Implementation of hydrostatic constraint in the software Diva: Theory and applications

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Abstract

Gridding data is a frequently demanded process in geophysics; it consists in determining the value of a given field on a regular grid, starting from measurements of this field at sparse locations (*e.g.* Fig. 1). The software Diva implements the Variational Inverse Method (VIM) to solve the gridding problem. We present here an additional tool permitting to remove hydrostatic instabilities generated by the data gridding itself when working on successive horizontal layers.

1 Introduction

Construction of 3D temperature (T) and salinity (S) fields for model initialization, climatological analysis or graphical purposes is generally performed by a stacking of 2D layers of interpolated/analysed T and S fields.
This process may generate problems, particularly in regions void of data: the horizontal analysis of two layers, performed independently, sometimes lead to density fields that are *statically unstable*. The elimination of such instabilities *ab initio* is the purpose of the present work.
Instability of the water column is characterised by the *Brunt-Väisälä frequency*, N, defined as:

$$N^2 = -\frac{g}{\rho_0}\frac{\partial \rho}{\partial z} \qquad (1)$$

where g is the acceleration of gravity, ρ the density, ρ_0 a reference value of density and z the vertical coordinate counted positive upward.
The next sections present the theory behind the instability elimination and a realistic application on Mediterranean Sea data.

2 Theory

As vertical coupling is relatively weak in the ocean at most places, the selected approach is to keep analysis in two dimensions and perform some actions that 1. decrease the instabilities between layers; 2. keep the rest as untouched as possible.
The strategy consists of adding data from a layer to the upper layer in order to restore stability and works in an iterative way, according to the following steps:

1. Make an analysis of the deepest layer of interest, $k = 1$
2. For layer k located above layer $k-1$:
   (a) analyse T_k and S_k and compute N from (1), discretized between the levels in each grid point and using a local linear expansion of density around a reference state T_0, S_0:

$$\rho = \rho_0 \left[1 - \alpha(T - T_0) + \beta(S - S_0)\right] \qquad (2)$$
$$N^2 = \frac{g}{\Delta z}\left[\alpha(T_k - T_{k-1}) - \beta(S_k - S_{k-1})\right] \qquad (3)$$
$$\Delta z = z_k - z_{k-1} \qquad (4)$$

   (b) if the layers are *too unstable*, add pseudo-data to layer k and start again step (a).
3. proceed to next layer k, up to the surface layer.

Considering this method, two questions have to be answered:

- **where** do we have to place the pseudo-data?
- **what value** of T and S do we have to assign them?

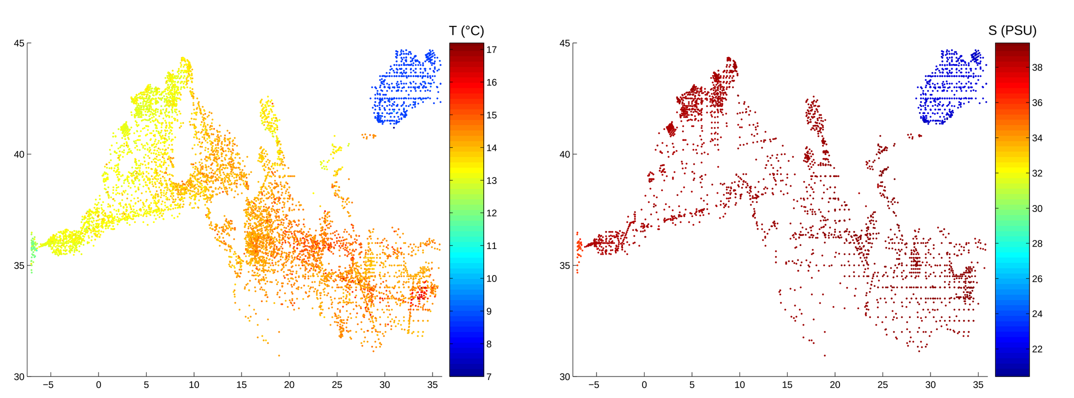


FIGURE 1: *Initial distribution of temperature (left) and salinity data at 400 m depth for the month of April.*

2.1 Characterisation of instabilities

Instabilities can be characterised by the reference value N_r^2, computed as:

$$N_r^2 = \max\left\{\mathrm{rms}(N_+^2), \mathrm{rms}, (N_-^2)\right\}, \qquad (5)$$

N_+^2 and N_-^2 are the N^2 values that are positive and negative, respectively.
A grid point can by qualified as strong instability when

$$N^2 \leq \varepsilon_3 N_r^2, \qquad (6)$$

where ε_3 is a relative threshold.

2.2 Pseudo-observation location

Since the analysis typically propagates the data over a distance comparable to the *correlation length* L, pseudo-data are first added in the location of the grid point with strongest instability and consider that surrounding points do not need any pseudo-data for this iteration.
In practice, all grid points that lie within a distance mL with respect to the pseudo-data point as "treated" are flagged.
Fig. 2 shows the positions of the pseudo-data added for the example.

2.3 Pseudo-observation values

The pseudo-data we want to add is characterised by a temperature $\tilde{T}_k$ and a salinity $\tilde{S}_k$ that have to satisfy the stability condition

$$\frac{g}{\Delta z}\left[\alpha(\tilde{T}_k - T_{k-1}) - \beta(\tilde{S}_k - S_{k-1})\right] = \tilde{N}^2. \qquad (7)$$

$\tilde{N}^2$ is a slightly positive value so as to ensure stability and that we relate to the reference value according to

$$\tilde{N}^2 = \epsilon N_r^2 \qquad (8)$$

where ϵ is a relative measure of the stability we impose locally.
Eq. (7) is the first equation to determine $\tilde{T}_k$ and $\tilde{S}_k$. The second equation can be determined by combining two different approaches:

1. *Mixing* approach:
   Mimicking convective adjustment, we suppose that the new water mass is a mix of (T, S) at levels k and $k-1$:

$$(\tilde{T}_k, \tilde{S}_k) = (T_k, S_k) + \eta\left[(T_k, S_k) - (T_{k-1}, S_{k-1})\right]. \qquad (9)$$

2. *Minimal perturbation* approach:
   Minimizing the combined effect of perturbations δT and δS on the density by considering the objective function:

$$J = w(n_T)\,\alpha^2 \delta T^2 \;+\; w(n_S)\,\beta^2 \delta S^2, \qquad (10)$$

where $w(n)$ is a decreasing function of n, n_T, n_S are the number of T and S data in layer k, respectively.

3 Results

3.1 Data

We illustrate the method by considering T and S observations extracted from Medar/MedAtlas II for the month of April, interpolated on 25 levels. We concentrate our attention on level $n°$ 12, corresponding to a depth of 400 m and characterised by the following numbers:

salinity data number	3577
temperature data number	8263
total number of wet points	31722

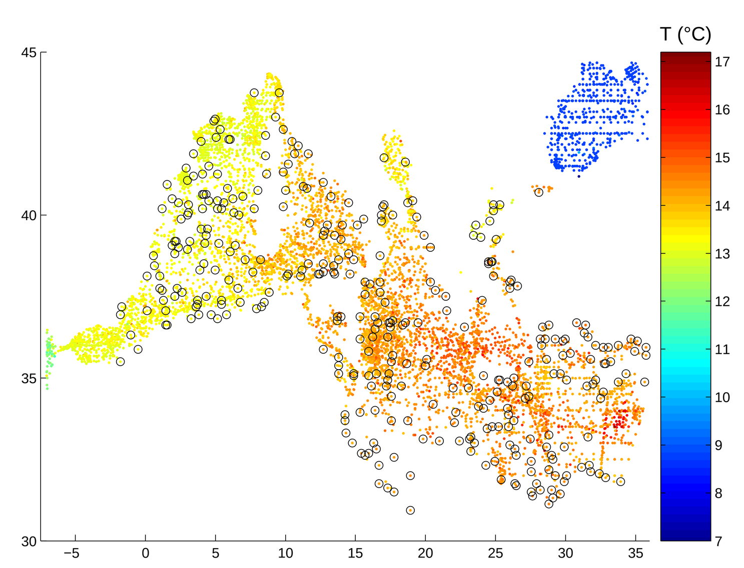


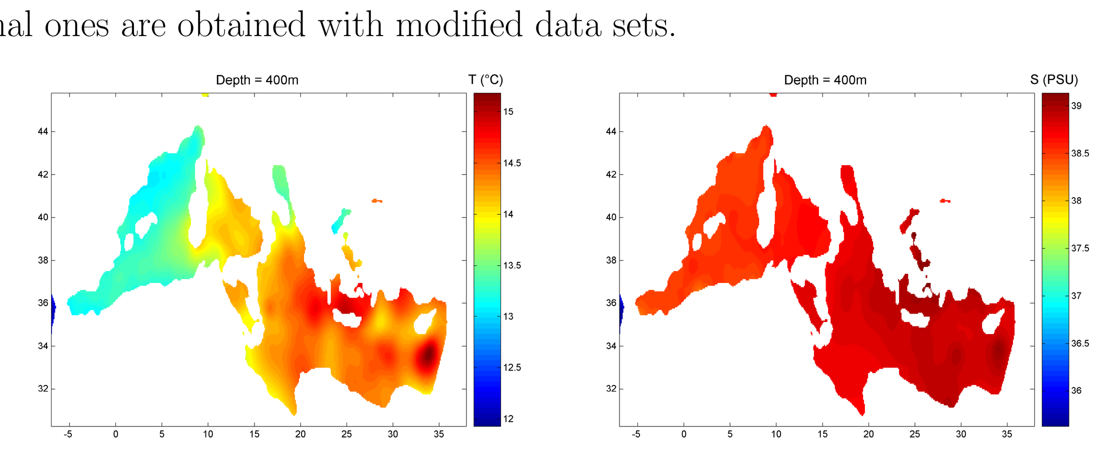
FIGURE 2: *Localisation of added data.*

3.2 Algorithm results

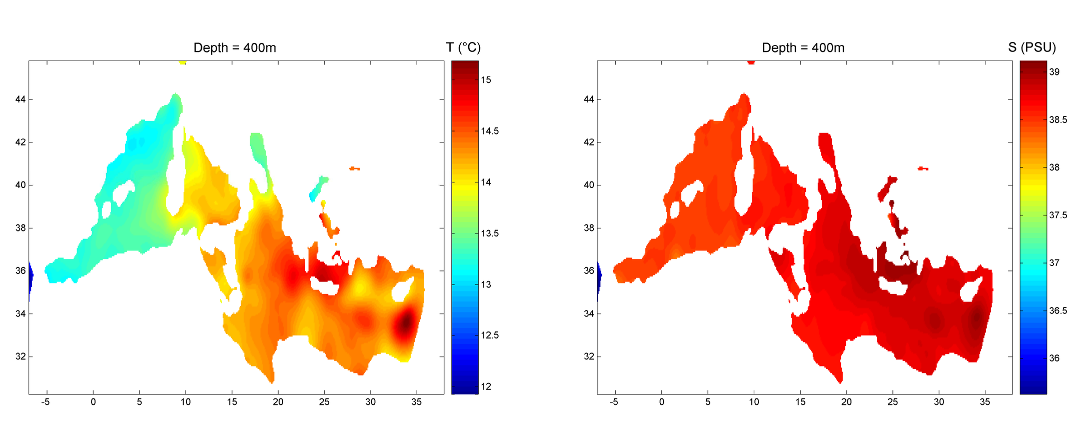
The instability elimination algorithm converges in six iterations. Note that the number of remaining instabilities has not to be zero, since weak instabilities are admissible.

Iteration	01	02	03	04	05	06
Nb N^2 instabilities	22424	17502	13358	9893	7252	5719
rms(N_+^2)	3.0E-6	2.7E-6	2.5E-6	2.4E-6	2.3E-6	2.1E-6
rms(N_-^2)	5.4E-6	2.4E-6	1.4E-6	1.1E-6	8.6E-7	7.0E-7
maximum negative N2	-1.7E-5	-1.7E-5	-1.4E-5	-2.5E-5	-1.1E-5	-7.5E-6
Nb of added data	92	68	62	52	41	0

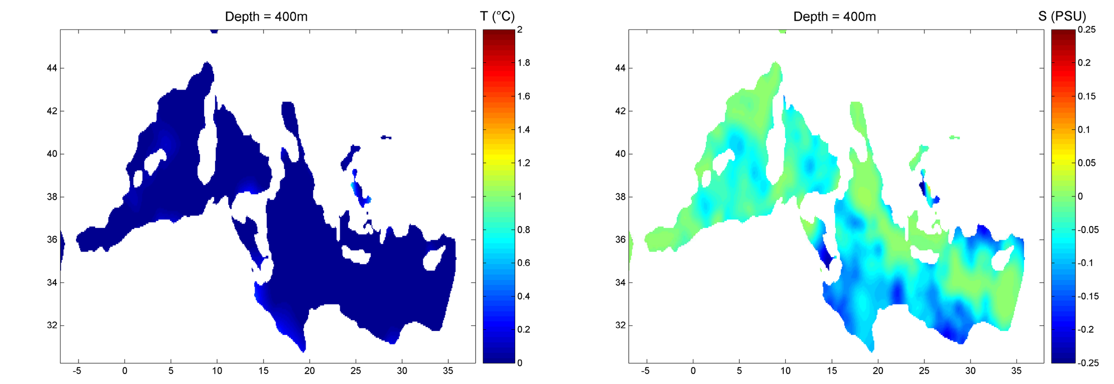
Fig. 3 shows the initial and final analysis for temperature and salinity; the final ones are obtained with modified data sets.



(a) Initial T field (b) Initial S field



(c) Final T field (d) Final S field



(e) Difference between T fields (f) Difference between S fields

FIGURE 3: *First analysis, final analysis and differences for temperature and salinity between them at 400 m.*

3.3 Numerical results: cross sections

The following figures show the first and final analysis of temperature and salinity, for three different latitudes, highlighting the 3D nature of the process.

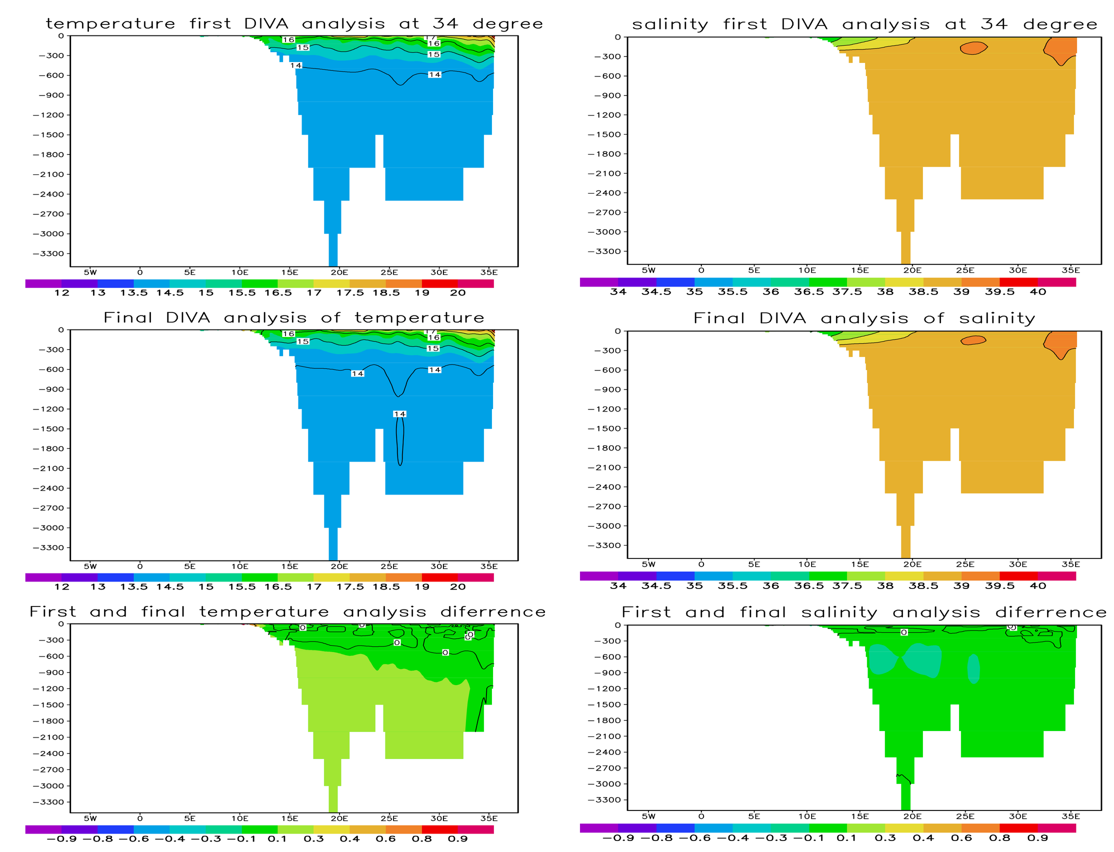


FIGURE 4: *Latitude 34°N: first analysis, final analysis and differences.*

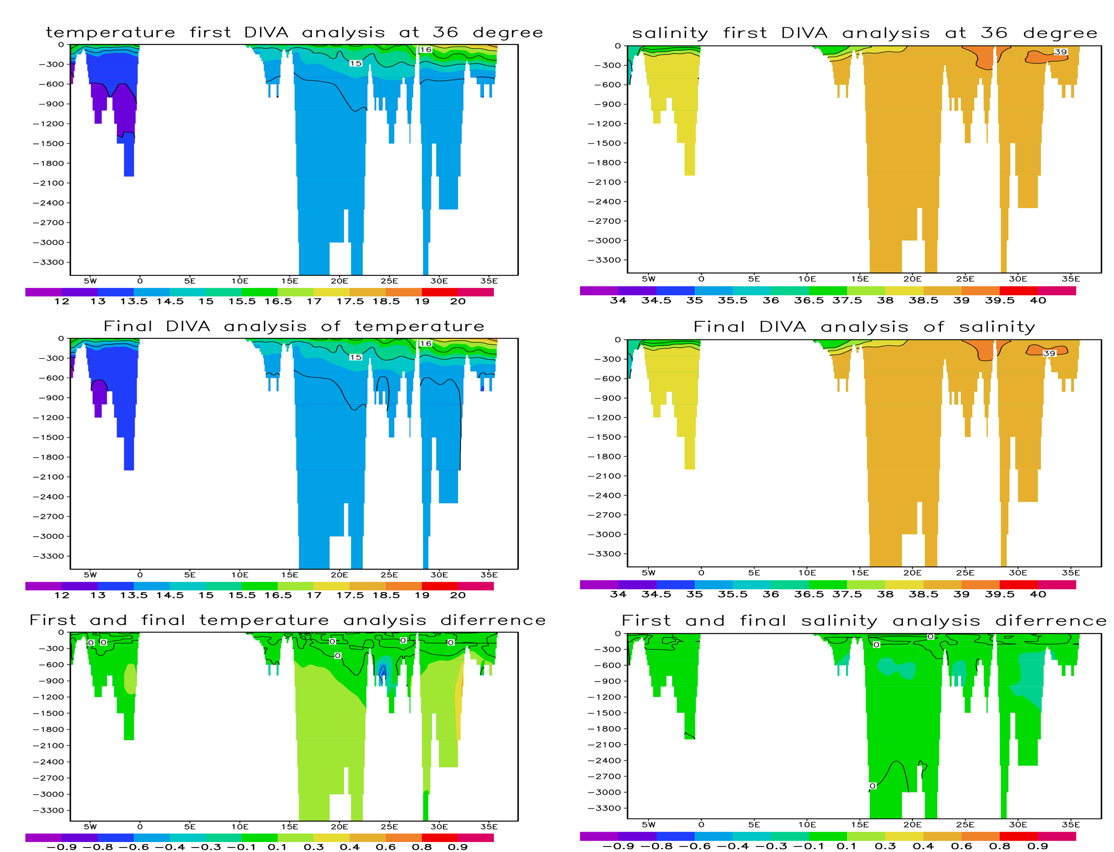


FIGURE 5: *Latitude 36°N: first analysis, final analysis and differences.*

4 Conclusions

The "Hydrostatic Constrain Stabilization" (HC) algorithm presented here showed a great efficiency on different 3D temperature and salinity data sets. It is now a reliable tool to generate stable fields for model initialization, as well as to generate new stable data sets. The efficiency of the HC algorithm resids in the fact that it adds a minimum number of pseudo-data with optimised value.
In near future, we expect to further improve our results by considering the Diva error field calculation on data in the HC algorithm, in order determine with more precision the locations and the layers where data will have to be added.

Acknowledgments

Diva was developed during E.U. MODB project and improved within the working package "Geostatistic analysis tools" (JRA4) of SeaDataNet project, an Integrated Infrastructure Initiative of the E.U. Sixth Framework Programme.
We thank the participants to the Diva workshops in Liège (November 2006) and in Calvi (November 2007) for their numerous valuable comments which help us to improve the software.