

## A MULTIAXIAL CONCRETE MODEL FOR APPLICATIONS IN STRUCTURAL FIRE ENGINEERING

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**Abstract.** *Temperature-dependent material models are required in numerical softwares dedicated to the nonlinear analysis of structures in fire. Although structural concrete is widely used in civil engineering, proper modelling of its thermo-mechanical behaviour remains a challenging issue for engineers mainly because of the complexity of the phenomena that result from the microcracking process in this composite material and because of the lack of numerical robustness of the models. This paper presents a new multiaxial concrete model developed for the analysis of concrete structures in fire. The multiaxial model is based on a plastic-damage formulation and incorporates an explicit term for transient creep strain. After implementation in a finite element software for structural fire engineering calculations, numerical simulations have been performed to highlight the ability of the model to capture some of the main phenomena that develop in concrete (permanent strains, degradation of the elastic properties, unilateral effect) as well as the ability to be used for the fire analysis of large-scale structural elements.*

### 1 INTRODUCTION

In performance-based analysis, the response of structures subjected to thermo-mechanical loading is assessed by temperature-dependent calculations. These calculations require a general stress-strain relationship for modelling the behaviour of concrete at elevated temperatures. The concrete model should fulfil the criteria of reliability, accuracy and numerical robustness. Accuracy and reliability ensure that the model captures properly the behaviour of the concrete material in all the situations of stress and temperature in the applicability domain. Numerical robustness ensures that the model is applicable to complex and large-scale engineering problems. In addition, the model should contain a limited number of parameters that can be identified from elementary tests. The present research aims at developing a multiaxial model able to reproduce the phenomenological behaviour of concrete at elevated temperature and satisfying to the requirements of structural fire engineers.

The behaviour of concrete at the macroscopic level results from the initiation and growth of microcracks in the cementitious matrix. The microcracking process causes softening behaviour, stiffness degradation and unilateral effect. These observed phenomena can be captured by models within the framework of continuum damage mechanics. On the other hand, concrete exhibits inelastic volumetric expansion in compression, referred to as dilatancy in the literature. Proper modelling of dilatancy is very important for simulating concrete structures under multiaxial loading [1]. Dilatancy can be modelled by the development of plastic strains in concrete. Therefore, combination of the elastoplasticity theory with the damage theory results in an efficient strategy for modelling the mechanical behaviour of concrete.

Constitutive models for concrete at ambient temperature based on plastic-damage formulation have been proposed by several authors. Although concrete experiences different microcracking in different directions, models developed for structural applications usually combine plasticity with isotropic damage [1, 2, 3], in order to avoid the inherent complexities of numerical algorithms required by most of the anisotropic damage models [4, 5]. The isotropic damage process can be characterized by one scalar, several scalars or a tensor. The use of different scalars to capture the damage process in concrete [1] is consistent with the experimentally observed different damage mechanisms developing in tension and in compression; a minimum of two scalar variables are necessary to describe these different damage mechanisms. Some authors have proposed a fourth-order damage tensor to characterize the state of isotropic damage in concrete [5, 6], because a fourth-order tensor is required to capture the unilateral effect. As a result, even for isotropic damage, proper description of the damage state in concrete requires a fourth-order tensor based on two scalar variables. Stress-based plasticity may be formulated either in the effective stress space [1, 2, 5] or in the nominal (damaged) stress space [3, 7]. Effective stress is meant as the average micro-level stress applied to the undamaged volume of the material whereas nominal stress is meant as the macro-level stress and is defined as force divided by the total area. Formulation of the plastic response in the effective stress space relies on the assumption that plastic flow occurs in the undamaged material micro-bounds by means of effective quantities [8]. This formulation allows for decoupling the plastic part from the damage part in the computation process; computation of the plastic response then constitutes a standard elastoplastic problem in the effective stress space. As a result, the combination of stress-based plasticity formulated in the effective stress space and isotropic damage constitutes an interesting approach for modelling the behaviour of concrete.

At elevated temperatures, the situation is even more complex due to additional phenomena that develop in heated concrete. Elevated temperatures are the cause of degradations at the micro-level that result in loss of stiffness and strength of the material. Moreover, a particular phenomenon appears in concrete subjected to elevated temperatures: the transient creep strain. Physically, the transient creep strain is the difference in strain between concrete that is heated under load and concrete that is loaded at elevated temperature; this strain develops during first-time heating and is irrecoverable [9]. Transient creep strain has to be incorporated in any constitutive model dedicated to fire-exposed structural concrete. Due to the high complexity of the many phenomena involved, few multiaxial models coupling plasticity and damage have been proposed for the mechanical behaviour of concrete at elevated temperature; an example of such model combining stress-based plasticity in the effective stress space and isotropic damage, extended at elevated temperature, has been proposed by Nechnech *et al.* [10]. Yet, research efforts are still required to give further insight into concrete modelling at elevated temperature and to extend the latest developments of ambient temperature models to elevated temperature; this is the purpose of the present study.

## 2 PRESENTATION OF THE MODEL

### 2.1 Assumptions

The mechanical behaviour of concrete at elevated temperatures is captured by constitutive relationships between the total strain tensor and the stress tensor. The total strain  $\underline{\underline{\epsilon}}_{tot}$  is decomposed into free thermal strain  $\underline{\underline{\epsilon}}_{th}$ , transient creep strain  $\underline{\underline{\epsilon}}_{cr}$ , elastic strain  $\underline{\underline{\epsilon}}_{el}$  and plastic strain  $\underline{\underline{\epsilon}}_p$  according to Eq. (1).

$$\underline{\underline{\epsilon}}_{tot} = \underline{\underline{\epsilon}}_{th} + \underline{\underline{\epsilon}}_{cr} + \underline{\underline{\epsilon}}_{el} + \underline{\underline{\epsilon}}_p \quad (1)$$

The sum of the elastic strain and the plastic strain is referred to as instantaneous stress-related strain  $\underline{\underline{\epsilon}}_{\sigma}$ . Basic creep strain is not taken into account here but this term could easily be added to the strain decomposition.

The characterization of plastic response is formulated in the effective stress space. The elastic strain tensor is related to the effective stress tensor  $\underline{\underline{\sigma}}$  by means of the fourth-order isotropic linear-elastic stiffness tensor  $\underline{\underline{C}}_0$ , see Eq. (2).

$$\underline{\underline{\sigma}} = \underline{\underline{C}}_0 : \underline{\underline{\varepsilon}}_{el} = \underline{\underline{C}}_0 : (\underline{\underline{\varepsilon}}_{\sigma} - \underline{\underline{\varepsilon}}_p) \quad (2)$$

The plastic response accounts for the development of irreversible strains in the material. Yet, the degradation of the elastic properties resulting from the development of microcracks is not addressed at this stage; the unloading stiffness in the effective stress space remains equal to the isotropic linear-elastic stiffness.

Concrete exhibits different damage mechanisms in tension and in compression. In this model, a tensile damage scalar and a compressive damage scalar are adopted to capture the phenomenological effects induced by microcracking in concrete under tension and compression, respectively. Based on the work by Wu *et al.* [5], these two damage scalars lead to a fourth-order damage tensor employed to characterize the state of isotropic damage in concrete. The use of a fourth-order damage tensor allows for appropriate description of the unilateral effect inherent to concrete behaviour. Mapping of the effective stress  $\underline{\underline{\sigma}}$  into the nominal stress  $\underline{\underline{\sigma}}$  is performed by this fourth-order isotropic damage tensor  $\underline{\underline{D}}$  according to Eq. (3), where  $\underline{\underline{I}}$  is the fourth-order identity tensor.

$$\underline{\underline{\sigma}} = \left( \underline{\underline{I}} - \underline{\underline{D}} \right) : \underline{\underline{\sigma}} \quad (3)$$

It is assumed that the plasticity and damage phenomena are coupled and evolve simultaneously in the material; therefore, the two phenomena are driven by the same internal variables in the model.

Finally, the new concrete model is a fully-3D constitutive model that can be used with solid finite elements or with shell finite elements; in the latter case, the plane stress version of the model is used.

## 2.2 Plasticity

A composite yield surface is used for capturing the concrete non-symmetrical behaviour in tension and in compression; a Rankine yield criterion is used to limit the tensile stresses and a Drucker-Prager yield contour is used for compression. The equations of the composite yield surface are written in terms of effective stresses, see Eq. (4).

$$f_t(\underline{\underline{\sigma}}, \kappa_t) \leq 0 \quad ; \quad f_c(\underline{\underline{\sigma}}, \kappa_c) \leq 0 \quad (4)$$

In Eq. (4),  $\kappa_t$  and  $\kappa_c$  are the tensile and compressive hardening parameters, respectively.

Plastic flow rules have to be postulated to govern the evolution of plastic flow when the effective stress state reaches the yield surfaces. As concrete is a frictional material, in which dilatancy occurs when loaded in compression, a non-associated flow rule is adopted in compression. The plastic flow rules, in combination with the Kuhn-Tucker and consistency conditions, allow for calculation of the accumulated plastic strains in tension and compression; these accumulated plastic strains are chosen as internal variables in the model. The hardening parameters depend on the accumulated plastic strains and therefore are induced by plastic flow; these hardening parameters govern the evolution of the yield surface through the definition of the hardening laws.

## 2.3 Damage

The isotropic state of damage of concrete is addressed by a fourth-order damage tensor which is calculated from the tensile damage scalar  $d_t$  and the compressive damage scalar  $d_c$  as given by Eq. (5). In this equation, the fourth-order projection tensors based on the eigenvalues and eigenvectors of the effective stress tensor are noted  $\underline{\underline{P}}^+$  and  $\underline{\underline{P}}^-$ .

$$\underline{\underline{D}} = d_t \underline{\underline{P}}^+ + d_c \underline{\underline{P}}^- \quad (5)$$

The projection tensors allow for a decomposition of the effective stress tensor into positive and negative components. As a result, the tensile damage scalar only affects the positive part of the effective stress tensor whereas the compressive damage scalar only affects the negative part of the effective stress tensor. Hence, the unilateral effect is captured without the need for an additional parameter.

By assumption, damage mechanism is coupled to plasticity in the model. Consequently, there is no specific threshold for damage and the evolution laws for tensile and compressive damage are driven by the accumulated plastic strains (in tension and compression, respectively). Once convergence has been obtained in the plastic return mapping algorithm, update of the damage variables is thus an explicit calculation.

#### 2.4 Transient creep strain

In the numerical calculation process, computation of the increment in transient creep strain is performed at the beginning of the time step ( $s$ ), separately from the computation of elastic and plastic strains. The Explicit Transient Creep (ETC) Eurocode model, developed at University of Liege for uniaxial relationships [11, 12], is extended to the multiaxial case by adopting the methodology proposed by de Borst and Peeters [13], see Eq. (6). In Eq. (6),  $f_{ck}$  is the compressive strength at 20°C; the function  $\Phi(T)$  is the transient creep function given in Table 1 and the fourth order tensor  $\underline{\underline{H}}$  is given by Eq. (7).

The material parameter  $\gamma$  that appears in Eq. (7) can be taken equal to Poisson's ratio [10], in accordance with Thelandersson's multiaxial data [14].

$$\Delta \underline{\underline{\varepsilon}}_{cr} = \left[ \phi \left( T^{(s)} \right) - \phi \left( T^{(s-1)} \right) \right] \left[ \underline{\underline{H}} \otimes \left( \underline{\underline{\sigma}}^- \right)^{(s-1)} / f_{ck} \right] \quad (6)$$

$$H_{ijkl} = -\gamma \delta_{ij} \delta_{kl} + 0.5(1 + \gamma) \left( \delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk} \right) \quad (7)$$

Table 1. Transient creep function  $\Phi(T)$ .

T [°C]	20	100	200	400	600	800
$\Phi$ [-]	0.0000	0.0010	0.0018	0.0049	0.0274	0.0733

Accordingly, it is assumed that the process of transient creep does not induce any anisotropy. The negative part of the effective stress tensor is considered in Eq. (6) because, on the one hand, transient creep strain is assumed to occur only in compression and, on the other hand, this mechanism occurs in the undamaged part of the material.

Computation of the transient creep strain increment takes into account the stress-temperature history. Between step ( $s$ ) and ( $s-1$ ), there is an increment in transient creep strain, which value is computed by Eq. (6), if and only if the three following conditions are fulfilled: the temperature has increased between step ( $s$ ) and ( $s-1$ ), the negative part of the (converged) effective stress at time ( $s-1$ ) is non-null, and the material is in the ascending branch of the constitutive relationship. It is thus assumed that the positive part of the effective stress tensor does not induce transient creep strain. Besides, the transient creep strain is irreversible at both load and temperature decrease.

#### 2.5 Model parameters

The model contains ten material parameters that can be obtained by three basic tests: uniaxial compression test until failure comprising one unloading-reloading at peak stress, biaxial compression test until peak stress, and uniaxial tension test until failure.

Concrete subjected to elevated temperatures exhibits thermo-mechanical degradation of its properties of strength and stiffness; this effect is taken into account through proper temperature dependency of the

material parameters. The evolution laws of the parameters with temperature are taken from design codes such as the Eurocode, when available, or from experimental data published in the literature.

### 3 VALIDATION OF THE MODEL

The concrete model has been validated against experimental data. The ability to capture the concrete behaviour for simple loading cases, e.g. uniaxial tension and compression, at both ambient and elevated temperature, is not demonstrated in this paper, due to the need to be concise. However two experimental tests are simulated here to demonstrate the ability of the new concrete model to capture the main phenomena observed in concrete material and the ability to be used for the simulation of structural members in fire. The numerical simulations have been conducted with the software SAFIR [15].

#### 3.1 At ambient temperature

A test of uniaxial tension followed by uniaxial compression on a concrete sample has been simulated using the new concrete model and the results are compared against experimental data [16], see Figure 1. The numerical results obtained with an elastoplastic concrete model are also plotted on Figure 1. The new concrete model succeeds at capturing the development of permanent strains, the degradation of the elastic properties and the stiffness recovery due to crack closure (unilateral effect), whereas elastoplastic models can only capture the development of permanent strains.

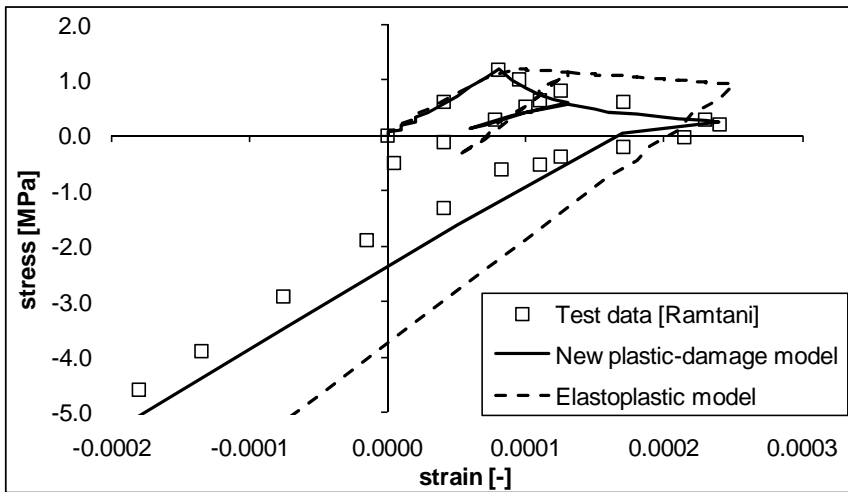


Figure 1. Numerical simulation of a uniaxial tension-compression test.

#### 3.2 At high temperatures

The concrete model is validated at the structural level against experimental results on a reinforced concrete flat slab in fire. The experimental test has been carried out at BRANZ and is described in a paper by Lim et al. [17]. The tested slab is 3.30 m wide by 4.30 m long with a clear span between the supports in the long and the short directions of 4.15 m and 3.15 m, respectively. The slab is simply supported at all four edges with the edges horizontally unrestrained. The flat slab is 100 mm thickness and is reinforced by 200 mm<sup>2</sup>/m steel reinforcement in each direction. The yield strength of the steel used in the slab is 565 MPa whereas the concrete compressive strength on cylinder is 37 MPa. The concrete cover is 25 mm. The slab was subjected to ISO fire exposure for 3 hours while carrying a constant uniformly distributed live load equal to 3.0 kPa. The slab, which deformed into double curvature, survived the 3 h ISO fire exposure without collapse.

Numerical simulation of this experiment has been performed with the software SAFIR. First, the thermal analysis is conducted to determine the temperature distribution in the concrete slab during fire. The thermal properties for concrete are taken from Eurocode 1992-1-2. Siliceous concrete was chosen, with a density of 2400 kg/m<sup>3</sup> and a water content of 72 kg/m<sup>3</sup>. The emissivity was taken as 0.7 and the coefficient of convection was 25 W/m<sup>2</sup>K. Temperatures in the slab were recorded during the test at the heated surface, at the unheated surface and at 55 mm depth within the slab. Figure 2 gives the comparison between the temperatures predicted by SAFIR and the measured temperatures at these locations; predicted and measured temperatures agree well.

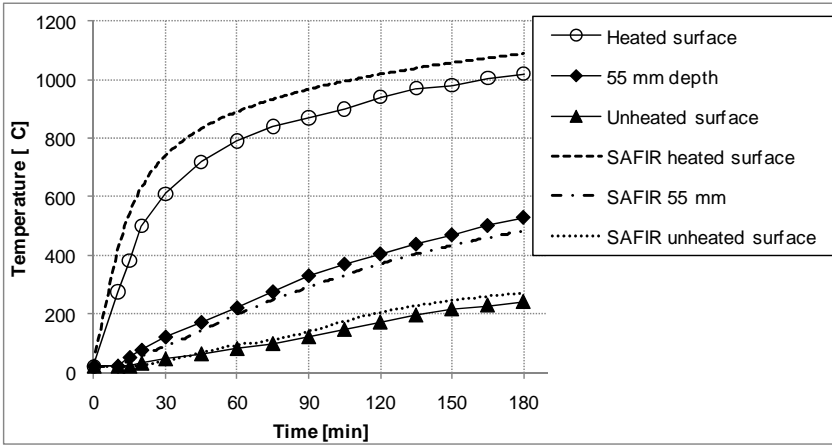


Figure 2. Computed and measured temperatures in the concrete slab.

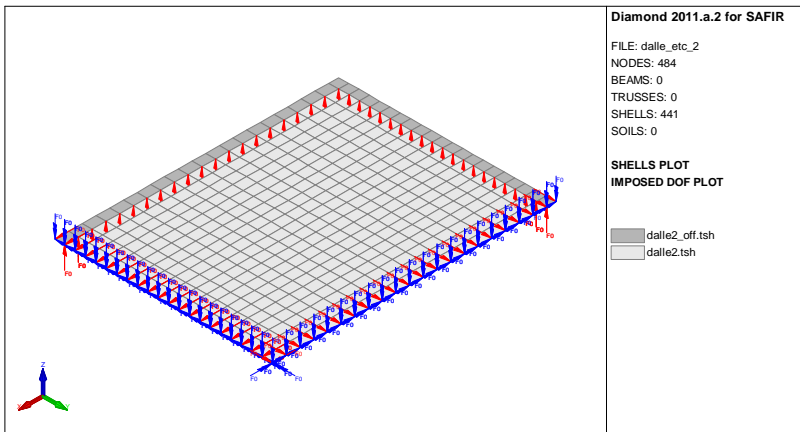


Figure 3. Finite elements model of the flat slab.

Then, the structural analysis is carried out to determine the structural behaviour of the reinforced concrete slab in fire. Shell finite elements are used for modelling the slab. Only a quarter of the full slab was modelled to take advantage of the symmetrical load and support conditions, see Figure 3. The slab is subjected to a uniformly distributed load of 5.4 kPa which represents the sum of the self-weight, 2.4 kPa, and the live load, 3.0 kPa. This applied load of 5.4 kPa corresponds to a load ratio of approximately 0.40 for this slab. The temperature evolution in the slab is given by the SAFIR thermal analysis. The concrete

model presented in this paper is used for the thermo-mechanical behaviour of concrete whereas the material model for the steel reinforcement is taken from Eurocode 1992-1-2. The concrete compressive and tensile strength are 37.0 MPa and 1.0 MPa. The other material parameters of the concrete model are calibrated on elementary tests and no additional calibration is required on the concrete slab.

The predicted and measured vertical deflection at mid-span of the slab in fire is shown in Figure 4. The results of the numerical simulation using the new concrete model agree with the experimental results. The numerical simulation has also been performed using the elastoplastic concrete model currently implemented in SAFIR; however this numerical simulation suddenly stopped before the end of the calculation due to numerical problems in the integration of the concrete law. Numerical robustness is a major issue in concrete modelling and an important requirement in structural fire engineering, as typical applications include large structural elements subjected to complex stress-temperature histories. The present example illustrates the efforts that have been made to ensure the numerical robustness of the new concrete model.

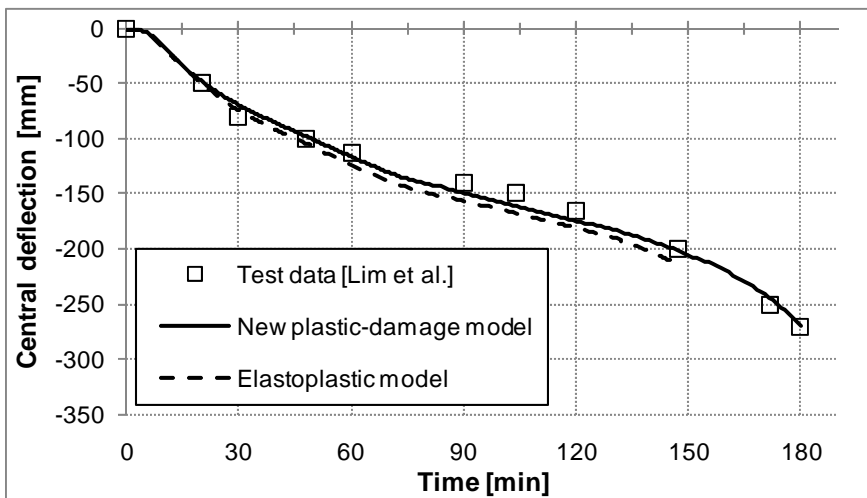


Figure 4. Computed (SAFIR) and measured values of the mid-span vertical deflection.

#### 4 FIRE ANALYSIS – REHABILITATION OF A CONCRETE SHELL ROOF

In the framework of a concrete building rehabilitation, the fire resistance of a shell roof structure was studied for *ICB-Ingénieurs Conseils en Bâtiments*. This practical example is used here to illustrate the ability of the concrete model to be used for structural fire engineering applications.

The SAFIR finite elements model of the structure is shown on Figure 5. Only half of the roof has been modelled with proper symmetry conditions imposed at the edge of the model. As the shell structure has variable thickness, 10 different sections have been used in the model with a thickness varying from 160 mm to 100 mm. The steel tie beams have been modelled using beam finite elements. The concrete shell roof is reinforced in both directions by means of steel reinforcement mesh. The concrete compressive strength is 20 MPa, the reinforcement steel yield strength is 400 MPa and the tie beams yield strength is 235 MPa. The fire analysis of the structure is performed under self-weight loads.

At room temperature, collapse of the structure arises by yielding of the steel tie beams when the applied loads exceed the load bearing capacity of the structure. Yielding of the steel beams leads to decrease in their stiffness; therefore, the horizontal force transmitted by the concrete shell roof is not equilibrated any more by the steel tie beams and the structure collapses. Figure 6 shows the membrane forces in the structure loaded at room temperature. The applied loads are transmitted by the concrete shell

through compressive forces toward the supports and the steel tie beams. The concentration of the forces at the location of the steel tie beams is clearly visible in Figure 6.

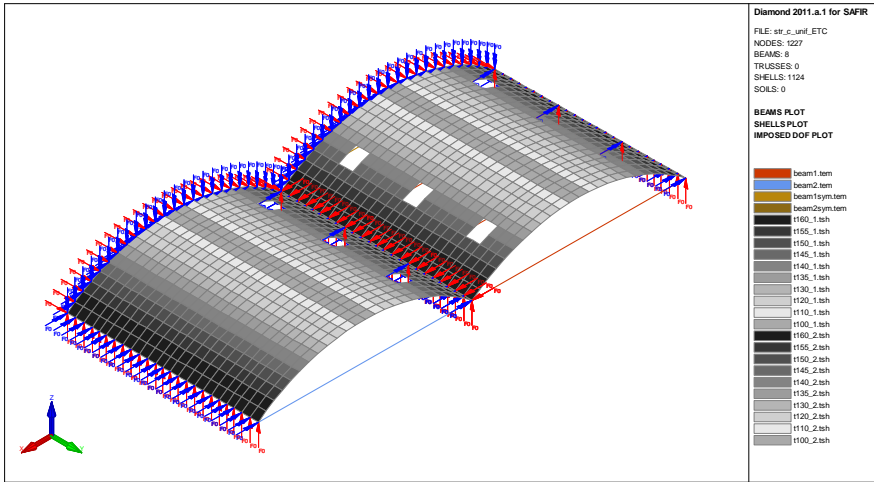


Figure 5. SAFIR structural model of the shell roof subjected to fire.

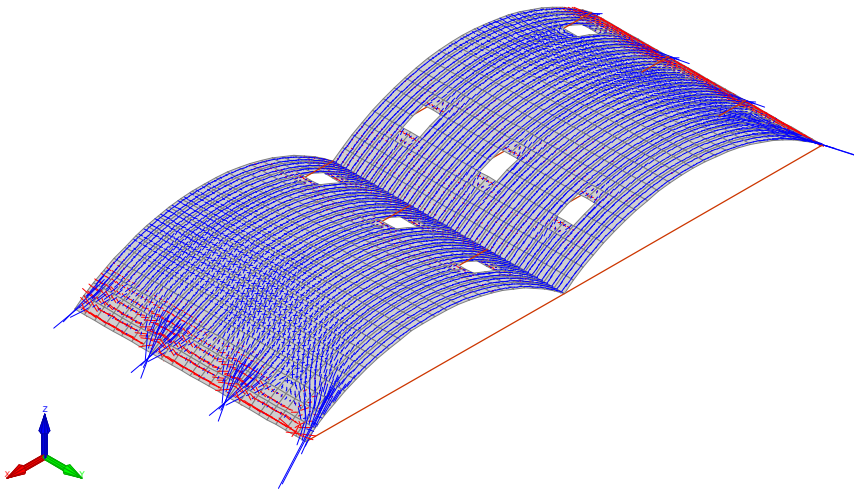


Figure 6. Membrane forces in the structure loaded at room temperature (SAFIR model).

At elevated temperature, the temperature increase in the steel tie beams leads to thermal expansion and decrease in stiffness and strength of these beams. As a consequence, the concrete shells lose their horizontal supports and the structure stiffness decrease. Finally, collapse arises as shown in Figure 7. If the steel tie beams are left unprotected, the fire resistance of the structure is lower than 15 minutes. It is found that collapse arises at a time when the temperature in the steel tie beams reaches approximately 550°C; therefore an efficient way to improve the fire resistance of the structure is to limit the temperature increase in the steel tie beams.

The robustness of the multiaxial concrete model is highlighted by this example as it was possible to simulate the structural behaviour of a large structure in fire until collapse. The consistency of the concrete

model for capturing the behaviour of a complex structure is highlighted by the analysis of the membrane forces distribution on Figure 6.

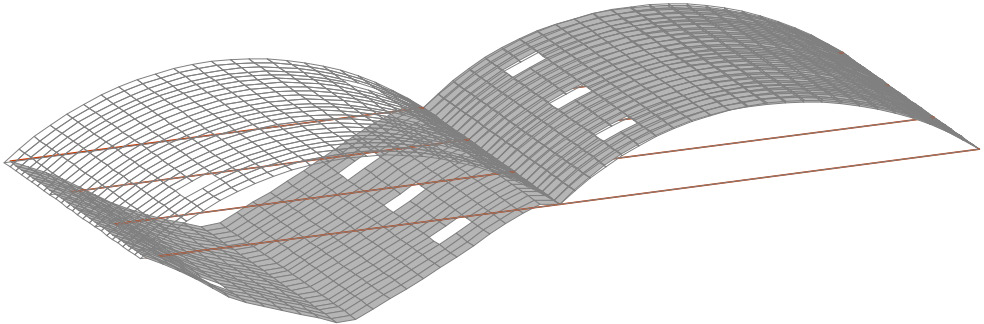


Figure 7. Deformed shape at collapse of a concrete shell roof subjected to fire (SAFIR model).

## 5 CONCLUSION

A plastic-damage concrete model has been developed and implemented in the software SAFIR for the analysis of structures in fire. The model is able to capture the phenomenological behaviour of concrete at ambient and elevated temperature and is sufficiently robust to be used in large-scale fire analysis. The model parameters can be identified by three elementary tests: uniaxial compression, uniaxial tension and biaxial compression. Transient creep strain is explicitly computed and takes into account the stress-temperature history in the material.

Validation of the new model has been performed against experimental data given in the literature at ambient and elevated temperature. It has been shown that the plastic-damage model accurately captures the unilateral effect due to the closing of the tensile cracks during unloading from tension to compression, whereas elastoplastic models are unable to capture this effect. At the structural level, the model has been validated by comparison against a test carried out at BRANZ on a reinforced concrete flat slab, 3.30 m wide by 4.30 m long, subjected to ISO fire exposure during 3 hours. Finally, the ability of the model to be used in large-scale structural simulations has been illustrated by presenting the fire analysis of a concrete shell roof, in the framework of the rehabilitation of a concrete building.

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