2-c_1).Y - (s-r).\int_0^Y F(x)dx + (s-r-c_2).u.\left[\int_0^{\frac{Y}{u}} F(x)dx + E[x]\right]
\end{aligned}$$

Again this function is the sum of concave functions so we can use the first order conditions. We obtain that Y^* is the solution to the following equation :

$$c_2 - c_1 - (s-r).F(Y) + [s - r - c_2].F(\tfrac{Y}{u}).\square$$

Proof of Proposition 2

The proof of Proposition 2 is similar to the proof of proposition 1.

Proof of Theorem 2

Using Proposition 2, we analyse the profit function, $\Pi(Y)$, for each case :

– If $c_2 + r \leq a + p$, then :

$$\begin{aligned}
\Pi(Y) &= -c_1.Y + \int_0^Y [(p-r).x + a.Y].f(x)dx + \int_Y^{+\infty} [(p-r).x + a.x - c_2.(x-Y)].f(x)dx \\
&= -c_1.Y + (p-r).E[x] + a.Y.F(Y) + c_2.Y.(1-F(Y)) + a - c_2.\int_Y^{+\infty} x.f(x)dx \\
&= (c_2-c_1).Y + (p-r).E[x] + (a-c_2).(E[x] + \int_0^Y F(x)dx)
\end{aligned}$$

This function is a sum of concave functions so it is concave. As a consequence, we can use the first order condition, and obtain :

$$Y^* = F^{-1}(\tfrac{c_2-c_1}{c_2-a}).$$

– If $c_2 + r.m < a + p.m$, then :

$$\begin{aligned}
\Pi(Y) &= -c_1.Y + \int_0^Y [(p-r).x + a.Y].f(x)dx + \int_Y^{\frac{Y}{u}} (p-r+a).Y.f(x)dx \\
&\quad + \int_{\frac{Y}{u}}^{+\infty} [(p-r).u.x + a.u.x - c_2.(u.x - Y)].f(x)dx \\
&= -c_1.Y + a.Y.F(Y) + (p-r).\left[Y.F(Y) - \int_0^Y F(x)dx\right] \\
&\quad + (p-r+a).Y.(F(\frac{Y}{u}) - F(Y)) + c_2.Y.(1 - F(\frac{Y}{u})) \\
&\quad + (p-r+a-c_2).u.\left[E[x] - \frac{Y}{u}.F(\frac{Y}{u}) + \int_0^{\frac{Y}{u}} F(x)dx\right] \\
&= (c_2-c_1).Y - (p-r).\int_0^Y F(x)dx + (p-r+a-c_2).u.\left[E[x] + \int_0^{\frac{Y}{u}} F(x)dx\right]
\end{aligned}$$

Again this function is the sum of concave functions so we can use the first order conditions. We obtain that Y^* is the solution to the following equation :

$$c_2 - c_1 - (p-r).F(Y) + [(p-r) + a - c_2].F(\tfrac{Y}{u}) = 0.\square$$

Proof of Proposition 3

In the first case, if $c_2 + r \leq a + p$, we have :

$$\begin{aligned}
Q^*(Y) &= \int_0^Y xf(x)dx + \int_Y^{+\infty} xf(x)dx \\
&= E\left[x\right]
\end{aligned}$$

$$\begin{aligned}
Z^*(Y) &= \int_0^Y 0.f(x)dx + \int_{m.Y}^{+\infty} (x-Y).f(x)dx \\
&= E\left[x\right] - Y.F(Y) + \int_0^Y F(x)dx - Y(1-F(Y)) \\
&= E\left[x\right] + \int_0^Y F(x)dx - Y
\end{aligned}$$

And in the second case, we have :

$$\begin{aligned}
Q^*(Y) &= \int_0^Y x.f(x)dx + \int_Y^{\frac{Y}{u}} Y.f(x)dx + \int_{\frac{Y}{u}}^{+\infty} u.x.f(x)dx \\
&= Y.F(Y) - \int_0^Y F(x)dx + Y.(F(\frac{Y}{u}) - F(Y)) + u.\left[E\left[x\right] - \frac{Y}{u}.F(\frac{Y}{u}) + \int_0^{\frac{Y}{u}} F(x)dx\right] \\
&= u.E\left[x\right] - \int_0^Y F(x)dx + u.\int_0^{\frac{Y}{u}} F(x)dx \\
Z^*(Y) &= \int_0^{\frac{Y}{u}} 0.f(x)dx + \int_{\frac{Y}{u}}^{+\infty} (u.x-Y)f(x)dx \\
&= u.\left[E\left[x\right] - \frac{Y}{u}.F(fracYu) + \int_0^{\frac{Y}{u}} F(x)dx\right] - Y.(1-F(\frac{Y}{u})) \\
&= u.\left[E\left[x\right] + \int_0^{\frac{Y}{u}} F(x)dx\right] - Y
\end{aligned}$$

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