A Mixed-Integer Heuristic for the Structural Optimization of a Cruise Ship.

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Abstract

A heuristic approach is proposed to solve the structural optimization problem of a cruise ship.

The challenge of optimization is to define the scantling of the structure of a ship in order to minimize the weight or the production cost. The variables are the dimensions and positions of the constitutive elements of the structure: they are discrete by nature. The objective functions are nonlinear functions. The structure is submitted to geometric constraints and to structural constraints. The geometric constraints are linear functions and the structural constraints are implicit functions requiring a high computation cost. The problem belongs to the class of mixed-integer nonlinear problems (MINLP).

A local heuristic of the type "dive and fix" is combined with a solver based on approximation methods. The solver is used as a black-box tool to perform the structural analysis and solve the nonlinear optimization problems (NLP) defined by the heuristic. The heuristic is designed to always provide a discrete feasible solution. Experiments on a real-size structure demonstrate that the optimal value of the mixed-integer problem is of the same magnitude as the optimal value of the optimization problem for which all the variables can take continuous values.

1. Introduction

In the domain of naval architecture, structural optimization of a cruise ship occurs at the stage of the proposal, the earliest phase of a project. Preliminary ship sizing and structural design always pose difficult problems to designers. They have to make the most adequate choices within a very short period of time. This happens in numerous industries working with large projects whose characteristics are that the product is unique and has to be custom-designed at the very beginning of the project or even before the client's order (i.e. naval or spatial structures).

The decisions taken during this preliminary design phase will greatly influence the subsequent steps of the production. Indeed, the preliminary structural design drastically limits the choices of production techniques and fixes the main constitutive elements of the structure. The constraints to take into account are the customer requirements concerning the ship characteristics such as, for example, speed or capacity.

The problem, as we formulate it, is to define the scantling of the constitutive elements of a structure modeled as a transversal cross-section of a cruise ship, composed of stiffened panels. The optimization is performed in order to minimize either the weight, the production cost or a combination of these two objectives. This choice has a great influence on the resulting structure. The design variables are the dimensions and positions of the constitutive elements of the structure: they have discrete values by nature. The structure is subject to geometric constraints and structural constraints. The geometric constraints ensure the feasibility of the structure (e.g. lower bounds on steel thickness) and the structural constraints model the response to solicitations and stresses.

The cost, the weight and the geometric constraints are nonlinear functions. The structural constraints are defined by implicit functions and their evaluation requires a high computation cost. This evaluation can be done thanks to analytical methods or to the use of simulation of mechanical models, such as finite element methods. The resulting model belongs to the class of mixed-integer nonlinear programming problems (MINLP).

2. Problem formulation

A ship is a large and complex steel structure whose study requires simplifications. The front and the back of the boat have particular shapes that are the object of other specific researches (Mesh models, finite element methods, etc.). We are concerned here with the structure between these two parts. The ship is considered as the repetition of similar structural pieces (Figure 1). Each piece is left-right symmetric such that the basic element to optimize is a half piece, as shown on figure 3. The basic element to optimize is an assembly of stiffened panels (fig. 2). The number, the arrangement, the width and the length of the panels are given data. The problem formulation used in this work is an adaptation of the model presented by Rigo for the optimization of stiffened structures (Rigo 2001a, 2001b, 2001c).

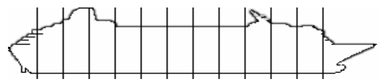

Fig.1: Decomposition of a ship into more simple elements

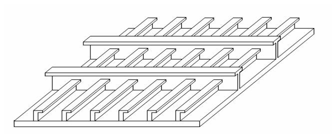

Fig.2: A stiffened panel

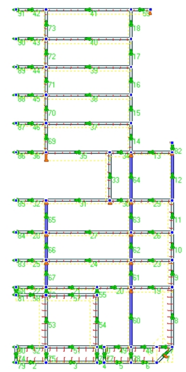

Fig.3: Cross section of a structure

2.1. Variables

Let P be the collection of panels of the structure. Each panel $p \in P$ of the structure is characterized by the plate thickness, as well as by the spacing and the dimensions of the members. The design variables apply to each stiffened panel p of the structure (see figure 4):

- δ^p is the plate thickness,
- h^p_x, d^p_x, w^p_x are the dimensions of web and flange of the longitudinals/stiffeners fitted along the X direction,
- h^p_y, d^p_y, w^p_y are the dimensions of web and flange of the transverse frames fitted along the Y direction,
- Δ^p_x is the spacing between two longitudinals/stiffeners fitted along the X direction,
- Δ^p_y is the spacing between two transverse frames fitted along Y,

Note that t^p_x and t^p_y are not design variables.

For convenience we use also the notation (v_1^p, v_2^p, v_3^p, v_4^p, v_5^p, v_6^p, v_7^p, v_8^p, v_9^p) to represent the vector of variables (δ^p, h^p_x, d^p_x, w^p_x, Δ^p_x, h^p_y, d^p_y, w^p_y, Δ^p_y) for each panel $p \in P$. We denote by $v_i = v_i^p$: $p \in P$ the vector of values assumed by variable v_i^p over all the panels, for each i=1,...,9. We also use the notation v to denote the vector (v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9).

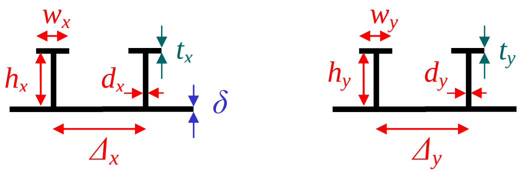

Fig.4: Design variables along x and y directions

The thickness and dimensions of the elements (δ^p, h^p_x, d^p_x, w^p_x, h^p_y, d^p_y, w^p_y) fall within a finite set of standard thicknesses and dimensions, as they are found in a catalog by a manufacturer. Therefore, we define for each variable and for each panel a discrete set of admissible values. The extreme values of a set are fixed by the technological constraints and we use a realistic discretization step. The definition of the admissible sets (one set for each panel and each variable) is of the following type:

$$v_i^p \in D_i^p \text{ with } D_i^p = \{ v_i^p{}_{min},\ v_i^p{}_{min} + \text{step}_i,\ v_i^p{}_{min} + 2*\text{step}_i,\ \ldots,\ v_i^p{}_{min} + n_i^p*\text{step}_i = v_i^p{}_{max}\}$$
$$i \in \{1,2,3,4,6,7,8\}$$

The number of members that have to be equidistantly fixed on a panel is integer by nature. Therefore, the spacing between two frames along the x direction (Δ^p_x) may take its value in a finite set of values. For each panel we determine the set of admissible spacings using the length of the panels L and the admissible number of members (n^p_{min},…n^p_{max}) for the panel p. The definition of the admissible set (one for each panel) is of the following type:

$$v_5^p \in D_5^p \text{ with } D_5^p = \{ v_5^p{}_{min}, \ldots, v_5^p{}_{max} \},\ v_5^p{}_{min} = L/n^p_{max},\ \ldots,\ v_5^p{}_{max} = L/n^p_{min}$$

We can not define such discrete sets of admissible values for the spacing of the members along the y direction : there are some correspondance constraints between the spacings of the longitudinal members of some panels and the widths of these panels are not identical. Thus, it is not possible to have a discrete set for the spacing of the longitudinals that would fit for every panel. It follows that, for each panel p, the spacing of the longitudinals may take its value in a set D_9^p that is a continuous interval : $v_9^p \in [v_9^p{}_{min}, v_9^p{}_{max}]$.

2.2. Objective functions

Two objective functions are modeled: the weight and the production cost of the structure. These are nonlinear functions in terms of the design variables.

- **Weight objective function :**

$$F = \gamma L \sum_p B^p \left\{ \delta^p + \frac{h^p_x d^p_x + w^p_x t^p_x}{\Delta^p_x} + \frac{h^p_y d^p_y + w^p_y t^p_y}{\Delta^p_y} \right\}$$

where L is the length of the panel according to the X coordinate (m), B^p is the width of panel p according to the Y co-ordinate (m) and γ is the specific weight (N/m3).

- **Cost objective function**

The cost is composed of three elements: the cost of raw materials (plates, bars, etc), the cost of manpower used for the construction of the entire structure and the cost of the consumables necessary for the manufacturing process (energy, welding materials, etc.). A complete expression of the cost objective function is given in [Rigo 2001c], it has the generic expression F_c :

$$F_C = \sum_p F_c^p$$

$$\begin{aligned} F^p{}_C = \ & a_1 \delta^2 + a_2 \delta + a_3 \\ & + b_1 \frac{d_x^2 h_x}{\Delta_x} + b_1 \frac{d_x w_x t_x}{\Delta_x} + b_2 \frac{d_x h_x}{\Delta_x} + b_2 \frac{w_x t_x}{\Delta_x} + b_3 \frac{d_x}{\Delta_x} + b_4 \frac{1}{\Delta_x} \\ & + c_1 \frac{d_y^2 h_y}{\Delta_y} + c_1 \frac{d_y w_y t_y}{\Delta_y} + c_2 \frac{d_y h_y}{\Delta_y} + c_2 \frac{w_y t_y}{\Delta_y} + c_3 \frac{d_y}{\Delta_y} + c_4 \frac{1}{\Delta_y} \\ & + a_4 \frac{1}{\Delta_x \Delta_y} \end{aligned}$$

where the specific coefficients depend on the application (efficiency of the shipyard, unitary cost of the materials, etc.). The user may choose to optimize the structure using the weight, the cost or a combination of these two objective functions.

2.3. Constraints

The constraints of the mathematical model are classified into three types: technological, geometrical and structural. We present the generic formulation of each constraint. The set of constraints for any specific application model is a subset of these generic constraints.

- **Technological constraints**

These constraints set bounds on the design variables. The lower bounds are usually determined by technical limitations (for example a lower bound for a thickness variable to limit the impact of corrosion) and the upper bounds are usually set by to production requirements (for example handling capabilities).

$$v_i^p{}_{min} \le v_i^p \le v_i^p{}_{max}$$

- **Geometrical constraints**

These constraints limit the values of some ratios between the design variables to ensure that the structure is feasible and reliable, they originate from regulations and norms. An example is to fix a maximum ratio between the dimensions and the thickness of members (frames, stiffeners) or to link the dimensions of two distinct types of members of a panel (frame height, stiffener height). An example is $\delta^p - 2 * d^p_x \le 0$. These constraints can be expressed by linear inequalities of the type:

$$a_i v_i^p - b_j v_i^p \le 0$$

- **Structural constraints**

As external loads and forces are applied to the structure, some resultant effects such as displacements, deformations and internal stress occur. The complexity of the behavior models leads to the impossibility of explicitly drawing the relationships between the parameter studied (deflection, stress, etc…) and the design variables (element dimensions and position). The evaluation of these resultant effects (and of their derivatives with respect to the design variables) is possible at expensive computational cost using analytical approaches or finite element methods (FEM). Given the values of the design variables, the displacements, deformations and internal stress are computed for several loading scenarios.

The structural constraints define the maximum admissible values of these resultant effects in order to limit the apparition of physical phenomenon such as yielding, buckling, ruin, etc. They are computed at a set of points defined by the user for each panel. The generic mathematical expression of these constraints is :

$$c_j(v) \leq c_{j\ max}$$

with v the design variables and $c_j(v)$ the value of the effect (displacement or stress). For any fixed value of the design variable v, a structural analysis is performed to compute the value of the constraints. For this calculation, we utilize the LBR-5 software. LBR5 is based on an analytic method to solve the systems of differential equations of stiffened panels [Rigo 2005].

2.4. Compact mathematical model

The problem to be solved has the following generic formulation :

$$\begin{aligned} &\text{Minimize } f(v) \\ &\text{s.t.} \quad g(v) \leq 0 \\ &\qquad c(v) \leq c_{max} \qquad\qquad (P1) \\ &\qquad v_i^p \in D_i^p \qquad i=1,\dots,8 \\ &\qquad v_9^p \in C_9^p, \end{aligned}$$

with v the vector of all variables, v_i a sub-vector of variables (i=1, ...,9), f(v) a nonlinear function, g(v) linear functions and c(v) implicit functions, D_i are discrete sets and C_9^p is a continuous interval.

This structural optimization problem creates several difficulties: it involves mixed (continuous and discrete) variables, a nonlinear objective function and implicit structural constraints with a nonlinear behavior. To solve this problem we will use a relaxation of P1 where the discretization constraints are removed such that each variable may take its value in a continuous interval $C_i^p = [v_i^p{}_{min}, v_i^p{}_{max}]$, where $v_i^p{}_{min}$ and $v_i^p{}_{max}$ are the extreme values of D_i^p. We obtain the following formulation :

$$\begin{aligned} &\text{Minimize } f(v) \\ &\text{s.t.} \quad g(v) \leq 0 \\ &\qquad c(v) \leq c_{max} \qquad\qquad (P2) \\ &\qquad v_i^p \in C_i^p \qquad i=1,\dots,9 \end{aligned}$$

We first present the resolution method available for the relaxed problem P2. A candidate solution (maybe not feasible) is initially considered and a structural analysis is performed. This analysis requires an important amount of computing time for real-size structures. Then an explicit local approximation of the problem is built using the output values of the structural analysis. This approximate problem is a conservative convex problem (any solution of the approximation is also a solution of the original problem). We apply an optimization algorithm based on a dual method [see Fleury, 1993 ; Schmidt and Fleury, 1980] to obtain the optimal solution of the approximate problem. This new solution is introduced in the original problem and a new structural analysis is performed to check its feasibility. These successive steps of optimization and structural analysis may be iterated a number of times fixed by the user: usually about 10 iterations are performed to obtain a solution that is satisfactory for the designer. We use the LBR5 software developed by Rigo [Rigo, 2001a] to perform the structural analysis and the optimization of the non linear approximate problems.

We conclude this section with a proposition based upon the experience of the designers. In structural optimization problems each design variable a has a direction in which its value can change that tends to satisfy the constraints of the problem. For example an increase of the thickness of an element or a decrease of the spacing between members does not affect the feasibility of a feasible solution and may lead to a feasible solution, starting from an unfeasible one. Let's call a change in this direction a

"positive change" then the proposition can be stated as follows :
Any "positive change" of the value of some or all of the design variables of a feasible solution always defines a feasible solution. (Proposition 1)
While not mathematically proven, this proposition always seems to hold in practice.

3. Heuristic Algorithm

3.1 Local search framework

We now turn to the more interesting case where the set D_i^p are discrete. The main challenge is to build a heuristic that always provides a discrete feasible solution of good quality to the problem P1, while requiring a very small number of structural analyses. The quality of the heuristic solution may be evaluated by comparison to the optimal value of the relaxed problem P2, where all the variables may take continuous values. This can also be compared with the value obtained by a single-step rounding procedure applied to the solution of the relaxed problem P2.

We use a local search heuristic inspired by the work of Fischetti and Lodi [Fischetti and Lodi, 2003] who experimented with a "relax and fix" heuristic for the solution of large MIP (Mixed Integer Programming Problems). This heuristic uses a generic MIP solver as a black-box "tactical" tool to explore suitable solution subspaces defined and controlled at a "strategic" level by a simple external branching framework. The "relax and fix" heuristic acts as described below. The variables are partitioned into disjoint sets of decreasing importance. A succession of MIPs are defined and solved iteratively. In the first MIP, the integrality requirement is imposed on the variables of the subset of greatest importance and the integrality constraint is relaxed on all the other variables. The resulting sub-problem is solved to optimality and the optimal values of the integer variables are fixed. The integrality constraint is imposed only on the variables of the next group in order of importance to form the MIP problem of the next iteration. The iterations stop when all the values of the variables are fixed. The *local branching* procedure introduced by Fischetti and Lodi consists in adding to the MIP model, at each iteration, a linear constraint that imposes a minimum percentage on the number of variables to fix at this iteration.

Our approach is a similar two-stage approach: an external heuristic framework acts as a "strategic" tool to control at a "tactical" level the definition and the optimization of the sub-problems. At the strategic level the "relax and fix" heuristic is replaced with a "dive and fix" heuristic. This heuristic for mixed integer linear problems is presented in [Pochet and Wolsey, 2006]. Initially the heuristic solves a linear relaxation of the problem. Then a succession of linear relaxations of the problem are solved: at each step a selection of variables are rounded and their values are fixed, this defines the next linear sub-problem. The iterative process ends when all the variables have been rounded to integer values. A main difference between the "relax and fix" heuristic and the "dive-and-fix" heuristic is the nature of the sub problems solved: the "relax and fix" heuristic adds some constraints and solves a MIP sub-problem while the "dive and fix" heuristic fixes the values of some variables and solves continuous sub-problems.

Applied to the structural optimization problem, the "dive-and-fix" procedure fixes the values of some variables while the discretization constraints are relaxed for the other variables. This defines a nonlinear sub-problem (instead of the linear problems discussed in Pochet and Wolsey): the procedure applied to select the variables and fix their values is described in the sequel of this paper. The LBR5 black box tool performs the structural analysis for the current solution and solves the current approximation to optimality.
We consider the mixed discrete-continuous nonlinear and implicit problem P1. The variables of the

problem are grouped according to their physical meaning, they represent the dimensions of the panels and the dimensions and the spacing of the stiffening members. There are 9 variables for each panel p : (δ^p, h^p_x, d^p_x, w^p_x, Δ^p_x, h^p_y, d^p_y, w^p_y, Δ^p_y) as described at Section 2.1, which we symbolize as (v_1^p, v_2^p, v_3^p, v_4^p, v_5^p, v_6^p, v_7^p, v_8^p, v_9^p) for convenience. We first consider the group of variables v_i: this is the vector composed of all the variables with index i, it may represent for example the thickness of all the panels of the structure. The variables in the vectors $v_1 ... v_8$ take their values in discrete sets as defined earlier and the vector v_9 has real values in a finite interval. We now consider only the eight groups of discrete variables. The groups v_i are sorted by order of decreasing importance: the importance of each group is defined by the designer according to their importance in the production process. This importance is roughly linked to the sensitivity of the objective and the constraints with respect to these variables. For clarity we assume that the order of importance is $v_1 ... v_8$. Finally we note v the vector of all the variables of the problem.

An initial solution is given by the designer: the values of the variables represent the dimensional characteristics of the structure. This solution may be feasible or not, discrete or not. We consider that the bounds and the linear explicit constraints are always respected. A non-feasible initial solution is allowed as the optimization algorithm used to solve the nonlinear sub-problems may start with any solution to derive a feasible solution to the initial problem. Given an initial solution v^0 the heuristic starts by computing an optimal solution v^{NLP} of the relaxed NLP problem P2, where all the variables are free (no variable has its value rounded and fixed).

At each iteration k, the heuristic starts with the solution of the previous iteration v^{k-1}. The sub-vector v_i^{k-1} of greatest importance among the free variables is selected and the values of these variables are fixed according to a rounding procedure (described below) to form the solution v^K. A structural analysis is performed at v^K by the LBR-5 software, it computes the value of the structural constraints for the solution v^k. These values are used to build an explicit approximated problem NLP^k. The LBR5 optimization module is then applied to solve the NLP^k problem. If the NLP^k problem appears to have no feasible solution, a relax procedure (described below) is applied to free the variables that have been fixed at the previous iteration and the algorithm moves to the next iteration. If a feasible solution for NLP^k is obtained then the algorithm moves to the next iteration (diving). This iterative scheme is repeated until all discretization constraints are satisfied.

The round and the relax procedures are the core of the dive-and-fix heuristic. They act jointly to define which regions of the solution space will be explored. They control the creation of the nonlinear sub problems P^k at each iteration by defining how the values for the variables are rounded and fixed. Three variations have been implemented and tested. The first one is a “closest rounding” procedure, the second one is a “up & down rounding” procedure and the third one, the “intensified closest rounding” procedure, may be seen as an enhancement of the closest rounding procedure.

3.2. Closest rounding procedure

The dive-and-fix heuristic based on the closest rounding procedure is the following. The algorithm starts by computing an optimal solution v^{NLP} of the NLP problem, i.e. the problem where all discretization constraints have been removed and where all the variables are free. At each iteration the group of free variables of greatest importance is selected and each variable v_i of this group is individually rounded to its closest value. The values of all the variables of this group are fixed. This forms the initial solution v^k of the iteration and the local approximated problem P^k is built from this solution. Three situations may occur at this point: either P^k has a feasible optimal solution and all the discretization constraints are satisfied, in which case the heuristic stops or all the discretization constraints are not satisfied and the algorithm moves to the next iteration (diving). Third, no feasible

solution is found for P^k, which implies that the problem is too constrained. A relax step is introduced to free the values of the group of variables v_i. This backtracking step allows the heuristic to consider an alternative rounding procedure to create an alternative solution $v^{k'}$. The alternative rounding procedure chosen here is to round up all the variables of the group selected for rounding. If proposition (1) holds this procedure is guaranteed to always provide a solution $v^{k'}$ for which $P^{k'}$, the approximate problem, has a feasible solution. A complete run of the algorithm generates at least 8 and at most 16 iterations. Each iteration involves a given number (fixed between 10 and 15 by the LBR5 user) of structural analysis : a complete discrete optimization using this procedure thus involves 160 up to 240 structural analysis.

3.3. Up & down rounding procedure

This procedure differs from the previous one in such a way that at each iteration k, the considered nonlinear sub-problem produces exactly two nonlinear sub-problems: P^k_{Up} and P^k_{Down} respectively obtained by rounding up or rounding down all the variables of the currently selected group of variables. A key difference with the closest rounding is that all the variables of a group are always rounded in a common way. A complete execution of the heuristic with this rounding procedure involves the creation and the resolution of exactly 16 sub-problems.

The heurisitic using this procedure, as well as the one using the closest rounding procedure, may look similar to a branch and bound method (see [Pochet and Wolsey, 2006]) but there are two important aspects that prevent us from using an exact branch and bound method. First, we are dealing with an implicit problem and each step of the local search defines and optimizes only a local approximation of the original structural problem. Therefore, no upper bounds can be obtained and used in a branch and bound tree. Second, as the definition of each local approximation requires a computationally expensive structural analysis, we prefer to operate on groups of variables instead of individual variables. This decision reduces the number of sub-problems but not induce a complete enumeration of the solution space as in a branch and bound approach.

3.4. Intensified closest rounding procedure.

We want to enhance the closest rounding procedure to intensify the local search towards more promising regions of the solution space. The heuristic is modified in order to analyze more sub-problems and to use some lower rounding. The improved heuristics act exactly as described with the closest rounding procedure except that when a discrete feasible solution is found the algorithm performs some backtracking instead of stopping.

The backtracking is an iterative procedure that relaxes the fixed values of one or several groups of variables of a current solution to create a new solution on which the diving process is applied. The backtracking considers the solution before the last rounding step on v_i. If this step was a closest rounding, this operation is replaced with a lower rounding and the diving starts again with the new solution. If the last rounding step was an upper rounding (meaning that a closest rounding on this group of variables has previously led to an unfeasible solution) or a lower rounding (meaning that the closest rounding was by chance equivalent to a lower rounding or that backtracking has already been applied to this sub-problem), then the values fixed at this step are relaxed and the procedure backtracks to the sub-problem where the rounding step was applied on the group of variables v_{i-1}.

This enhanced heuristic may create up to 2^8 sub-problems and may be highly time consuming. It can be stopped at any time by the user or can be set to stop when a time limit is reached.

The framework of the three heuristics is presented in Figure 5.

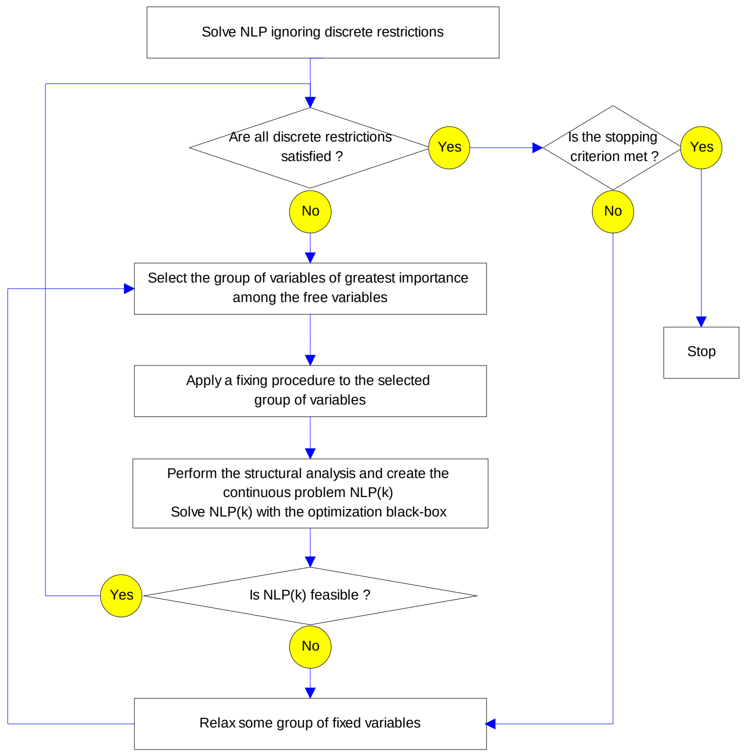

Fig. 5: Flowchart for the heuristic procedure.

4. Enhancements of the heuristic

Sorting the groups of variables.

Usually in the dive-and-fix heuristics the choice of the variables to be fixed is value-driven: the variables whose values are close to a discrete admissible value are chosen to be rounded and fixed. In our heuristic the variables are partitioned in groups according to their physical meaning. We have performed several experiments with other orders of the groups of variables and the results obtained using the order of the groups proposed intuitively by the designers outperforms the results with other orders. This observation suggests that the methods usually presented in the literature to select the variables to round (i.e. rounding the variables whose values are close to the discrete admissible value) would not provide good results as some variables of structural importance could be fixed too early. In such situations unfeasible solutions may appear early and the search process may terminate prematurely.

Balancing the workload between the number of sub-problems created at the strategic level and the number of iterations performed to solve a nonlinear problem at the tactical level.

Within the LBR5 black-box, each nonlinear (continuous) sub-problem is optimized thanks to an iterative method, each iteration implies the resolution of a computationally expensive structural analysis. Usually, the number of iterations in LBR5 is fixed by the user to 10 or up to 15 for difficult problems. This number of iterations allows a convergence of the optimization method even starting with unfeasible solutions. In practice, we have observed that after 5 iterations, a solution of good quality is usually found. We add a dynamic control of the iterative process : the number of iterations to solve the nonlinear sub-problems is set to 5 and 10 extra iterations are allowed if no feasible solution is found after the 5 initial iterations.

Tolerance for the rounding rule

A major characteristic of both heuristics is that the rounding procedure is applied simultaneously on all variables of a group. Although this "group-based" approach is a key to limit the number of sub-problems optimized and the number of structural analyses performed, it may appear too restrictive when using the up- or down-rounding. We may want to independently consider the variables whose values are very close to a discrete admissible value. We define a tolerance parameter so that if the value of a variable selected to be rounded (up or down) falls within a tolerance interval of a discrete admissible value then it is fixed to this value even if this does not comply with the active rounding procedure. The interval for this rule is defined using (*tolarr* * *step*) where *tolarr* is the tolerance parameter and *step* is the difference between two consecutive values of the admissible domain of the variable. For example, a variable *delta* is rounded to its closest admissible value if the following condition is true:

(modulo (*delta*, *step*)<= *tolarr***step*) or (modulo (*delta*, *step*)>= (1-*tolarr*)**step*)

Tolerance for the feasibility of the solution

The optimization method applied to solve approximate nonlinear problems is a dual approach that allows some constraint violation and tries to reduce these violations as much as possible. An initial solution may be unfeasible. The result of the optimization of the approximate problem may not satisfy exactly all the constraints of the original problem: a structural analysis is performed to check that the solution is feasible for the original problem. We use a tolerance factor to accept solutions of the nonlinear problem for which the constraints violation is less than a given percentage of the value of the constraint. Allowing to consider these non feasible solutions of the approximated problem as a starting point for sub-problems allows the diving process to continue and leads to the creation of sub-problems for which feasible solutions of good quality are sometimes found.

5. Computation experiments

The heuristic has been tested to find discrete values for the design variables of the structure of a real ship build by a major European Shipyard. A partial model of the ship structure is composed of 68 panels whose structural characteristics have to be optimized. The problem has 460 discrete variables and 52 continuous variables. The solution space for the discrete variables contains more than $10e^{360}$ solutions. There are 1709 constraints. A nonlinear analysis of the relaxed problem (with all continuous variables) takes about 10 minutes on a workstation with a processor pentium 3GHz and 2GB of RAM. Using the same machine, a run of the "dive-and-fix" heuristic may last from one to thirty hours, depending on the settings and the number of sub-problems generated. One may notice that the computing time is mainly due to the structural analysis.

Solving the relaxed problem (P2) with a cost objective function provides a value of magnitude 904197 (in cost units). When the solution of the nonlinear problem is rounded by a designer, the values of the objective functions increases by about 3%. We implemented a straightforward method to obtain a discrete solution. First a simultaneous rounding of all the variables down to their closest admissible value is performed. The solution produced by this method appears to be unfeasible. Then we try rounding the variables to their closest admissible discrete value. This again leads towards an unfeasible solution. Finally, rounding of all the variables up to their closest discrete value gives a feasible discrete solution with cost value of 942469, that is an increase of 4.2%.

The optimization of the relaxed problem P2 with the weight objective function provides an objective function of magnitude 6244916 (in weight units). A discrete solution is computed thanks to the same procedure as above. Rounding down yields a weight value of 6414353, i.e. a increase of 2.7%.

We ran the heuristic with the three rounding procedures. The results are presented in Table 1. We computed the percentage of loss due to the discretization of the problem. Two orders of importance, o1 and o2 provided by the designer, were used to sort the groups of variables. Other orders have been tested and only lead to only unfeasible sub-problems or provide results that are significantly worse. When using the heuristics the increase from 1.4% up to 2.2% for the weight objective function and from 0.3% up to 0.8% for the cost objective function, depending on the method used (See Table 2).

Table 1. Results for dive-and-fix heuristic

Order of the groups	Systematic Rounding		Closest Rounding		Enhanced Closest Rounding		
	Value of the objective	Nb of sub-problems	Value of the objective	Nb of sub-problems	Value of the objective	Nb of sub-problems	Nb of solutions
Weight objective function							
o1	6338793	117	6381804	10	6352028	131	64
o2	6333208	147	6349315	10	6334737	160	52
Cost objective function							
o1	906901	22	908348	11	907375	53	26
o2	911293	24	912187	11	911459	56	26

Table 2. Results : percentage of increase due to discretization

Order of the groups	Systematic Rounding	Closest Rounding	Intensified Closest Rounding
	% of increase	% of increase	% of increase
Weight objective function			
o1	1.5	2.2	1.7
o2	1.4	1.7	1.4
Cost objective function			
o1	0.3	0.5	0.4
o2	0.8	0.9	0.8

The basic heuristic using the closest rounding procedure provides results of good quality really fast. The number of sub-problems generated is around 10 and is guaranteed to be less than 16. The basic heuristic provides results of better quality using the up & down rounding procedure but the number of sub-problems generated is much larger and the run may take up to 30 hours. The enhanced heuristic may produce a number of sub-problems as large as the basic heuristic with up & down rounding and produces results of the same quality. Any one of these two methods may be interrupted at any time by the user or a time limit may be imposed. An interesting feature is that 50% of the solutions generated

with these two methods are feasible discrete solutions. The designer may choose among those solutions the one that complies with some constraints that are not expressed in the model. He may also use this list of about 60 discrete solutions to choose the one that provides low values for both the weight and the objective function.

6. Conclusion

This paper has presented a basic heuristic and an enhanced heuristic to solve the structural design of a cruise ship. This is a mixed integer nonlinear problems with implicit constraints and discrete variables. To evaluate if a structure complies to the structural constraints a computationally expensive structural analysis has to be performed. The heuristic method proposed is a two-stage local search heuristic. At a strategic level a “dive-and-fix” method controls the definition of nonlinear sub-problems. The generation of the explicit sub-problems and their optimization are performed at a tactical level, using the LBR5 software as a black-box. Other structural analysis and optimization methods could be chosen. Two rounding procedures have been proposed and tested for the basic heuristic and one for the enhanced heuristic. Any heuristic is guaranteed to always provide a solution using a small number of structural analysis and a reasonable amount of time, if proposition (1) holds. The heuristics have been tested on a real ship structure. The solutions of the heuristics shows very similar values for the objective functions and always outperform a “hand-made” rounding or an automatic single step rounding of the solution of the continuous problem. The designer may choose among the proposed heuristics the one that is fast to give a discrete result or the one that generates an important number of discrete feasible solutions in a more important period of time.

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