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SUBPROBLEM FINITE ELEMENT METHOD FOR MAGNETIC MODEL REFINEMENTS

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***Abstract*— Model refinements of magnetic circuits are performed via a subproblem finite element method. A complete problem is split into subproblems with overlapping meshes, to allow a progression from source to reaction fields, ideal to real flux tubes, 1-D to 2-D to 3-D models, perfect to real materials, with any coupling of these changes. Its solution is the sum of the subproblem solutions. The procedure simplifies both meshing and solving processes, and quantifies the gain given by each refinement on both local fields and global quantities.**

Introduction

The finite element (FE) subproblem method (SPM) provides advantages in repetitive analyses and helps improving the solution accuracy [1-6]. It allows to benefit from previous computations instead of starting a new complete FE solution for any geometrical, physical or model variation. It also allows different problem-adapted meshes and computational efficiency due to the reduced size of each SP. A general framework allowing a wide variety of refinements is herein presented. It is a FE SPM based on canonical magnetostatic and magnetodynamic problems solved in a sequence, with at each step volume sources (VSs) and surface sources (SSs) originated from previous solutions. VSs express changes of material properties. SSs express changes of boundary conditions (BCs) or interface conditions (ICs). Common and useful changes from source to reaction fields, ideal to real flux tubes (with leakage flux), 1-D to 2-D to 3-D models, perfect to real materials, and statics to dynamics, can all be defined via combinations of VSs and SSs. The developments are performed for the magnetic vector potential FE formulation, paying special attention to the proper discretization of the constraints involved in each SP. The method is illustrated and validated on various problems.

Sequence of Coupled Subproblems

Canonical Magnetodynamic/static Problem

A complete problem is split into a series of SPs that define a sequence of changes, with the complete solution being replaced by the sum of the SP solutions. Each SP is defined in its particular domain, generally distinct from the complete one and usually overlapping those of the other SPs. At the discrete level, this aims to decrease the problem complexity and to allow distinct meshes with suitable refinements. No remeshing is necessary when adding some regions.

A canonical magnetodynamic/static problem p, to be solved at step p of the SPM, is defined in a domain Ω_p, with boundary $\partial\Omega_p = \Gamma_p = \Gamma_{h,p} \cup \Gamma_{b,p}$. The eddy current conducting part of Ω_p is denoted $\Omega_{c,p}$ and the non-conducting one $\Omega_{c,p}{}^C$, with $\Omega_p = \Omega_{c,p} \cup \Omega_{c,p}{}^C$. Massive inductors belong to $\Omega_{c,p}$, whereas stranded inductors belong to $\Omega_{c,p}{}^C$. The equations, material relations and boundary conditions (BCs) of problem p are

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$$\mathrm{curl}\,\boldsymbol{h}_p = \boldsymbol{j}_p\,, \quad \mathrm{div}\,\boldsymbol{b}_p = 0\,, \quad \mathrm{curl}\,\boldsymbol{e}_p = -\partial_t \boldsymbol{b}_p\,, \tag{1a-b-c}$$
$$\boldsymbol{h}_p = \mu_p^{-1}\,\boldsymbol{b}_p + \boldsymbol{h}_{s,p}\,, \quad \boldsymbol{j}_p = \sigma_p\,\boldsymbol{e}_p + \boldsymbol{j}_{s,p}\,, \tag{2a-b}$$
$$\boldsymbol{n} \times \boldsymbol{h}_p|_{\Gamma_{h,p}} = \boldsymbol{j}_{f,p}\,, \quad \boldsymbol{n} \cdot \boldsymbol{b}_p|_{\Gamma_{b,p}} = \boldsymbol{f}_{f,p}\,, \quad \boldsymbol{n} \times \boldsymbol{e}_p|_{\Gamma_{e,p} \subset \Gamma_{b,p}} = \boldsymbol{k}_{f,p}\,, \tag{3a-b-c}$$

where $\boldsymbol{h}_p$ is the magnetic field, $\boldsymbol{b}_p$ is the magnetic flux density, $\boldsymbol{e}_p$ is the electric field, $\boldsymbol{j}_p$ is the electric current density, μ_p is the magnetic permeability, σ_p is the electric conductivity and $\boldsymbol{n}$ is the unit normal exterior to Ω_p. Note that (1c) is only defined in $\Omega_{c,p}$ (as well as $\boldsymbol{e}_p$), whereas it is reduced to the form (1b) in $\Omega_{c,p}{}^C$. It is thus absent from the magnetostatic version of problem p. Further (3c) is more restrictive than (3b) in their homogeneous forms. Equations (1b-c) are fulfilled via the definition of a magnetic vector potential $\boldsymbol{a}_p$ and an electric scalar potential v_p, leading to the $\boldsymbol{a}_p$-formulation, with

$$\mathrm{curl}\,\boldsymbol{a}_p = \boldsymbol{b}_p\,, \quad \boldsymbol{e}_p = -\partial_t \boldsymbol{a}_p - \mathrm{grad}\, v_p\,, \quad \boldsymbol{n} \times \boldsymbol{a}_p|_{\Gamma_{b,p}} = \boldsymbol{a}_{f,p}\,. \tag{4a-b-c}$$

For various purposes, some paired portions of Γ_p can define double layers, with the thin region in between exterior to Ω_p [2-5]. They are denoted $\gamma_p{}^+$ and $\gamma_p{}^-$ and are geometrically defined as a single surface γ_p with ICs, fixing the discontinuities ($[\,\cdot\,]_{\gamma_p} = \cdot\,|_{\gamma_p{}^+} - \cdot\,|_{\gamma_p{}^-}$)

$$[\boldsymbol{n} \times \boldsymbol{h}_p]_{\gamma_p} = [\boldsymbol{j}_{f,p}]\,, \quad [\boldsymbol{n} \cdot \boldsymbol{b}_p]_{\gamma_p} = [\boldsymbol{f}_{f,p}]\,, \quad [\boldsymbol{n} \times \boldsymbol{e}_p]_{\gamma_p} = [\boldsymbol{k}_{f,p}] \;\text{ and }\; [\boldsymbol{n} \times \boldsymbol{a}_p]_{\gamma_p} = [\boldsymbol{a}_{f,p}]\,. \tag{5a-b-c-d}$$

With the definitions $(\boldsymbol{n}_{\gamma_p} \equiv \boldsymbol{n}|_{\gamma_p}) = -(\boldsymbol{n}^+ \equiv \boldsymbol{n}|_{\gamma_p{}^+}) = (\boldsymbol{n}^- \equiv \boldsymbol{n}|_{\gamma_p{}^-})$ for the normal $\boldsymbol{n}$ in different contexts, one has, e.g. for (5a),

$$[\boldsymbol{n} \times \boldsymbol{h}_p]_{\gamma_p} = \boldsymbol{n}_{\gamma_p} \times \boldsymbol{h}_p|_{\gamma_p{}^+} - \boldsymbol{n}_{\gamma_p} \times \boldsymbol{h}_p|_{\gamma_p{}^-} = -(\boldsymbol{n}^+ \times \boldsymbol{h}_p|_{\gamma_p{}^+} - \boldsymbol{n}^- \times \boldsymbol{h}_p|_{\gamma_p{}^-})\,. \tag{6}$$

The fields $\boldsymbol{h}_{s,p}$ and $\boldsymbol{j}_{s,p}$ in (2a-b) are volume sources (VSs). The source $\boldsymbol{h}_{s,p}$ is usually used for fixing a remnant field in magnetic materials. The source $\boldsymbol{j}_{s,p}$ fixes the current density in inductors. With the SPM, $\boldsymbol{h}_{s,p}$ is also used for expressing changes of permeability and $\boldsymbol{j}_{s,p}$ for changes of conductivity, or for adding portions of inductors [2-6]. For changes in a region, from μ_q and σ_q for problem q to μ_p and σ_p for problem p, the associated VSs $\boldsymbol{h}_{s,p}$ and $\boldsymbol{j}_{s,p}$, limited to the modified regions, are

$$\boldsymbol{h}_{s,p} = (\mu_p^{-1} - \mu_q^{-1})\,\boldsymbol{b}_q\,, \quad \boldsymbol{j}_{s,p} = (\sigma_p - \sigma_q)\,\boldsymbol{e}_q\,, \tag{7a-b}$$

for the total fields to be related by the updated relations $\boldsymbol{h}_q + \boldsymbol{h}_p = \mu_p^{-1}\,(\boldsymbol{b}_q + \boldsymbol{b}_p)$ and $\boldsymbol{j}_q + \boldsymbol{j}_p = \sigma_p\,(\boldsymbol{e}_q + \boldsymbol{e}_p)$.

The surface fields $\boldsymbol{j}_{f,p}$, $\boldsymbol{f}_{f,p}$ and $\boldsymbol{k}_{f,p}$ in (3a-c), and $\boldsymbol{a}_{f,p}$ in (4c), are generally zero for classical homogeneous BCs. The discontinuities (5a-d) are also generally zero for common continuous field traces. If nonzero, they define possible surface sources (SSs) that account for particular phenomena occurring in the thin region between $\gamma_p{}^+$ and $\gamma_p{}^-$ [2-5]. This is the case when some field traces in a problem q are forced to be discontinuous. The continuity has to be recovered after a correction via a problem p. The SSs in problem p are thus to be fixed as the opposite of the trace solution of problem q.

Each problem p is to be constrained via the so defined VSs and SSs from parts of solutions of other problems. This is a key element of the SPM, offering a wide variety of possible corrections, as shown hereafter.

The complete solution is

$$\boldsymbol{u} = \sum\nolimits_{p \in P} \boldsymbol{u}_p\,, \text{ with } \boldsymbol{u} \equiv \boldsymbol{h}, \boldsymbol{b}, \boldsymbol{j}, \boldsymbol{e}, \ldots \tag{8}$$

with P an ordered set of SPs. A correction can become a significant source for any of its source problems, which is inherent to large perturbation problems. In this case, an iterative process between the related SPs has to be done till convergence up to a desired accuracy [4]. In addition to the iterations between SPs, classical inter-problem iterations are needed in nonlinear analyses. Each solution $\mathbf{u}_p$ can then be calculated as a series of corrections $\mathbf{u}_{p,i}$ (with i the sub-SP (SSP) index), i.e.

$$\boldsymbol{u}_p = \sum\nolimits_i \boldsymbol{u}_{p,i} = \boldsymbol{u}_{p,1} + \boldsymbol{u}_{p,2} + \ldots \tag{9}$$

Finite Element Weak Formulations

The weak $\boldsymbol{a}_p$-formulation of the canonical problem p is obtained from the weak form of the Ampère equation (1a), i.e. [2-5],

$$(\mu_p^{-1} \operatorname{curl} \boldsymbol{a}_p, \operatorname{curl} \boldsymbol{a}')_{\Omega_p} + (\boldsymbol{h}_{s,p}, \operatorname{curl} \boldsymbol{a}')_{\Omega_p} - (\boldsymbol{j}_{s,p}, \boldsymbol{a}')_{\Omega_p} + (\sigma_p \partial_t \boldsymbol{a}_p, \boldsymbol{a}')_{\Omega_{c,p}}$$
$$+ \langle \boldsymbol{n} \times \boldsymbol{h}_{s,p}, \boldsymbol{a}' \rangle_{\Gamma_{h,p}} + \langle \boldsymbol{n} \times \boldsymbol{h}_p, \boldsymbol{a}' \rangle_{\Gamma_{b,p}} + \langle -[\boldsymbol{n} \times \boldsymbol{h}_p]_{\gamma_p}, \boldsymbol{a}' \rangle_{\gamma_p} = 0, \ \forall \boldsymbol{a}' \in F_p^1(\Omega_p), \quad (10)$$

where $F_p{}^1(\Omega_p)$ is a curl-conform function space defined on Ω_p, gauged in $\Omega_{c,p}{}^C$, and containing the basis functions for $\boldsymbol{a}$ as well as for the test function $\boldsymbol{a'}$ (at the discrete level, this space is defined by edge FEs; the gauge is based on the tree-co-tree technique); $(\cdot,\cdot)_\Omega$ and $\langle \cdot,\cdot \rangle_\Gamma$ respectively denote a volume integral in Ω and a surface integral on Γ of the product of their vector field arguments. The surface integral term on $\Gamma_{h,\mathrm{p}}$ accounts for natural BCs of type (3a), usually zero.

The term on the surface $\Gamma_{b,p}$ with essential BCs on $\boldsymbol{n}\cdot\boldsymbol{b}_p$ is usually omitted because it does not locally contribute to (10). It can be used for post-processing a solution, a part of which $\boldsymbol{n} \times \boldsymbol{h}_p|_{\Gamma_{b,p}}$ having to act further as a SS [2-5].

Some parts of a previous solution $\boldsymbol{a}_q$ serve as VSs and SSs in a subdomain $\Omega_{s,p} \subset \Omega_p$ of the current problem p. At the discrete level, this means that this source quantity $\boldsymbol{a}_q$ has to be expressed in the mesh of problem p, while initially given in the mesh of problem q. This is done via an L^2-projection [8] of its curl limited to $\Omega_{s,p}$, i.e.

$$(\operatorname{curl} \boldsymbol{a}_{q\text{-}p}, \operatorname{curl} \boldsymbol{a}')_{\Omega_{s,p}} = (\operatorname{curl} \boldsymbol{a}_q, \operatorname{curl} \boldsymbol{a}')_{\Omega_{s,p}}, \forall \boldsymbol{a}' \in F_p^1(\Omega_{s,p}), \quad (11)$$

where $F_p{}^1(\Omega_{s,p})$ is a gauged curl-conform function space for the p-projected source $\boldsymbol{a}_{q\text{-}p}$ (the projection of $\boldsymbol{a}_q$ on mesh p) and the test function $\boldsymbol{a}'$.

VSs for changes of material properties and inductors – A change of material properties from SP q to SP p is taken into account in (10) via the source integrals $(\boldsymbol{h}_{s,p}, \operatorname{curl} \boldsymbol{a}')_{\Omega_p}$ and $(\boldsymbol{j}_{s,p}, \boldsymbol{a}')_{\Omega_p}$. The VSs $\boldsymbol{h}_{s,p}$ and $\boldsymbol{j}_{s,p}$ are respectively given by (7a) and (7b), i.e. $\boldsymbol{h}_{s,p}=(\mu_p^{-1}-\mu_q^{-1})\boldsymbol{b}_q$ and $\boldsymbol{j}_{s,p}=(\sigma_p-\sigma_q)\boldsymbol{e}_q$ with (4a-b), $\boldsymbol{b}_q=\operatorname{curl}\boldsymbol{a}_q$ and $\boldsymbol{e}_p=-\partial_t \boldsymbol{a}_p$ (v_p can be generally omitted). A change of current density directly defines the related VS $\boldsymbol{j}_{s,p}$. At the discrete level, the source primal quantity $\boldsymbol{a}_q$, initially given in mesh q, is projected in the mesh p via (11), with $\Omega_{s,p}$ limited to the modified regions.

SSs for changes of BCs or ICs – Both essential and natural BCs or ICs are to be considered, respectively (4c) or (5d), and (3a) or (5a). Essential conditions can be directly applied, whereas natural conditions gain at being applied in an indirect way, avoiding to locally evaluate the weak trace of $\boldsymbol{h}_p$ on the related surfaces. Usually, the trace discontinuity of $\boldsymbol{h}_p$ is defined as the opposite of the trace discontinuity of a previous solution $\boldsymbol{h}_q$ (or a sum of previous solutions), i.e.

$$[\boldsymbol{n} \times \boldsymbol{h}_p]_{\gamma_p} = -[\boldsymbol{n} \times \boldsymbol{h}_q]_{\gamma_q \equiv \gamma_p}, \text{ or } [\boldsymbol{n} \times \boldsymbol{h}_p]_{\gamma_p} = -\boldsymbol{n} \times \boldsymbol{h}_q|_{\gamma_q^+ \equiv \gamma_p^+}, \quad (12a\text{-}b)$$

or respectively, in the related integral term in (10),

$$\langle -[\boldsymbol{n} \times \boldsymbol{h}_p]_{\gamma_p}, \boldsymbol{a}' \rangle_{\gamma_p} = \langle [\boldsymbol{n} \times \boldsymbol{h}_q]_{\gamma_q}, \boldsymbol{a}' \rangle_{\gamma_q \equiv \gamma_q}, \text{ or } \langle -[\boldsymbol{n} \times \boldsymbol{h}_p]_{\gamma_p}, \boldsymbol{a}' \rangle_{\gamma_p} = \langle \boldsymbol{n} \times \boldsymbol{h}_q, \boldsymbol{a}' \rangle_{\gamma_q^+ \equiv \gamma_q^+}. \quad (13a\text{-}b)$$

The RHS term of (13a-b), on this weak form, is just a term that occurs in the weak formulation of SP q, of form (10) written for SP $\mathrm{p} \equiv q$. It can thus be naturally expressed via the volume integrals of this formulation, i.e. usually, with $\Gamma_{b,q}=\gamma_p$, e.g.

$$\langle \boldsymbol{n} \times \boldsymbol{h}_q, \boldsymbol{a}' \rangle_{\gamma_q} = -(\mu_q^{-1} \operatorname{curl} \boldsymbol{a}_q, \operatorname{curl} \boldsymbol{a}')_{\Omega_p \equiv \Omega_q} - (\sigma_q \partial_t \boldsymbol{a}_q, \boldsymbol{a}')_{\Omega_{c,p} \equiv \Omega_{c,q}}. \quad (14)$$

The weak nature of the natural BC or IC is thus conserved. At the discrete level, the volume integrals in (14) are limited to one single layer of FEs touching γ_q for (13a) or $\gamma_q{}^+$ for (13b) (thus respectively on both sides or on one side of γ_q), because they involve only the associated traces $\boldsymbol{n} \times \boldsymbol{a}'|_{\gamma_q}$ or $\boldsymbol{n} \times \boldsymbol{a}'|_{\gamma_q^+}$. The source $\boldsymbol{a}_q$, initially in mesh q, has to be projected in mesh p via (11), with $\Omega_{s,p}$ limited to the FE layer, which thus decreases the computational effort of the projection process.

Various Correction Schemes

Various correction schemes, appropriate to practical magnetic system analyses, can benefit from the developed SPM.

Change of material properties

A typical problem is that of a region put in an initially calculated source field $\boldsymbol{b}_1$ (SP 1, Fig. 1). The associated SP 2 is solved in its proper mesh, with the added core and its surrounding region, and VSs (2a) and/or (2b) limited to this core, where μ and/or σ are modified, from μ_1 to μ_2 and/or σ_1 to σ_2. Such changes can occur when adding or suppressing materials or portions of those, in, e.g., shape optimization, non-destructive testing, moving systems [6].

A change from a linear to a nonlinear magnetic material is a particular case of a change or permeability (Fig. 2), requiring classical nonlinear iterations for the related nonlinear SP.

From the so calculated field corrections, the associate corrections of global quantities inherent to magnetic models, i.e. fluxes, magnetomotive forces (MMFs), currents and voltages, can be evaluated.

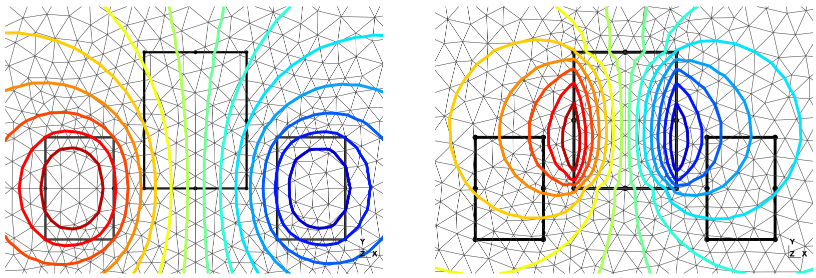

Fig. 1. Field lines for an inductor alone ($\boldsymbol{b}_1$, *left*) and for an added core ($\boldsymbol{b}_2$, $\mu_{r,core} = 100$) (*right*); distinct meshes are used for SPs 1 and 2.

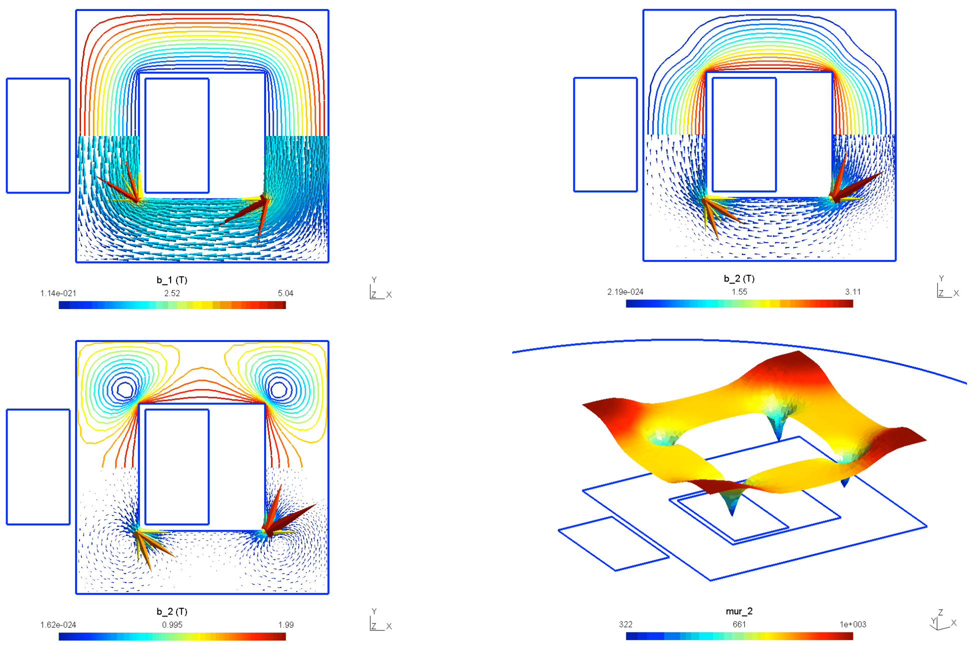

Fig. 2. Field lines and magnetic flux density for a linear model ($\boldsymbol{b}_1$, $\mu_{r,1}$=1000, *top left*) and its non-linear correction ($\boldsymbol{b}_2$, *top right*); another non-linear correction ($\boldsymbol{b}_2$, from $\mu_{r,1}$=780, *bottom right*); final relative permeability ($\mu_{r,2}$, *bottom right*).

Change from ideal to real flux tubes

A SP q can first consider ideal tubes [7], i.e. surrounded by perfect flux walls through which $\boldsymbol{n} \cdot \boldsymbol{b}_q|_{\gamma_q}$ is zero and $\boldsymbol{b}_q$ and $\boldsymbol{h}_q$ outside are zero [3-4]. The complementary trace $\boldsymbol{n} \times \boldsymbol{h}_q|_{\gamma_q}$ is unknown and non-zero. Consequently, a change to a permeable flux wall defines a SP p with SSs opposed to this non-zero trace. This change can be done simultaneously with a material change (Fig. 3): a leakage flux solution $\boldsymbol{b}_3$ can complete an ideal distribution $\boldsymbol{b}_1$ while knowing the source $\boldsymbol{b}_2$ proper to the inductor; this allows independent overlapping meshes for both source and reaction fields.

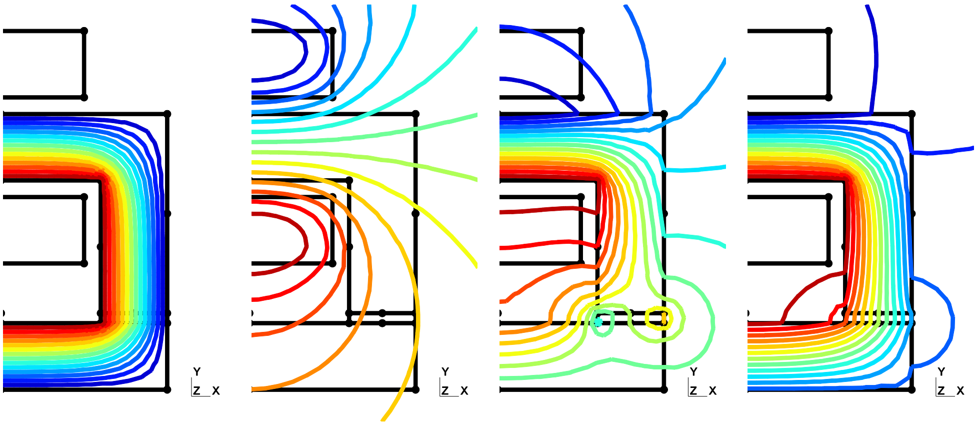

Fig. 3. An electromagnet: field lines in an ideal flux tube ($\boldsymbol{b}_1$, $\mu_{r,core} = 100$), for the inductor alone ($\boldsymbol{b}_2$), for the leakage flux ($\boldsymbol{b}_3$) and for the total field ($\boldsymbol{b}=\boldsymbol{b}_1+\boldsymbol{b}_2+\boldsymbol{b}_3$) (*left to right*).

Change from 1-D to 3-D

The SPM can be applied for coupling solutions of various dimensions, starting from simplified models, based on ideal flux tubes defining 1-D models, that evolve towards 2-D and 3-D accurate models [5].

Series connections of models of lower dimensions are direct applications requiring such changes. A violation of ICs when connecting two models can be corrected via SSs in opposition to the unwanted discontinuities. An accurate series connection of two 1-D flux tubes needs a changes from 1-D to 2-D (Fig. 4). Also, the connection of 2-D models needs a 3-D correction model for an accurate consideration of 3-D effects (Fig. 5).

Change from ideal to real flux tubes can be extended to allow a dimension change, e.g. from 2-D to 3-D (Fig. 6): a 2-D solution is first considered as limited to a certain thickness in the third dimension, with a zero field outside; on the other side, another independent SP is solved. Changes of ICs on each side of this portion, via SSs, then allow the calculation of 3-D end effects. The successive corrections of the flux linkage, from 1-D to 3-D, are shown in Fig. 7.

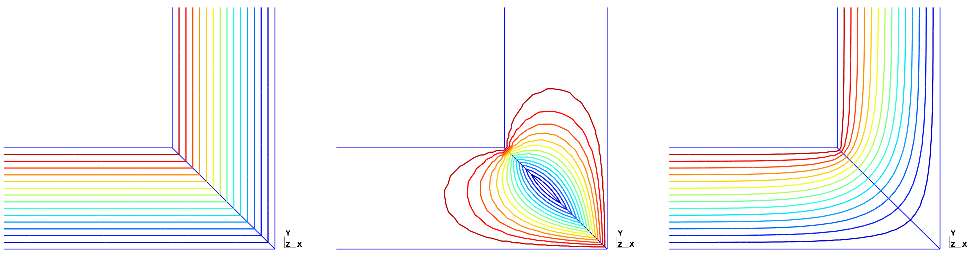

Fig. 4. Series connection of two flux tubes: field lines in 1-D ideal flux tubes ($\boldsymbol{b}_1$, *left*), 2-D local correction at the junction ($\boldsymbol{b}_2$, *middle*) and complete 2-D solution ($\boldsymbol{b}=\boldsymbol{b}_1+\boldsymbol{b}_2$, *right*).

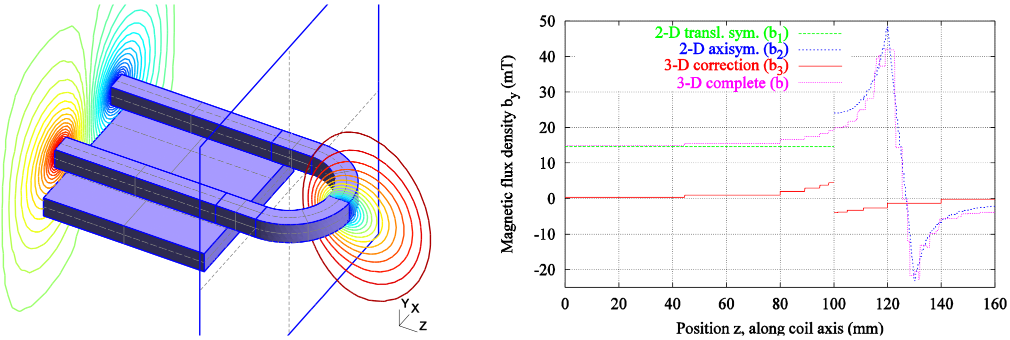

Fig. 5. Field lines (*left*) generated by a stranded inductor (half geometry): solution of a 2-D plane model in the XY plane (z = 0) ($\boldsymbol{b}_1$, portion on the *left*) and of a 2-D axisymmetrical model in the YZ plane (x = 0) ($\boldsymbol{b}_2$, portion on the *right*); the interface between the two portions is shown. Magnetic flux density (*right*) along the coil axis in the 3-D system: $\boldsymbol{b}_1$ for the 2-D plane model (implicitly extended as a constant up to z = 100 mm), $\boldsymbol{b}_2$ for the 2-D axisymmetrical model, $\boldsymbol{b}_3$ for the 3-D correction and $\boldsymbol{b}$ for the complete 3-D model. The solution $\boldsymbol{b}_1$+$\boldsymbol{b}_2$+$\boldsymbol{b}_3$ is generally obtained with a higher accuracy than $\boldsymbol{b}$ for a lower computational cost thanks to the coupling of meshes, some of lower dimensions.

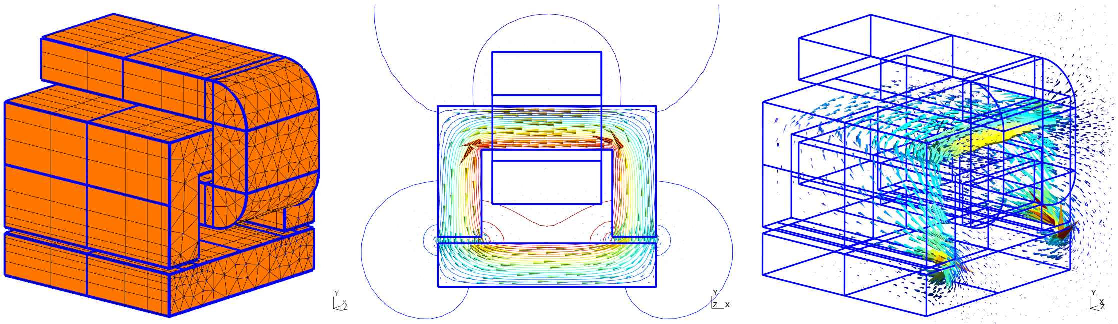

Fig. 6. 3-D model of an electromagnet (*left*), 2-D cross section and solution (magnetic flux density and field lines) (*middle*), 3-D correction of the magnetic flux density (*right*).

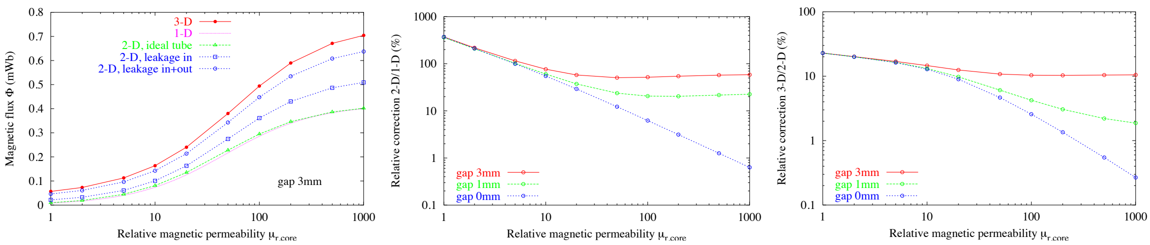

Fig. 7. Inductor flux linkage versus the core magnetic permeability (air gap thickness of 3 mm) updated after each model refinement (*left*); flux linkage relative correction from 1-D to 2-D models (*middle*) and from 2-D to 3-D models (*right*) versus the core magnetic permeability for different air gap thicknesses.

Change from perfect to real materials

A SP q can first consider perfect conducting (resp. magnetic) materials [2], with $\sigma_q \rightarrow \infty$ (resp. $\mu_q \rightarrow \infty$), in which case the trace $\boldsymbol{n} \cdot \boldsymbol{b}_q|_{\gamma_q}$ (resp. $\boldsymbol{n} \times \boldsymbol{h}_q|_{\gamma_q}$) on its boundary is zero and $\boldsymbol{b}_q$ (resp. $\boldsymbol{h}_q$) inside is zero. The complementary trace $\boldsymbol{n} \times \boldsymbol{h}_q|_{\gamma_q}$ (resp. $\boldsymbol{n} \cdot \boldsymbol{b}_q|_{\gamma_q}$) is unknown and non-zero. Consequently, a change to a finite σ_p (resp. μ_p) defines a SP p with SSs opposed to this non-zero trace.

As an illustration, an induction heating core-inductor system is initially modeled with a perfectly conducting core, then corrected for a finite conductivity (Fig. 8). After correction, a very good accuracy, in particular in the vicinity of the conductor corners, is obtained (Fig. 9), also in comparison with the impedance BC (IBC) technique. Initially perfectly conducting inductors can be also considered, followed by the correction of their eddy current density for an accurate calculation of impedance and Joule losses (Fig. 10).

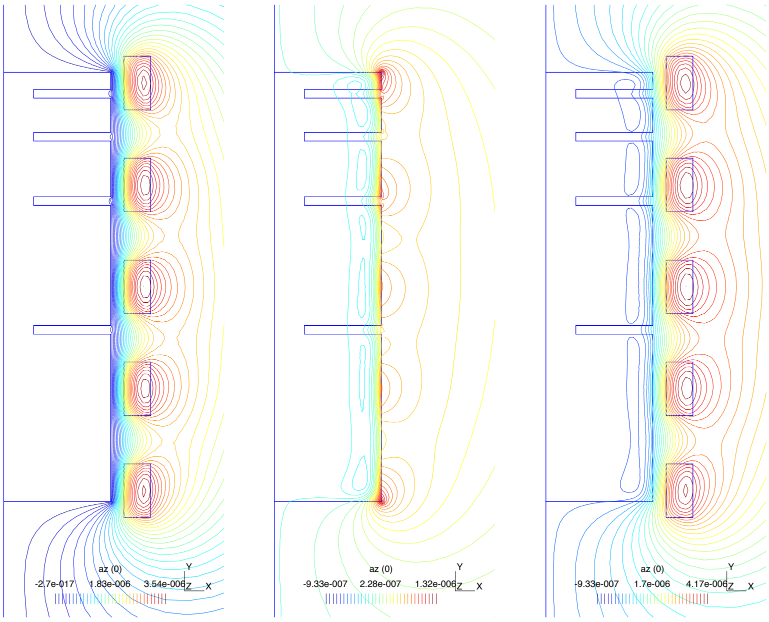

Fig. 8. Core-inductor system: magnetic flux lines for the perfectly conducting core (non-magnetic) ($\boldsymbol{b}_1$; *left*), the finite conductivity correction (aluminium core) ($\boldsymbol{b}_2$; *middle*) and the complete solution ($\boldsymbol{b}=\boldsymbol{b}_1+\boldsymbol{b}_2$; *right*).

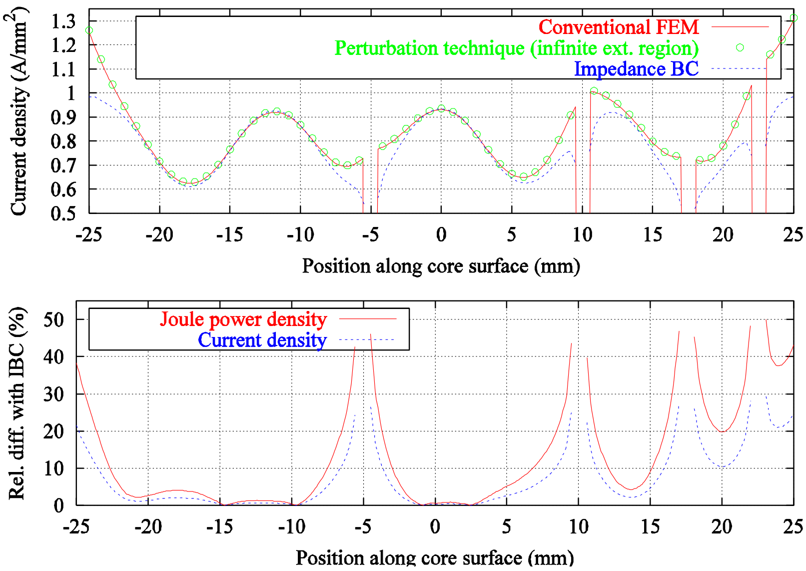

Fig. 9. Eddy current density along the (conductive non-magnetic) core surface for the conventional FE solution, the SPM and the IBC technique (*top*); relative difference between solutions of the last two techniques (*bottom*).

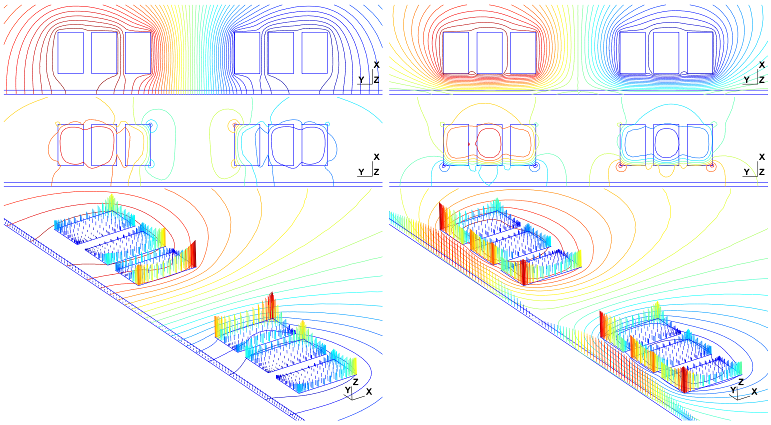

Fig. 10. Transverse flux system (3-turn inductor above a half plate, with perpendicular flux horizontal symmetry axis below); a low (*left*) and a high (*right*) electric conductivity are considered for the plate; from *top* to *bottom*: magnetic flux lines (phase 0) for the reference solution $\boldsymbol{b}_1$ with a perfectly conductive inductor, the perturbation solution $\boldsymbol{b}_2$ and the perturbed solutions $\boldsymbol{b}$; *bottom*: current density distribution (modulus).

Conclusions

The developed SP FE method splits magnetic problems into SPs of lower complexity with regard to meshing operations and computational aspects. This allows a natural progression from simple to more elaborate models, while quantifying the gain given by each model refinement and justifying its utility.

Approximate problems with ideal flux tubes are accurately corrected when accounting for leakage fluxes and material changes. Also, reference solutions related to limit behaviors of conductors (perfectly conductive or magnetic nature) can be used in several subproblems for accurate calculation of the field distribution in real materials and the ensuing losses. This allows efficient parameterized analyses on the electric and magnetic characteristics of the conductors in a wide range, i.e. on the parameters affecting the skin depth. Nonlinear analyses, e.g. with temperature dependent conductivities, could then benefit from this. The method is naturally adapted to parameterized analyses on geometrical and material data.

All the constraints involved in the subproblems have been carefully defined in the associated FE formulations, respecting their inherent strong and weak nature. As a result, local fields and global quantities, i.e. flux, MMF, reluctance, voltage, current, resistance, are efficiently and accurately calculated.

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