Solving the m-TSP Problem with Stochastic or Time Dependent Demands

by

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Abstract

There are many examples of problems in transportation where some elements are uncertain. In the distribution of goods as well as systems responding to calls for emergency, demands typically occur in a random fashion. Transportation systems have thus to be created in face of uncertainty about future levels of demands, making strategic decisions difficult to take. Similarly, traffic conditions vary randomly over time and travel routes are usually designed in face of uncertainty about traffic conditions, hence about effective travel times. Stochastic models, i.e. models that take uncertainty explicitly into account, have thus a central role to play in transportation.

Even if decision makers realize the existence of uncertainty, most used decisions models are deterministic, i.e. assume all data to be known with certainty. The reason for such a choice is that stochastic models are typically more difficult to solve than the deterministic ones. The common tendency is to solve very detailed deterministic models. As perfect forecasting does not exist, real data are often different from the data used in the models. This may result in poor decisions being made.

To avoid such pitfalls, one may first advocate the use of stochastic models when uncertainty tends to play a significant role. In particular, it may be wise to solve a more simplified version of a model, but, at the same time, allow for some data to be random. (For a general presentation of stochastic programming, see Birge and Louveaux (1997)). Next, one needs more research effort in view of developing efficient solution methods of stochastic programs in transportation. Typical solution methods currently include the design of heuristics, asymptotic analysis of algorithms and some exact methods in location and routing. The present research goes into this last direction. For a recent methodological survey on stochastic routing, see Gendreau, Laporte, Séguin(1996). Other important areas of transportation where stochastic models play a significant role include airline yield management, (Brumelle and McGill,1993), dynamic vehicle allocation problems (Frantzeskakis and Powell,1990), the location of hazardous materials (Boffey and Karkazis ,1995) or the location of emergency units (Berman et al,1985), to cite a few examples.

In this paper, we analyze the variations in travel (and or service) times. Following Malandraki and Daskin (1992), we observe that, in a congested urban environment, fluctuations in traffic density may cause fluctuations in travel speed that result in variations of travel times. One component is the variation due to accidents, weather conditions or any other random events. This variation is best modelled as a random travel time. This randomness may be included into all types of transportation models. In this paper, we will consider the example of the TSP. To be more precise, we will thus consider the variant of the TSP with stochastic travel time, which will be denoted by STTSP (for Stochastic Travel Time TSP). Another component of this variation, which may cause travel times to increase dramatically during rush hours, is the temporal variation that results from the hourly, daily or seasonal cycles in the average traffic volumes. This variation is typically modelled as a variable travel time and results in a Time Dependent TSP, denoted by TDTSP.

The classical m-TSP consists of finding the optimal routes for m identical vehicles, starting and leaving at the depot, such that every client is visited exactly once. There are several ways to include the effect of random or variable travel times into the model. Typically, one considers a

threshold value, say T, corresponding to the maximal duration of a route. One way to model uncertainty is to request that the probability of exceeding the time T is low. This has been studied in Laporte et al (1989).

Here, we address the case where a penalty is paid for excess duration. The penalty may be proportional to the amount of overtime (time in excess of T). It may correspond for instance to extra payments to the driver, which are usually proportional to the excess duration. In other cases, the penalty is paid whenever the vehicle misses the target time T. The penalty is independent of the amount of overtime. One such example is considered in Laporte et al (1993), where the penalty is proportional to the value of the truckload. The (STTSP) TDTSP then consists of minimizing the cost of travelling the planned routes plus the (expected) penalty for overtime.

For the stochastic case, such an assessment belongs to the family of a priori optimization. In routing problems, a priori optimization consists in defining a planned route as well as a simple rule to cope with uncertainty. See Bertsimas et al (1990) for a description of other a priori optimization problems and their asymptotic analysis. In the present context, it means that planned routes are not reoptimized along the way. In other words, the idea is to find routes which are robust in view of their expected performance.

Earlier works that incorporate variability or stochasticity in the travel times include analysis of some heuristic solutions. The TDTSP has been studied in Malandraki and Daskin (1992). The travel time on an arc is represented by a step function, with a number of different time intervals. In each of these intervals, the travel time is constant. As an example, an arc may receive three time intervals, one for peak hours in the morning, one for normal conditions in the day and one for the peak hours in the evening. Obviously, the step function on an arc (i,j) may be different from the step function on arc (j,i).

In the STTSP, the travel time on an arc is described by a random variable. Laporte et al.(1993) consider the case where the penalty is proportional to the truckload value. Verweij et al. (2003) consider the case where the penalty is proportional to the excess duration. They discuss an implementation of the Sample Average Approximation method.
It is the purpose of this paper to study each one of these variations and to show that they can be solved through advanced implementations of the integer_L-shaped algorithm of Laporte and Louveaux(1993). In terms of the objective function, this method can handle both types of penalties (proportional or not proportional to the excess duration) and the two types of modelling (TDTSP and STTSP). It is simply required that the (expected) penalty can be computed for any given route.

For the TDTSP, this simply means considering the two directions (clockwise or anticlockwise) to compute the minimal time and the potential penalty. An interesting additional feature is that the representation of the variability in the travel time is not restricted to a piecewise constant function (such as the one used in Malandraki and Daskin (1992)). A more general function can be used, such as a piecewise linear function. Such a representation has the advantage that, for any arc and any arrival time at a node of an arc, the departure time can be made a monotone function of the arrival time.

For the STTSP with proportional penalty, the expected penalty term depends on $E(\xi-T)^+$, where ξ is the sum of travel times along the route. This calculation is standard in probability, provided the distribution of ξ can be computed. This is typically the case with summable distributions such as Normal or Poisson for which analytical expressions for $E(\xi-T)^+$ are available .
For the STTSP with not proportional penalty, the expected penalty term depends on $\Pr(\xi \geq T)$, which is easily computable under the same conditions. In the stochastic case, the expected penalty is independent of the direction followed.

The two additional ingredients for an efficient implementation are the development of good lower bounds on the penalty term and the development of lower bounding functionals that also hold at fractional points (see also Laporte et al (2003)). These aspects are problem dependent and are discussed in more details in the paper.

We illustrate here one example for the case with proportional penalty. Consider the STTSP with travel times following independent normal distributions. To obtain a good lower bound for the penalty term of the STTSP, we solve two auxiliary problems. The first one consists of finding the TSP with minimal expected travel time. The second one consists of finding the TSP with minimal total variance. Now consider the normal variable ξ whose expectation (resp. variance) is the solution of the first (resp. second) auxiliary problem. It is easy to show that the value of $E(\xi\text{-}T)^+$ for this particular ξ gives a lower bound on the expected penalty. Even if this calculation seems to be costly (solving two deterministic TSP before starting the STTSP), this approach pays in practice. Firstly, because the bound is of reasonably good quality. Second, because the various solutions obtained while finding them can be evaluated for the STTSP. Similarly, adding lower bounding functionals prove also to be efficient.

A more involved calculation is required for the m-STTSP (m-TSP with stochastic travel times). We will discuss these bounds and report on the results of numerical experiments.

References

Berman O., R.C. Larson, S.S. Chiu (1985) Optimal server location on a network operating as a M/G/1 queue, Operations Research 33, 746-770

Bertsimas D., P. Jaillet, A. Odoni (1990) A priori Optimization, Operations Research 38, 1019-1033.

Birge J., F. Louveaux (1997) Introduction to stochastic programming, Springer-Verlag, New York.

Boffey B., J. Karkazis eds (1995) The location of hazardous materials facilities, Location Science, Vol. 3, n. 3

Brumelle S.L., J.I. McGill (1993) Airline seat allocation with multiple nested fare classes, Operations Research 41, 127-137

Dror M., P. Trudeau (1986) Stochastic vehicle routing with modified savings algorithm, European Journal of Operations research 23, 228-235

Frantzeskakis L., W. Powell (1990) A successive linear approximation procedure for stochastic dynamic vehicle allocation problems, Transportation Science 24, 40-57

Gendreau M., G. Laporte, R. Séguin(1996), Stochastic vehicle routing, European Journal of Operations Research 88,3-12.

Laporte G., F. Louveaux (1993), The integer L-shaped method for stochastic integer programs with complete recourse, Operations Research Letters 13, 133-142

Laporte G., F.Louveaux, V.Lambert (1993),Designing money collection routes through bank branches, Computers and Operations Research 20,783-791

Laporte G., F. Louveaux, H. Mercure, (1989), Models and exact solutions for a class of stochastic location-routing problems, EJOR 39, 71-78.

Laporte G., F.Louveaux, H.Mercure(1993), The vehicle routing problem with stochastic travel times, Transportation Science, 26,161-170

Laporte G., F. Louveaux, L. Vanhamme (2002), An integer L-shaped algorithm for the capacitated vehicle routing problem with stochastic demands, Operations Research 50, 415-423.

Malandraki C., M.S. Daskin (1992), Time Dependent Vehicle Routing Problems: Formulations, Properties and heuristic Algorithms, Transportation Science 26, 185-200

Ruszczynski A., A. Shapiro (eds), (2003), Stochastic programming, Handbooks in OR & MS, Vol. 10, Elsevier Science, Amsterdam

Verweij B., S. Ahmed, A.J. Kleywegt, G.L. Nemhauser, A. Shapiro (2003), The sample average approximation method applied to stochastic routing problems: a computational study. Computational Optimization and Applications 24,289-333