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ABSTRACT

The present study which relies on a simulation program integrating, on a section, the first three eigen modes of the vertical and torsional movements, concern a parametric approach to the phenomenon of galloping. Our aim is to discover the specific relations necessarily connecting the structural parameters of the lines liable to galloping. The determining factor will be shown to be the torsion of the conductors as a catalyst of the phenomenon. Three essential aspects will be discussed :
 -the use of four dimensionless parameters ; the mechanism of galloping depends exclusively on them in the case of bundle conductors.
 -a parametric study of the stability.
 -an original approach to the analytical predetermination of the maximum amplitudes.
 This will call some commonly accepted ideas into question. We will draw a critical range of the wind speed favourable to galloping.
 Our calculations are made on the basis of the aerodynamic curves produced by various laboratories for ice coatings observed in the field.
 Finally we will examine the effect of some anti-galloping devices (pendulums)

INTRODUCTION

Galloping is a phenomenon which affected the first high voltage overhead lines. It can appear when the meteorological conditions are favourable to ice formation around the conductors. The ice coating tends to develop dissymmetrically because the matter preferentially settles on the sides facing the wind or the precipitation. As a result the profile is rarely cylindrical and most often is excentered. Whereas on the cylindrical profile the aerodynamic force is reduced to the drag, f_D , parallel to the wind speed, on a dissymmetrical profile it is accompanied by a lift component, f_L , perpendicular to the wind direction, and by a couple F_M . The three values f_D, f_L, F_M are proportional to the square of the relative wind speed (v_r) seen by the conductor in movement :

$$f_D = v_r^2 k_D C_D(\varphi) \quad f_L = v_r^2 k_D C_L(\varphi) \quad F_M = v_r^2 k_M C_M(\varphi)$$

$$k_D = \frac{1}{2} \rho_{air} \phi_{cond} \quad k_M = \frac{1}{2} \rho_{air} \phi_{cond}^2$$

The aerodynamic and dimensionless coefficients C_D, C_L and C_M are even or odd functions of the yaw angle, φ , and of the relative wind with respect to the ice coating, as illustrated for two typical profiles by the diagrams of Fig. 1 & 2.

By modifying the relative wind, any movement of the conductor thus brings about drag, lift, and moment variations. Particular formation conditions can bring the yaw angle into areas in which the derivatives of the aerodynamic coefficients are such that the conductor movement associated with some eigen modes of oscillation goes together with lift variations in phase with speed ; the drag damping effect can then be compensated and there appears a sustained oscillation called galloping. Galloping can be insignificant, of low amplitude, or momentary ; but it can also bring the phase into clearing distance and sometimes it lasts so long that it ruptures

some components through fatigue. Although they are exceptional, such accidents are serious and some lines seem to be more prone to them than others. Even though the orientation with respect to the prevailing winds is definitely the major factor, an unfortunate combination of the mechanical parameters (sagging tension, the length of the span, the mass of the conductors, bundle geometry, number of spans per section) can also be regarded as likely to increase the undesirable effects of the climate constraints. The present work aims at establishing the conditions in which galloping will start in order to establish from the planning stage of a line that the stability ensured with an adequate safety margin and to foresee the nature and number of the additional anti-galloping devices required.

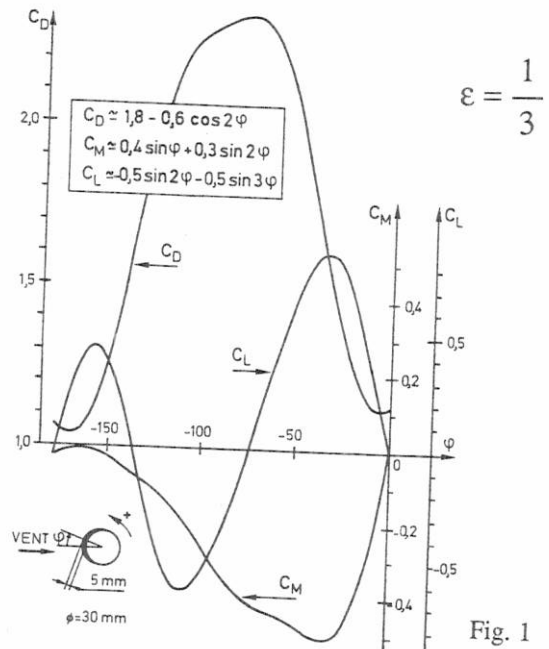


Fig. 1

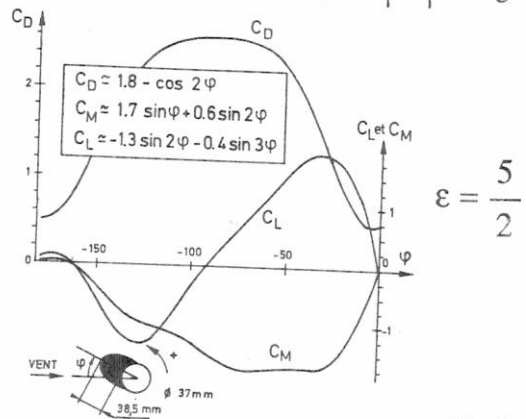


Fig. 2

Fig.1 et 2 coefficients of drag C_D , of lift C_L , and of moment C_M for two typical ice coating. (Fig.1 low excentricity, source O.NIGOL, Fig.2 high excentricity, source /11/)

MATHEMATICAL FORMULATION

The starting mechanisms outlined in the introduction and formalized by Den-Hartog et al.'s theory /1/ disregards the torsion of the conductors. But, as shown by experience, an oscillation of the torsion often appears when there is an oscillation in the vertical plane. For some authors this oscillation of the torsion is simply a response to the periodical pitching moment undergone by the ice coating exposed by its vertical movement to the pulsations of the relative wind /6/ ; they refused to admit the torsional movement as a destabilizing effect. It seems to us that such an opinion cannot be accepted without reservations, particularly because the condition which creates vertical galloping without torsion leads us to expect unstable areas clearly narrower than that in which the phenomenon of torsion is included. When only the fundamental mode is affected, the vertical displacement of the conductor, $y(z,t)$, and its rotation, $\vartheta(z,t)$, are expressed in the form :

$$y(z,t) = y_1(t) \sin \frac{\pi z}{L} \quad \vartheta(z,t) = \vartheta_1(t) \sin \frac{\pi z}{L}$$

The equations of dynamic equilibrium in sag and in torsion associated with the fundamental mode are then written, putting $\delta = \pi z / L$

$$\ddot{y}_1 + \omega_v^2 y_1 = \frac{2}{\pi} \int_0^\pi \left(\frac{k_D v_r^2}{m} (C_D(\varphi) \sin \alpha + C_L(\varphi) \cos \alpha) - g \right) \sin \delta \, d\delta \quad (1)$$

$$\ddot{\vartheta}_1 + b_1 \dot{\vartheta}_1 + \omega_t^2 \vartheta_1 = \frac{2}{\pi} \int_0^\pi \frac{k_M v_r^2}{I} C_M(\varphi) \sin \delta \, d\delta$$

v_r, α, φ functions of δ are defined by (see Fig. 3)

$$\begin{aligned} v_r \sin \alpha &= -\dot{y}_1 \sin \delta \\ v_r \cos \alpha &= U_0 \\ \varphi &= \vartheta_g + \vartheta_1 \sin \delta - \alpha \end{aligned} \quad (2)$$

By inserting the relations of (2) into (1) we obtain :

$$\ddot{y}_1 + \omega_v^2 y_1 = \frac{2}{\pi} \int_0^\pi \left(\frac{k_D v_r}{m} (-y_1 C_D(\varphi) + U_0 C_L(\varphi)) - g \right) \sin \delta \, d\delta \quad (3)$$

$$\ddot{\vartheta}_1 + b_1 \dot{\vartheta}_1 + \omega_t^2 \vartheta_1 = \frac{2}{\pi} \int_0^\pi \frac{k_M v_r^2}{I} C_M(\varphi) \sin \delta \, d\delta$$

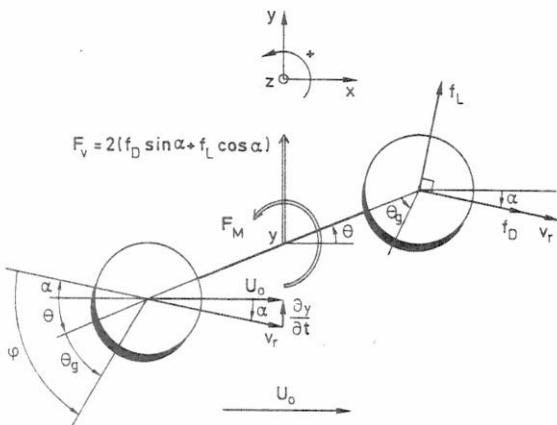


Fig.3 Definitions of the angles and of the applied forces.

If we take the highest mode into account we obtain similar equations, but

$$\delta = \frac{k\pi z}{L}$$

(k : rank of the mode) . When we consider the mechanical tensions the other spans of the sections must be taken into account . We must consider the displacement of the suspension strings (for further information see /7./) In the present paper the consideration of the other spans amounts to taking account all the possible ω_v obtained by modal analysis of the section.

STEADY-STATE STABILITY

The purpose of the study of steady-state stability is the detection of unstable equilibriums. For each value of the

layer angle and of the wind speed U_0 , the conductor takes up an equilibrium position, solution of the system of non-linear equations that is obtained by reducing the temporal derivatives in (3) to zero. Let $y_1 \epsilon, \vartheta_1 \epsilon$, be such a solution around which the stability of low amplitude discrepancies Δy_1 and $\Delta \vartheta_1$ will be studied.

Taking (2) into account, the equations (3) linearized around the equilibrium position become :

$$\Delta \ddot{y}_1 + \frac{k_D U_0}{m} (\bar{C}_D - \partial C_L) \Delta \dot{y}_1 + \omega_v^2 \Delta y_1 = \frac{k_D U_0^2}{m} \partial C_L \Delta \vartheta_1 \quad (4)$$

$$\Delta \ddot{\vartheta}_1 + b_1 \Delta \dot{\vartheta}_1 + (\omega_t^2 - \frac{k_M U_0^2}{I} \partial C_M) \Delta \vartheta_1 = \frac{k_M U_0}{I} \partial C_M \Delta \dot{y}_1$$

with : (cf Fig. 4)

$$\begin{aligned} \bar{C}_D &= \frac{2}{\pi} \int_0^\pi C_D \sin^2 \delta \, d\delta & \partial C_L &= \frac{2}{\pi} \int_0^\pi \frac{\partial C_L}{\partial \varphi} \sin^2 \delta \, d\delta \\ \partial C_M &= \frac{2}{\pi} \int_0^\pi \frac{\partial C_M}{\partial \varphi} \sin^2 \delta \, d\delta \end{aligned} \quad (5)$$

The aerodynamic coefficients or their derivatives appearing under the integral are calculated for :

$$\varphi = \vartheta_g + \vartheta_1 \epsilon \cdot \sin \delta$$

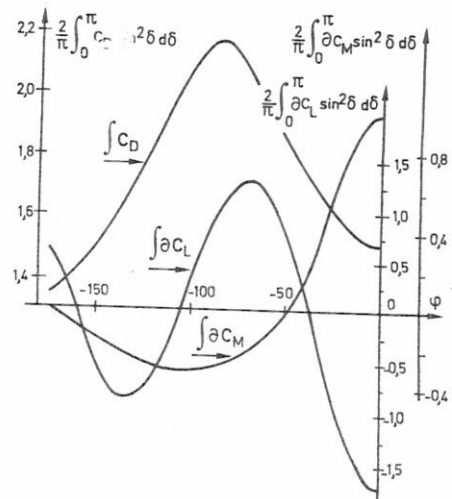


Fig. 4 Mean aerodynamic coefficients or their derivatives (corresponding to Fig. 1 coefficients)

For convenience's sake, system (4) will be written :

$$\begin{aligned} \Delta \ddot{y}_1 + a_1 \dot{\Delta y}_1 + a_2 \Delta y_1 &= a_3 \Delta \dot{\theta}_1 \\ \Delta \ddot{\theta}_1 + b_1 \dot{\Delta \theta}_1 + b_2 \Delta \theta_1 &= b_3 \dot{\Delta y}_1 \end{aligned} \quad (6)$$

Coefficient b_2 is the square of the eigen torsional pulsation in the presence of wind.

If there is no torsion only the first equation subsists, without right-hand member and the instability corresponds to $a_1 < 0$. Den-Hartog's condition /1/.

In a general way the stability of the system (6) is studied by expressing that the eigenvalues, solution of the characteristic equation, are with negative values. The characteristic equation is written :

$$\lambda^4 + (a_1 + b_1)\lambda^3 + (b_2 + a_2 + a_1 b_1)\lambda^2 + (a_1 b_2 - a_3 b_3 + a_2 b_1)\lambda + a_2 b_2 = 0 \quad (7)$$

By applying a Routh-Hurwitz's criterion to the equation we obtain - when all the calculations have been made - the conditions of stability.

$$a_2 b_2 > 0 \quad (8.1)$$

$$a_1 + b_1 > 0 \quad (8.2)$$

$$a_1 b_1^2 + (a_1^2 + b_2) b_1 + a_1 a_2 + a_3 b_3 > 0 \quad (8.3)$$

$$\begin{aligned} a_1 a_2 b_1^3 + a_1 [a_1 (a_2 + b_2) - a_3 b_3] b_1^2 \\ + \{ (b_2 - a_2) [a_1 (b_2 - a_2) - a_3 b_3] + a_1^2 (a_1 b_2 - a_3 b_3) \} b_1 \\ + a_3 b_3 [a_1 (b_2 - a_2) - a_3 b_3] > 0 \end{aligned} \quad (8.4)$$

The condition (8.1) is automatically checked, for the conductor cannot stay at a value of θ_{1g} for which $b_2 < 0$ and will spontaneously change into a stable angle of rotation for which $b_2 > 0$.

In the absence of torsional damping the condition (8.2) merges into Den-Hartog's criterion. But such an assumption is quite imaginary because it would place the system out of physical reality : as soon as the phenomenon of torsion appears it is always accompanied by a dissipation of energy, although sometimes at a low rate. Giving it a zero value is the wrong way of expressing the problem. It can clearly be observed from (8.2) that the torsional damping already takes its positive effect under the form of a positive term in (8.2).

The last two conditions arise from taking into account the phenomenon of torsion. New fields of instability appear that are unpredictable in function of Den-Hartog's criterion.

Before I give the results, notice that by defining a reduced amplitude by $\omega \Delta y / U_0$ the equations (4) and consequently the criteria of stability depend only - for particular aerodynamic coefficients - on the following four dimensionless coefficients :

$$P_1 = \frac{\omega_v}{\omega_t} \quad P_2 = \frac{k_M U_0}{2 \omega_t I} \quad \zeta_1 = \frac{k_D U_0}{b_1 m} \quad \xi = \frac{b_1}{2 \omega_t} \quad (9)$$

These coefficients are dealt with in /7/ to extract the important structural parameters :

-The torsional damping ξ (10)

-The relation between the vertical and torsional frequencies $P_1 = \omega_v / \omega_t$ (11)

-The relation between the diameters conductor/bundle :

$$P_3 = \frac{\sqrt{P_2}}{\zeta_1 \xi} = 200 \frac{\phi}{d} \quad (12)$$

⤴ (particular cases of ACSR conductors)

- The reduced wind speed P_4 :

$$P_4 = \frac{P_2}{\zeta_1 \xi} = 8 \frac{\phi U_0}{d^2 \omega_t} \quad (13)$$

-The reduced amplitude A_R :

$$A_R = 8 \frac{\phi \cdot A_{pk-pk}}{d^2 \cdot P_3} = \frac{A_{pk-pk}}{25 \cdot d} \quad (14)$$

⤴ (particular cases of ACSR conductors)

PARAMETRIC STUDY OF THE STABILITY

Neither the structural parameters nor the wind speed appear in Den-Hartog's condition, only the aerodynamic coefficients of drag and lift (or rather the derivative of the latter).

When applied to the profiles for which we have the aerodynamic curves this condition is never checked. The layer angles most likely to satisfy the condition are situated opposite the set of the wind ($\theta_g = 180^\circ$), where C_D is minimal, and the derivative of the lift positive (see our conventions of signs).

Under some conditions (irregularity and shape of the ice layer, turbulent wind, etc...) C_D may be low enough and the derivative of the lift high enough to create a small area of layer angle favourable to galloping. This was experimentally checked on artificial ice coating /3/.

The condition (8.4) seems to us more prone to account for the appearance of galloping : on the basis of the experimental curves we can see that the critical areas of layer angles are situated between 0 and -100° (upper quadrant, windward) and better fit the actual conditions of the formation of an ice coating. In the condition (8.4) the structural conditions of the line and the wind speed are the determining factors. We shall make a further analysis of it.

If we want to make it more suggestive the condition (8.4) can conveniently be converted into a graph parametered by the dimensionless values explained above. In function of the layer angle θ_g Fig. 5 and 6 thus give the value of ξ for which (8.4) is equal to zero. The values of ξ indicate the lower limits of the damping coefficient beyond which stability is maintained. Such a presentation is all the more justified since an a priori value of ξ cannot easily be determined, the data concerning torsional damping being still quite few.

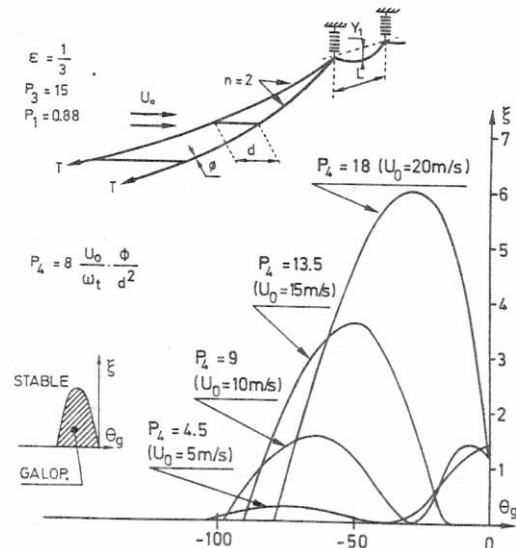


Fig. 5 Minimum torsional damping which ensure the stability of the bundle, vs. ice accretion angle and for some different P_4 values.

Fig. 5 is parametered in function of P_4 ; in order to make things clear the values of P_4 are accompanied by the corresponding U_0 values for the following data :
 $\omega_v = 1.23 \text{ rad/s}$, $\omega_t = 1.43 \text{ rad/s}$, $\phi = 32.3 \text{ mm}$, $I = 0.15 \text{ Kg.m}$,
 $m = 3.4 \text{ Kg/m}$, $L = 360 \text{ m}$, $\text{sag} = 8 \text{ m}$, $d = 0.45 \text{ m}$

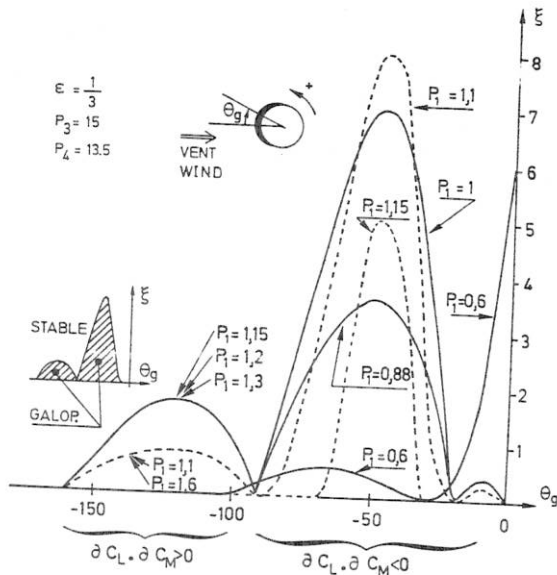
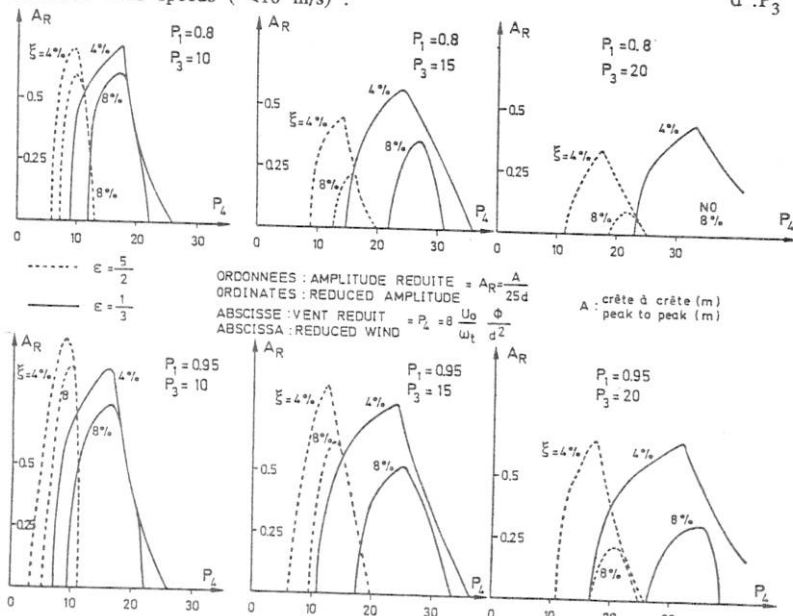


Fig. 6 Minimum torsional damping which ensure the stability of the bundle, vs. ice accretion angle and for different $P_1 = \omega_v / \omega_t$ values.

Fig.6 is parametered in function of P_1 , the value of which can be reduced by the presence of pendulum type devices. Without any devices, the value of P_1 remains slightly under 1 for all the bundled lines. ($P_1 = 0.95$). This confirms that torsion place a determining role in the conditions under which galloping occurs. We can see (see Fig. 5) that above a certain wind speed (or rather for certain values of P_4) the condition of instability is no longer respected. The critical speed of the Belgium line is of the order of 30 m/s for a low eccentricity of the coating and of 20 m/s for a high eccentricity. We shall come back to this point later when estimating the amplitudes. In the particular case examined, torsional dampings, even if they are small, are clearly sufficient to inhibit the phenomenon of galloping, at least for the most commonly observed wind speeds (<10 m/s).



PARAMETRIC STUDY OF THE AMPLITUDE

In Fig. 7 the reduced amplitude is plotted vs P_4 . The curves are parametered in function of the torsional damping (ξ) and of the eccentricity of the ice layer (ϵ). For each curve the relations of the frequencies (vertical/torsional) - P_1 - and of the diameters (conductor/bundle) - P_3 - are constant.

In order to eliminate the parameter of the ice layer angle, the whole windward upper quadrant will be scanned until we find the layer angle corresponding to the maximum amplitude, the only one to be preserved.

Observations :

1) As foreseen the amplitude increases as P_1 gets closer to the unit.
 2) When moving from 4% to 8% the torsional damping noticeably reduces the amplitude. The rate of increase becomes higher when P_3 becomes higher (remember $P_3 = 200 \phi / d$ for ACSR conductors)

3) The eccentricity of the ice has hardly any influence on the maximum amplitude. This remarkable conclusion can be drawn from the results of the calculations made on 2 very different ice layers (5 mm and 3.5 cm) ; their experimental aerodynamic curves were produced by two different laboratories (typical ice coatings). The range of variation of the wind yaw angle during the limit cycle thus seems to be independent on the eccentricity. Such a significantly wide variation of the yaw angle leads us to replace the specific values of the aerodynamic coefficients and of their derivatives by average values calculated on intervals of approximately 40° to 60° : if this is done, local irregularities are smoothed out and the curves become uniform. The yaw angle corresponding to the limit cycle is connected with the average values of the aerodynamic coefficients giving the zero value to the stability criterion.

4) The maximum amplitude moves towards lower wind speed as the ice coating eccentricity increases.
 5) The wind range connected with the appearance of galloping can be seen in Fig. 8. The wind range is affected only by ξ and the eccentricity.
 6) On the bases of the curves in Fig. 7 one can give an analytical expression of the maximum (peak to peak) amplitude in function of the reduced parameters. After calculating we obtain :

$$8 \frac{\phi \cdot A_{pk-pk}}{d^2 \cdot P_3} = (-5 \xi^2 - 0.016) \cdot P_3 + 0.5 P_1^2 + 0.7 \quad (15)$$

Numerical application : For conductors ACSR in bundles with stranded cables ($\xi = 6\%$) :

$$\frac{A_{pk-pk}}{25 \cdot d} = -0.034 P_3 + 0.5 P_1^2 + 0.7 \quad (16)$$

Without any anti-galloping devices, we have $P_1 = 0.95$ and the expression becomes :

$$A_{pk-pk} = -170 \cdot \phi + 29 \cdot d \quad (17)$$

Where we immediately obtain the maximum peak to peak amplitude in meters although we only know the diameter of one subconductor (ϕ in m.) and the diameter of the bundle (d in m., equal to the distance between subconductors in the case of twin bundle).

Fig. 7 Reduced amplitude A_R vs. reduced wind speed P_4 for several P_1 and P_3 and for two different eccentricity and two different torsional damping.

In a classical bundle it is thus better to increase the diameter of the conductors (for example by using a smaller number of conductors) and to decrease the diameter of the bundle - subject to the constraints imposed on the constructor by the corona effect, the phenomenon of kissing, and the subspan oscillations- .
The use of so-called 'smooth' conductors would enable us to reduce the maximum amplitude by significantly increasing ξ

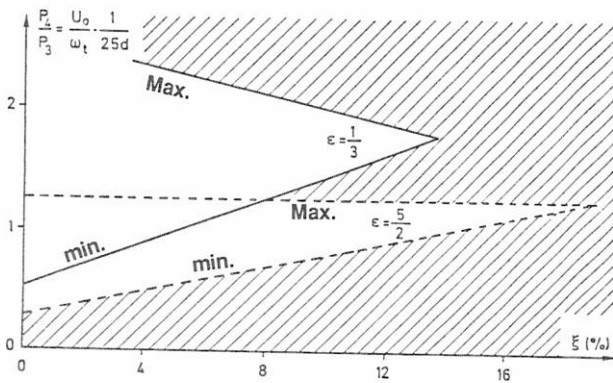


Fig. 8 Reduced wind speed ranges prone to galloping vs. torsional damping.

In some cases new wind ranges favourable to galloping can appear when the maximum amplitude is reduced by appropriate devices.

We should add that it is highly probable that ξ increases with the age of the conductor but it is difficult to quantify this effect.

7) In the case of several spans we simply have to use, in P_1 evaluation, the much more numerous eigen frequencies corresponding to the various modes (symmetrical, antisymmetrical, etc...); the wind speed must of course remain in credible range.

8) The treatment of a multi-modal galloping is more problematic for when we have several unstable modes only one of them can reach its maximum value. By reducing the ranges of P_4 open to galloping, torsional damping is again favourable to the limitation of the combined multi-loops galloping.

It can also be asserted that wind speed favours a certain amount of loops. Indeed, for a given range of P_4 , only one given frequency corresponds for a given wind.

EFFECT OF A DEVICE OF THE PENDULUM TYPE

Modification of the dimensionless parameters :

The presence of pendulums as suggested by /4/ increases the stiffness and the moment of inertia of the phase. It can be shown that in the cases with which we are dealing (bundle) the stiffness increase is greater than the increase of the moment of inertia and thus pendulums contribute to increase the torsional frequency .

The new frequency is estimated : it will be introduced into P_1 . Parameters P_3 and P_4 are also slightly modified by a factor k ($0.8 < k < 1$) depending the number and locations of pendulums /7/ (new $P_3 = k \cdot P_3$)

Expression 17 becomes (with pendulums bringing P_1 back from 0.95 to 0.8)

$$A_{pk-pk} = -170.k^2.\phi + 25.k.d \quad (18)$$

Note that the modification of the torsional frequency shifts the area favourable to galloping towards higher winds (see Fig. 8). The displacement of this area account for the fact that two phases of an experimental line - one with pendulums and the other without - do not always gallop simultaneously.

However, the phase fitted with pendulums will reach a lower maximum amplitude. On the other hand, the efficiency of the presence of pendulums cannot be tested at the same wind speeds when they are no pendulums.

Numerical application :

Diameter of the subconductor : 0.03 m

Distance between the two subconductors : 0.45 m
Natural ξ of the line : 6%

max. amplitude without pendulum, eq. (17) : 7.95 m

idem with pendulums, $P_1=0.8, k=0.8$, eq. (18) : 5.70 m

max. amplitude with pendulums and additional torsional damping, eq. (15) :

4.4 m	if $\xi = 8\%$
2.66 m	if $\xi = 10\%$

CONCLUSIONS

Trying to give a general explanation of galloping without taking into account the phenomenon of torsion is pointless. It alone accounts for the structural parameters and for the existence of a critical interval for the wind speeds that cannot be explained by Den-Hartog's theory. Remember that Den-Hartog's theory is based exclusively on the aerodynamic properties of the ice coating. Besides, given the shape of the aerodynamic curves, Den-Hartog's criterion is not likely to be checked if the ice coating is windward. Ice naturally increases on the side of the conductor facing the precipitation. We must admit that if galloping is to occur, the ice coating must turn because of the effect of the pitching moment thanks to a low torsional stiffness. Such a situation may appear with simple conductors but not with bundles which would then always be preserved against galloping. But this conclusion is contradicted by the facts.

Our study draws attention to a value rarely dealt with in the relevant publications, i.e. torsional damping. This value is the major factor in helping us to understand the comparative rarity of galloping and the gradual and even total disappearance of the conditions which created in the course of time (line ageing).

The ratio between the vertical and torsional frequencies is missing from Den-Hartog's criterion and yet we have just demonstrated its determining importance, which can be forefelt.

In the course of our study we showed the impact of putting in pendulums separating the torsional frequency from the vertical frequency.

The introduction of a parameter depending mostly on the simple ratio between the diameter of the conductor (ϕ) and the diameter of the bundle (d) means that this ratio should be increased, subject - of course - to the constructor's constraints connected with other electrical or mechanical effects (corona effect, subspan oscillations, kissing).

Finally, an analytical expression of the peak to peak amplitude is proposed as follows :

$$A_{pk-pk} = -a.\phi + b.d$$

where 'a' and 'b' are positive coefficients depending on the torsional damping (for 'a') and on the ratio between the vertical and torsional frequencies (for 'b').

This formula does not depend on the shape of the ice coating, which is accounted for in our report.

Finally the formula shows the importance of limiting the number of subconductors in the bundle, even if it means increasing the diameters of each subconductors and decreasing the bundle diameter.

Any reluctance to use conductors with large diameters is therefore unjustified (for bundle configuration).

We are continuing our research, particularly on the field of a single conductor per phase. Concerning bundles we have completed Nigol's theory /8/ by introducing a supplementary term into the expression for the torsional stiffness. The presence of such an analytically deduced term accounts for the discrepancies of more than 50% discovered by Nigol between his theoretical assumptions and some of his experimental results.

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