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A formulation of the Eurocode 2 concrete model at elevated temperature that includes an explicit term for transient creep

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ABSTRACT

The first objective of this paper is to highlight the capabilities and limitations of concrete uniaxial constitutive models at elevated temperatures for thermo-mechanical behavior modeling, depending on the implicit or explicit consideration of transient creep strain in the model. The characteristics inherent to the two types of models are described and compared. It appears that one of the major limitations of implicit models concerns the unloading stiffness. Based on numerical analysis performed on loaded concrete columns subjected to natural fire, it is shown that the stress–temperature paths experienced by structural concrete are varied and complicated and that concrete material models cannot handle properly these complex situations of unsteady temperatures and stresses without explicit consideration of transient creep.

The second objective of the paper is to propose a new formulation of the Eurocode 2 concrete material model that contains an explicit term for transient creep. The new model is implemented in the software SAFIR and validated against experimental data of the mechanical strain developed by concrete cylinders under different unsteady temperatures and loads. It is shown that the actual material behavior is better matched with the new explicit model than with the current implicit Eurocode 2 model. Finally, a comparison is given between experimental and computed results on a centrally loaded concrete column submitted to heating–cooling sequence.

Keywords: concrete, fire, constitutive model, transient creep strain, Eurocode

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1. INTRODUCTION

Numerical analysis taking into account extreme thermo-mechanical loading is now an accepted tool to assess the performance of building structures in accidental situations such as fire. For these numerical simulations, temperature dependent constitutive relationships must be available for the load bearing materials used in the structure such as, for example, steel and concrete.

For linear structural members such as beams and columns, steel and concrete uniaxial constitutive models have been available for many years [1–5]. In concrete, a particular phenomenon appears when subjected to high temperatures: the transient creep strain. Physically, the transient creep strain is the difference in strain between concrete that is heated under load and concrete that is loaded at elevated temperature; this strain develops during first-time heating and is irrecoverable [1,6]. This strain component depends on the temperature and on the stress applied during heating. The fact that it does not depend on time makes the term ‘‘transient creep’’ quite improper, but this term has imposed itself in the literature. Several uniaxial models of concrete integrating explicitly a term for transient creep strain have been proposed in the literature since the first works of Anderberg and Thelandersson [1] and Schneider [2]. Interesting state of the art reviews of the transient creep strain models can be found in recent literature, e.g. Li and Purkiss [7], Law and Gillie [8] and Youssef and Moftah [9]. In most of these models, the transient creep strain is linearly proportional to the applied stress and increases with temperature but not linearly [1,2]. In Anderberg’s model [1], the transient creep term is proportional to free thermal expansion. Yet, it is thought that the origins of transient creep are in the cement paste [8,10] and free thermal strain of concrete is dominated by that of the aggregate (Khoury reported in [11]). Khoury et al. considered that transient creep strain was physically independent of free thermal expansion [12]. Nielsen proposed a modification to the Anderberg’s formulation of transient creep strain in which transient creep strain is linearly proportional to temperature instead of the free thermal strain [13]. In Diederichs’ model ([14] reported in [7]) transient creep strain is proportional to the applied stress and to a third order function of temperature obtained by experimental data fitting. In Schneider’s model [2], the transient creep strain is also a function of the initial stress before heating, in addition to the applied stress, the temperature and the temperature dependent concrete modulus of elasticity and strength.

Other authors refer to Load Induced Thermal Strain (LITS) instead of transient creep strain. LITS is the sum of different strain components in heated concrete; it consists of transitional thermal creep, drying creep, basic creep and changes in elastic strains that are caused by the change in elastic modulus as temperature increases [8]. Transient creep refers to the sum of transitional thermal creep and drying creep, it is by far the largest component of LITS [11]. Terro [11] used the experimental results of Khoury et al. [12] to develop an empirical formula by data fitting for the Load Induced Thermal Strain. In Terro’s empirical formula, LITS is assumed to be a linear function of applied stress.

It is well-admitted in literature that transient creep has to be considered in any fire analysis involving concrete in compression [7,15]. However, the necessity of taking it into account by an explicit term in the strain decomposition has been questioned [16] and in the current Eurocode 2 (EC2) model [17], the transient creep has been incorporated implicitly in the stress–mechanical strain relationship. Law and Gillie [8] have recently shown that considering this term implicitly can have important implications on the Young modulus calculation of concrete but the implications on the behavior of a complete structure is still a pending question.

It has been shown [2] that the amount of transient creep may significantly depend on the type of concrete. It is possible to determine precisely the properties of a well-defined type of concrete to be used in well-defined conditions, usually for a very important project, e.g. the concrete vessel for a nuclear reactor that will be subjected to a well-defined fire scenario. For more general applications, generic properties of concrete have to be established. Generic properties are used, for instance, when the mechanical behavior of two structural systems has to be compared, with no reference to a particular concrete mix. Generic properties are also needed at the preliminary stage of a design, when no information is yet available on the particular mix that will be used. Generic properties are also required for determining the fire resistance of an element in a small project, where the cost to conduct experimental tests would by far outweigh the budget allocated for the design studies of the building.

The constitutive model of Eurocode 2 has imposed itself as one of the most widely used generic models in the last decade, in Europe and beyond. It has been proposed by a draft committee comprising several European experts, has proved to yield quite satisfactory results when applied to structural calculations (although most application were under ISO fire, which means under constantly increasing temperature) and it is well accepted by authorities and regulators. It was estimated that, if there is a chance to see a breakthrough in the utilization of explicit models, this could not be achieved by selecting one of the various particular models presented up to now, each with its own characteristics and some requiring particular tests to characterize different concrete mixes, but rather by proposing an explicit model that would yield the same results as the present Eurocode implicit model when used in the situation of transient test. This model could then be seen as a new formulation of the Eurocode model and be called Explicit Transient Creep Eurocode model (ETC Eurocode model). It should of course encompass the most widely accepted characteristics of transient creep.

The first objective of the study reported here was to highlight the capabilities and limitations of a uniaxial constitutive model for concrete depending on its implicit or explicit consideration of transient creep strain. The models that comprise an explicit transient creep term are denoted as ‘‘explicit models’’ whereas the models that consider the transient creep implicitly, such as the EC2 model, are denoted as ‘‘implicit models’’.

The second objective was, if this proved to be necessary, to derive an explicit model that would encompass the characteristics of most models presented up to now in the literature and, for the reasons explained above, that would be as close as possible to the present Eurocode 2 model.

2. GENERAL CHARACTERISTICS OF IMPLICIT AND EXPLICIT MODELS FOR TRANSIENT CREEP

2.1 Definitions

In implicit models, the total strain ε_{tot} is considered as the sum of free thermal strain ε_{th}, mechanical strain ε_{m}, and possibly basic creep strain ε_{cr} :

$$\varepsilon_{tot} = \varepsilon_{th} + \varepsilon_{m} \left(+\varepsilon_{cr}\right) \qquad (1)$$

Basic creep, defined as the strain that develops when only time is changing with all other conditions such as stress and temperature being constant, is generally omitted for the structural calculation of building structures in the fire situation [7].

In explicit models, the total strain is split into free thermal strain ε_{th}, instantaneous stress-related strain ε_{σ} and transient creep strain ε_{tr} (and possibly basic creep strain ε_{cr}):

$$\varepsilon_{tot} = \varepsilon_{th} + \varepsilon_{\sigma} + \varepsilon_{tr} \left(+\varepsilon_{cr}\right) \qquad (2)$$

The instantaneous stress-related strain can in turn be divided in elastic and plastic strains: $\varepsilon_{\sigma} = \varepsilon_{el} + \varepsilon_{p}$. From Eq. (1) and (2) it is clear that the mechanical strain is the sum of the instantaneous stress-related strain and the transient creep strain.

2.2 Implicit models

The stress is directly related to the mechanical strain, without calculation of the transient creep strain. In the EC2 model, for instance, the relationship at a given temperature *T* between the stress and the mechanical strain is given for the ascending branch by the following equation:

$$\frac{\sigma}{f_c(T)} = \frac{3\,\varepsilon_m^{\text{implicit}}}{\varepsilon_{c1,EC2}(T)\left(2+\left(\varepsilon_m^{\text{implicit}} / \varepsilon_{c1,EC2}(T)\right)^3\right)} \qquad (3)$$

with f_c the compressive strength and $\varepsilon_{c1,EC2}$ the peak stress strain (PSS) [17]. In this relationship, the value of the peak stress strain accounts for the transient creep strain. The relationship of Eq. (3) is represented at 500°C in Fig. 1.

The mechanical strain given by implicit models for a given stress-temperature state is the same, whether concrete has been heated and then loaded at constant temperature or loaded and then

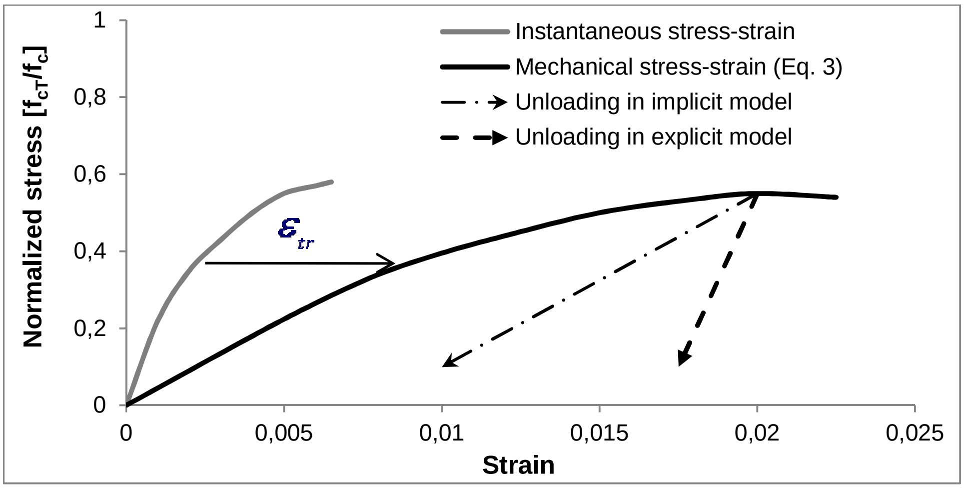

Figure 1. Strain components in implicit and explicit models at 500°C.

heated under constant stress and this is known not to correspond to experimental evidence. Another major limitation of implicit models is that transient creep strain is recovered during eventual unloading. This is because, at a given temperature, the elastic modulus used for unloading is taken as the initial tangent of the constitutive curve in terms of $(\varepsilon_m;\sigma)$ [8], see Fig. 1.

2.3 Explicit models

In explicit models, the stress σ is directly related to the instantaneous stress-related strain ε_σ. This relationship can be obtained experimentally at any temperature from a steady-state test [2] and the transient creep strain is obtained as the difference in strain between a steady state test and a transient test.

In the tests conducted to derive the constitutive models, either the temperature or the stress is constant, whereas the other variable is increased. It is important to note that, in real structures, the transient creep strain depends not only on temperature and stress but also on the stress–temperature path followed by the material. As a result, in explicit models, the relationship between the stress and the mechanical strain is not univocal at a given temperature as seems to be implied by Fig. 1. In explicit models, the transient creep strain is not recovered during unloading and/or cooling and the modulus for unloading at a given temperature is taken as the initial tangent to the instantaneous stress–strain curve.

3. POSSIBLE STRESS-TEMPERATURE PATHS IN A STRUCTURAL ELEMENT

The kind of demand that is being imposed on a material model may be quite different when it comes to modeling a structural element when it is used to model experimental tests conducted on cylinder with a quite simple stress–strain–temperature history. Because of transient thermal gradients inherent to concrete sections, different points in the structure are expected to experience different and complex stress–strain–temperature histories. The following simple example illustrates this aspect and will serve as a starting point to establish the demand imposed on a constitutive model. All simulations have been performed with the software SAFIR [18] and with the current thermal and mechanical models of Eurocode 2, i.e. with an implicit model. The results would of course be quantitatively different with another model but the exercise has been performed to show the trends, not to obtain precise values.

The model is a circular siliceous concrete column of 4 m height, with a section of 300 mm diameter reinforced with four 16 mm diameter rebars covered by 40 mm of concrete. The concrete has a compressive strength of 30MPa and a tensile strength of 3 MPa whereas the steel of the bars has a yield strength of 500MPa. The ultimate load of this column at room temperature is 2309 kN.

The temperature distribution in the sections was determined by a 2D nonlinear transient analysis. The column is first axially loaded with a load of 462 kN and then subjected to the natural fire curve shown in Fig. 2. No collapse occurs during the numerical simulation.

The stress–temperature paths observed at different points across the section at mid-level of the column are plotted in Fig. 3 (compression is positive). Points A to F are regularly distributed on a radius in the section, with point A at the center and point F at the surface.

It can be observed in Fig. 3 that the stress and temperature evolutions across the section during the fire are complex and significantly different depending on the position in the section. It is possible to extract five different situations from Fig. 3. For each of these situations, it is discussed whether explicit or implicit constitutive models are able to take into account accurately the transient creep strain.

Situation I: increasing stress and temperature. Transient creep strain develops because the temperature increases under stress. However, the transient creep strain is overestimated by implicit models because these models calculate at any time the transient creep strain on the base of the actual value of the stress. On the contrary, it is possible with explicit models to perform an incremental calculation of transient creep strain.

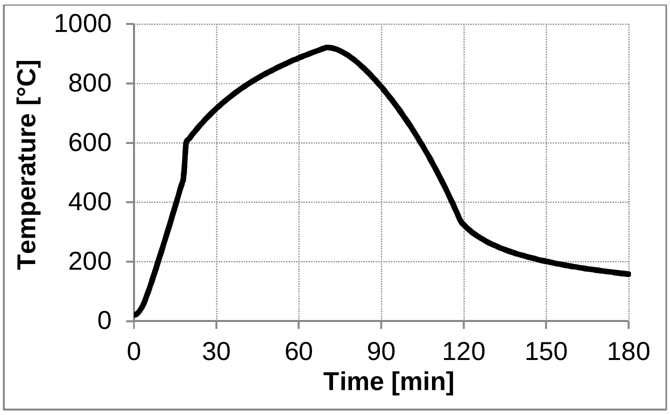

Figure 2. Natural fire curve applied to the column.

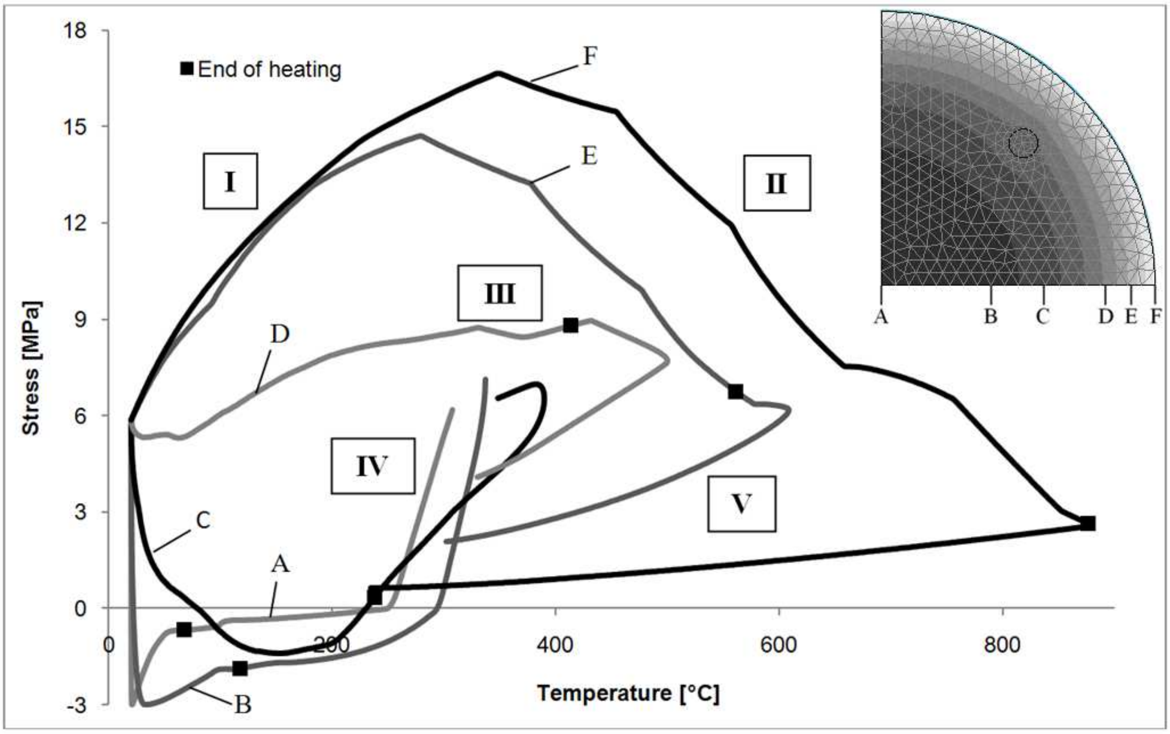

Figure 3. Stress-temperature path in different parts of the section.

Situation II: decreasing stress and increasing temperature. In situations where the stress decreases, explicit and implicit models lead to very different results because the unloading stiffness considered by both models is different. In implicit models, the transient creep strain is treated as reversible, which is in contradiction with its physical nature. Physically, additional transient creep strain is assumed to develop in the concrete material submitted to increasing temperature as long as the stress in the material remains in compression, even if the compressive stress is decreasing [7]. In other words, in explicit models, the transient creep strain is still incremented in situation II.

Situation III: (approximately) constant stress and increasing temperature. This situation corresponds to transient tests. Implicit and explicit models give the same mechanical strain for a given stress–temperature state reached after an evolution that matches situation III.

Situation IV: increasing stress and (approximately) constant temperature. This situation corresponds to steady-state tests. No transient creep develops. In explicit models, the mechanical strain reduces to the instantaneous stress-related strain. However in implicit models, transient creep strain is still implicitly included, leading to a highly underestimated stiffness.

Situation V: decreasing stress and decreasing temperature. Implicit and explicit models lead to different material behaviors because the unloading stiffness considered in the two models is different (see Fig. 1). In explicit models, the transient creep strain remains constant as it cannot be recovered and it does not develop under decreasing temperature. In implicit models, the transient creep strain decreases.

This example shows that implicit models reproduce correctly the behavior of concrete only in a very particular situation, when the temperature increases and the stress is constant (situation III), and this situation is not so common, even in a simple element subjected to the heating phase of a fire. This is even more the case during the cooling phase of the fire.

It is thus preferable to utilize an explicit model for the sake of precision of the stress and stiffness calculated at the local level, i.e. in every point of integration considered in the structure. Whereas the difference between the utilization of both types of model will be noticeable in the global behavior of the structural elements is another question.

4. EXPLICIT TRANSIENT CREEP FORMULATION OF THE EUROCODE MODEL

4.1 Assumptions

1) The new formulation was calibrated to yield the same mechanical strain as the EC2 model for a material first-time heated under constant stress (i.e. transient test). Eq. (1) and (2) leads to the following equation:

$$\varepsilon_m^{\text{implicit}} = \varepsilon_\sigma^{\text{explicit}} + \varepsilon_{tr}^{\text{explicit}} \tag{4}$$

2) The elastic modulus of the material is taken as the initial tangent to the ENV curve [3] with the minimum value of the PSS, $\varepsilon_{c1,\min}$. Indeed, the ENV relationship with $\varepsilon_{c1,\min}$ is based [19] on steady-state tests done by Schneider [20] that do not include transient creep strain, see Fig. 4. Relationships for the evolution of the elastic modulus with temperature presented by Felicetti and Gamabarova [21] (reported in [22]) are in line with the values given by ENV.

3) Transient creep models have been developed by several authors in literature and, generally, transient creep strain is proportional to the applied stress [1, 2, 11]. Adopting the same assumption, the formulation was developed according to the following equation:

$$\varepsilon_{tr} = \phi(T) \times \frac{\sigma}{f_{ck}} \tag{5}$$

where $\phi(T)$ is a nonlinear function of temperature and f_{ck} is the compressive strength at 20°C.

Concrete has a memory and concrete models have been developed in the literature to take into account the effect of the load history before heating [2] and during heating [23,24] on the deformation response to a change in stress and temperature increase. This effect has not been taken into account in the present model because of uncertainties on the hypothesis to be made for considering the load history during heating on the basis of experimental tests in which only the load level before heating was considered (with the load level maintained constant during heating).

4.2 Development of the Model

The initial stiffness (i.e. the tangent to the curve at 0 stress) of the material subjected to steady-state test must be equal to the ENV elastic modulus, written here as $E_{\text{ENV}}(T)$. In case of transient test, the new model must be calibrated on the EC2 model, so in particular the tangent to the curve at 0 stress must be the same as that of the EC2 curve, denoted as $E_{\text{EC2}}^{\text{implicit}}(T)$. Transient creep strain is defined as the difference between the "transient test" curve and the "steady-state test" curve. As transient creep has been assumed linearly stress-dependent, it is graphically obtained in Fig. 4 between the straight line of slope $E_{\text{EC2}}^{\text{implicit}}(T)$ and that of slope $E_{\text{ENV}}(T)$. Mathematically, it is given as follows:

$$\varepsilon_{tr}(T,\sigma)=\frac{\sigma}{E_{\text{EC2}}^{\text{implicit}}}-\frac{\sigma}{E_{\text{ENV}}}=\frac{2}{3}\frac{\left(\varepsilon_{c1,EC2}-\varepsilon_{c1,\min}\right)}{f_c/f_{ck}}\frac{\sigma}{f_{ck}}=\phi(T)\frac{\sigma}{f_{ck}} \qquad (6)$$

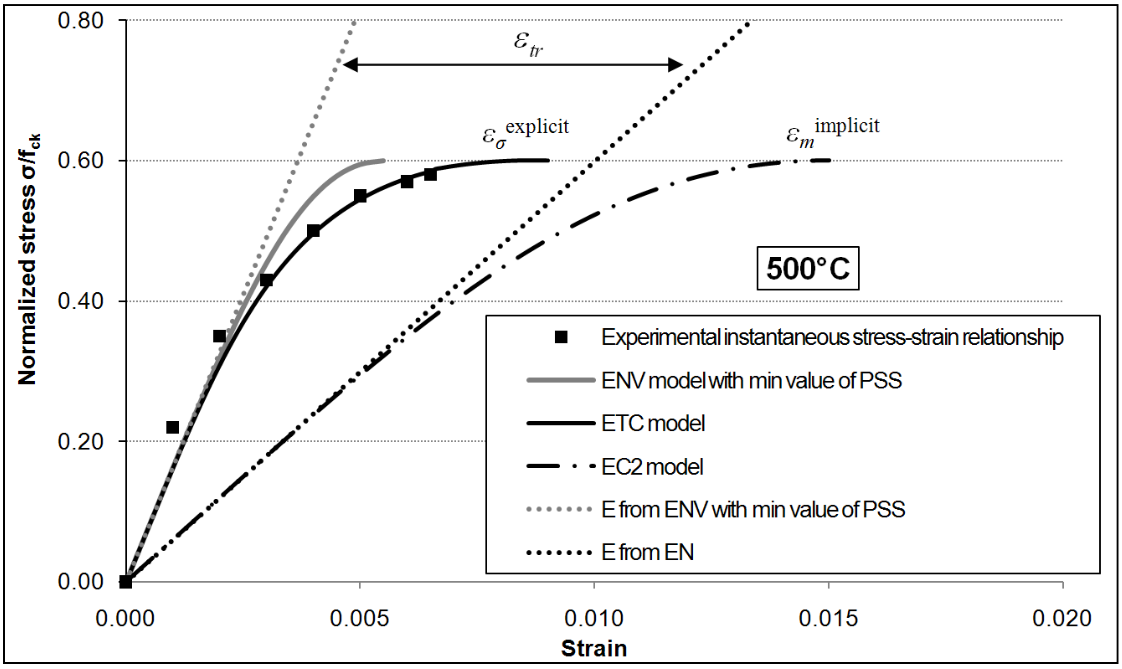

Figure 4. Comparison at 500°C of ENV [3], ETC and EC2 [17] models with experimental data from Schneider [20].

The instantaneous stress-strain relationship of the model, obtained as the difference between the EC2 relationship and the transient creep given by Eq. (6), is not exactly equal to the ENV relationship because the transient creep has been considered as linearly stress dependent. However, the initial stiffness of the new relationship is exactly equal to the ENV elastic modulus.

The function $\phi(T)$ is a growing function of temperature that is not reversible during cooling, as each of its components $\varepsilon_{c1,EC2}$; $\varepsilon_{c1,\min}$; f_c / f_{ck} is irrecoverable (Table 1

). This is in line with the definition of transient creep that is not recovered during the cooling phase. Fig. 5 compares the transient creep strain of the present model with experimental data and models given in the literature (reported in [9]) for the particular case of a specimen first subjected to a uniaxial compressive stress equal to 0.33 f_c and then heated at a constant rate. It can be seen that the present ETC model is reasonably close to the other models and to experimental data, and that the increase in transient creep strain with temperature is correctly reproduced by the ETC model. The quantitative discrepancy between experimental results and computed results could be expected since the ETC concrete model is a generic model, for the reasons given in the introduction, and consequently it cannot be calibrated to capture exactly the behavior of the particular concrete mix tested in the experiment in Fig. 5.

T(°C)	20	100	200	300	400	500	600	700	800
$\varepsilon_{c1,\min}$	0.0025	0.0025	0.0030	0.0040	0.0045	0.0055	0.0065	0.0075	0.0085
$\varepsilon_{c1,EC2}$	0.0025	0.0040	0.0055	0.0070	0.0100	0.0150	0.0250	0.0250	0.0250
f_c / f_{ck}	1.00	1.00	0.95	0.85	0.75	0.60	0.45	0.30	0.15
ϕ	0.0000	0.0010	0.0018	0.0024	0.0049	0.0106	0.0274	0.0389	0.0733

Table 1. Phi function and main parameters value for siliceous concrete at high temperature.

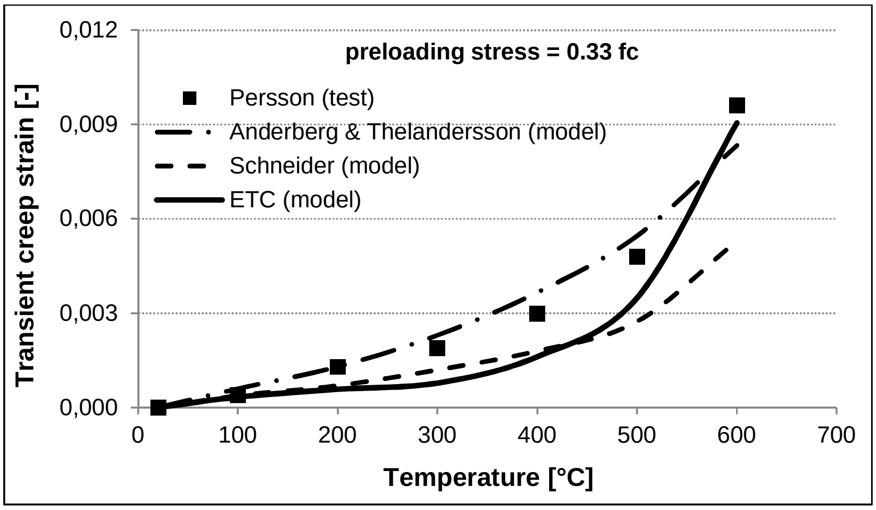

Figure 5. Comparison between different models of transient creep and experimental data.

In tension, the cracking behavior of concrete is described by a smeared-crack model, which means that neither the opening of the individual cracks nor the spacing between different cracks is present in the model. The stress–strain relationship is made of a second order ascending branch and a descending branch made of two curves, each of them being a third order function of the strain. If concrete has been loaded in tension and, in a later stage, the strain decreases, the unloading is made according to a damage model. This means that the path is a linear decrease from the point of maximum tensile strain in the loading curve to the point of origin in the stress–strain diagram plane.

4.3 Introduction of Transient Creep into a Finite Element Code

Let us assume that at time t_i the finite element code has converged, which means that the local state of equilibrium of the material is completely defined everywhere in the structure or, in terms of numerical modeling, at every integration point, i.e. the values of $\left\{\varepsilon_{tot,i}\,;\varepsilon_{p,i}\,;\varepsilon_{tr,i}\right\}$ and $\left\{\sigma_i\right\}$ are known. The values of the displacements at the nodes are also defined. Then suppose that, from time t_i to time t_{i+1}, the variation of the displacements of the nodes calculated by the finite element code produce an increment in total strain, that is noted $\Delta\varepsilon_{tot}$. The problem is to update to time t_{i+1} the basic variables describing the local state of the body in a manner that is consistent with the constitutive law. This process should also yield the tangent modulus of the constitutive law, to be used by the finite element code in the iteration process.

The total strain $\varepsilon_{tot,i+1} = \varepsilon_{tot,i} + \Delta\varepsilon_{tot}$ is calculated straightforwardly (some codes even give the total displacement, and strain, as a primary result).

By definition of the free thermal strain, this term is calculated directly as a function of the temperature at time t_{i+1}, $\varepsilon_{th,\mathrm{i+1}} = f(T_{\mathrm{i+1}})$.

This allows deriving the mechanical strain from $\varepsilon_{m,i+1} = \varepsilon_{tot,i+1} - \varepsilon_{th,i+1}^{\text{explicit}}$.

Next step is the calculation of the transient creep strain. For the step-by-step analysis of the structure, an incremental form of the transient creep strain term given by Eq. (6) has to be derived. Calculating the time derivative of this term with the chain rule and then applying a backward-Euler difference scheme yields the following approximation:

$$\varepsilon_{tr,i+1} = \varepsilon_{tr,i} + \left.\frac{\sigma}{f_{ck}}\right|_{i+1} \times \Delta\phi + \left.\frac{\phi}{f_{ck}}\right|_{i+1} \times \Delta\sigma \qquad (7)$$

where ϕ is a function of temperature.

However, by definition, an increment of stress $\Delta\sigma \neq 0$ at constant temperature ($\Delta\phi = 0$) induces no additional transient creep, so that there is no contribution of the term in $\Delta\sigma$. Eq. (7) can be rewritten to yield the following equation:

$$\varepsilon_{tr,i+1} = \varepsilon_{tr,i} + \left[\phi\left(T_{i+1}\right) - \phi\left(T_i\right)\right] \frac{\sigma|_{i+1}}{f_{ck}} \tag{8}$$

At each step, the transient creep term is incremented only if the temperature variation shows a first-time heating under compressive stress. As the function $\phi(T)$ is growing with temperature, the transient creep term can only increase. Transient creep strain is irreversible at both load and temperature decrease. At temperature decrease, there is no variation of transient creep strain. However, at load decrease, additional transient creep strain is assumed to develop in the concrete material submitted to increasing temperature as long as the stress in the material remains in compression. The increase in transient creep strain that develops in the concrete material submitted to (first-time) increase in temperature and submitted to given compressive stress is the same for loading and unloading [7]. Once the transient creep term at time t_{i+1} is known by Eq. (8), the instantaneous stress-related strain can be derived from the mechanical strain according to the following equation:

$$\varepsilon_{\sigma,i+1}^{\text{explicit}} = \varepsilon_{m,i+1} - \varepsilon_{tr,i+1}^{\text{explicit}} = \varepsilon_{tot,i+1} - \varepsilon_{th,i+1}^{\text{explicit}} - \varepsilon_{tr,i+1}^{\text{explicit}} \tag{9}$$

In the explicit transient creep formulation (ETC) of the model, the aim is to express the constitutive relationship in terms of the instantaneous stress-related strain, in order to treat the transient creep effects separately from the elastic and plastic effects. The relationship between σ_{i+1} and $\varepsilon_{\sigma,i+1}^{\text{explicit}}$ can be obtained by an adaptation of the current EC2 implicit relationship. First, it should be remembered that, by assumption, the mechanical strain considered in the EC2 model implicitly includes the transient creep strain that develops for a material first-time heated under constant stress, which can be expressed as follows:

$$\varepsilon_{m,i+1}^{\text{implicit}} = \varepsilon_{\sigma,i+1}^{\text{explicit}} + \phi(T_{i+1}) \frac{\sigma_{i+1}}{f_{ck}} \tag{10}$$

Then, the relationship between the stress σ_{i+1} and the instantaneous stress-related strain $\varepsilon_{\sigma,i+1}^{\text{explicit}}$ is obtained by inserting Eq. (10) into the EC2 implicit relationship given by Eq. (3). The resulting relationship is given by Eq. (11) where the subscripts *i+1* have been omitted to simplify the notation.

$$\frac{\sigma}{f_c(T)} = \frac{3\left(\varepsilon_\sigma^{\text{explicit}} + \phi(T)\,{}^{\sigma}\!/_{f_{ck}}\right)}{\varepsilon_{c1,EC2}(T)\left(2+\left(\dfrac{\varepsilon_\sigma^{\text{explicit}} + \phi(T)\,{}^{\sigma}\!/_{f_{ck}}}{\varepsilon_{c1,EC2}(T)}\right)^3\right)} \tag{11}$$

At a given step, knowledge of the instantaneous stress-related strain can theoretically give the stress from Eq. (11). However, it is not straightforward to extract σ from Eq. (11). Two methods can be applied: either a direct relationship $\sigma = f\left(\varepsilon_\sigma^{\text{explicit}}\right)$ can be derived that approximates Eq. (11) at each temperature or an algorithmic strategy can be implemented to solve Eq. (11) iteratively. The first method should probably be preferred in order to allow an easier generalization of the ETC model in three dimensions. It will be developed in Section 4.5.

Fig. 4 shows at 500°C the ETC relationship between the stress and the instantaneous stress-related strain ε_σ, expressed by Eq. (11), next to the EC2 relationship between the stress and the mechanical strain ε_m. The difference between the ETC curve and the EC2 curve is the transient creep strain given by Eq. (6). The experimental results for the instantaneous stress-strain relationship obtained by Schneider [20] are plotted and can be compared to the ETC constitutive curve.

4.4 Tangent Modulus of the Instantaneous Stress-Strain Curve

The tangent modulus of the stress–strain relationship has to be determined and, if Eq. (11) is used as constitutive relationship (for the second method mentioned in Section 4.3), the derivation is not straightforward. The tangent modulus calculation is performed using the change of variable of Eq. (12). The tangent modulus can then be expressed as a function of the new variable derivative of stress

$$\xi = \varepsilon_\sigma + \phi(T)\left(\sigma/f_{ck}\right) \quad ; \quad \frac{d\sigma}{d\varepsilon_\sigma} = \frac{d\sigma}{d\xi}\,\frac{d\xi}{d\varepsilon_\sigma} = \frac{d\sigma}{d\xi}\;\frac{1}{1-\dfrac{\phi(T)}{f_{ck}}\,\dfrac{d\sigma}{d\xi}} \tag{12}$$

After inserting the new variable ξ into Eq. (11), the term $d\sigma/d\xi$ can be calculated. Finally, the ETC tangent modulus calculation can be performed and after a few manipulations, it yields

$$\frac{d\sigma}{d\varepsilon_\sigma} = \frac{6\,f_c\left[1-\left(\xi/\varepsilon_{c1,EC2}\right)^3\right]}{\varepsilon_{c1,EC2}\left[2+\left(\xi/\varepsilon_{c1,EC2}\right)^3\right]^2 - 6\,\phi\,\dfrac{f_c}{f_{ck}}\left[1-\left(\xi/\varepsilon_{c1,EC2}\right)^3\right]} \tag{13}$$

The initial stiffness (elastic modulus), obtained by replacing ε_σ and σ by zero (thus $\xi = 0$) in Eq. (13), is equal to the ENV elastic modulus with the minimum value of PSS.

4.5 Development of a direct relationship between the stress and the instantaneous stress-related strain approximating Eq. (11)

A direct relationship $\sigma = f\left(\varepsilon_\sigma^{\text{explicit}}\right)$ with the same generic form as the current EC2 model can be derived as an approximation of Eq. (11) as

$$\frac{\sigma}{f_c(T)} = \frac{n\ \varepsilon_\sigma^{\text{explicit}}}{\varepsilon_{c1,ETC}(T)\left[(n-1)+\left(\dfrac{\varepsilon_\sigma^{\text{explicit}}}{\varepsilon_{c1,ETC}(T)}\right)^n\right]} \qquad (14)$$

where *n* is a parameter to be determined and $\varepsilon_{c1,ETC}(T)$ is the PSS for the ETC relationship, given by the following equation:

$$\varepsilon_{c1,ETC} = \varepsilon_{c1,EC2} - \phi\frac{f_c}{f_{ck}} = \frac{2\,\varepsilon_{c1,\min} + \varepsilon_{c1,EC2}}{3} \qquad (15)$$

The ETC tangent modulus and the ETC initial stiffness (elastic modulus) are obtained directly by derivation of Eq. (14). The parameter *n* has to be chosen to obtain the best possible correlation between Eq. (14) and Eq. (11). A single value of *n* was used for all temperatures. A good indication to calibrate the parameter *n* is to calibrate the ETC initial stiffness E_{ETC} on the ENV elastic modulus with the minimal value of the PSS. This is done using the following equation:

$$E_{ETC} = \frac{n\ f_c}{(n-1)\,\varepsilon_{c1,ETC}} = \frac{3\ f_c}{2\,\varepsilon_{c1,\min}} \Leftrightarrow \frac{n}{(n-1)} = \frac{3\,\varepsilon_{c1,ETC}}{2\,\varepsilon_{c1,\min}} = 1 + \frac{\varepsilon_{c1,EC2}}{2\,\varepsilon_{c1,\min}} \qquad (16)$$

Good correlation between Eq. (14) and Eq. (11) in the range of temperatures from 100°C to 1100°C is obtained using $n = 2$. The initial stiffness of the ETC model is close to the elastic modulus of the ENV with $\varepsilon_{c1,\min}$.

4.6 Characteristics of the ETC model

- The ETC model has the same generic form as the current EC2 implicit model;
- The ETC initial stiffness is close to the elastic modulus of ENV with minimal value of the PSS, which leads to an accurate representation of the elastic modulus of the material;

- The transient creep strain calculated with the ETC model is comparable to other models found in literature (Fig. 5);
- The instantaneous stress-strain relationships considered in the ETC model are consistent with experimental data obtained by steady-state tests (Fig. 4);
- The mechanical stress-strain relationships obtained with the ETC model for a material first-time heated under constant stress (transient tests) are calibrated to yield the same results as the present version of EC2.

5. EXPERIMENTAL VALIDATION AT THE MATERIAL LEVEL FOR UNSTEADY TEMPERATURES AND LOADS

The ETC model is validated by a comparison between experimental results and the computed values of the mechanical strain developed by concrete specimens subjected to unsteady temperatures and loads. The considered experiments are taken from Schneider et al. [24]. The specimens are axially unrestrained cylinders with 80 mm diameter and 300 mm height. In all cases, the temperature is constantly increasing at a heating rate of 2 °C/min. The compressive strength at 20°C is 38 MPa. The numerical calculations are performed with the software SAFIR [18] where the ETC model has been implemented.

The concrete cylinders are subjected to different stress–time relationships (which can be traduced in stress–temperature relationships, see first column in Fig. 6, because the temperature is increasing at constant heating rate). The aim is to highlight the influence of the explicit consideration of transient creep strain on the mechanical strain calculation. The mechanical strains computed using the ETC model and the EC2 implicit model are compared to the measured results, see second column in Fig. 6. The observations are put in relation with the theoretical considerations discussed in Section 3.

The first test corresponds to Situation I, i.e. simultaneously increasing stress and increasing temperature. The results given by the two models are very close to each other, see first row in Fig. 6.

The second test successively represents Situation I (increasing stress), Situation II (decreasing stress) and finally Situation III (constant stress). At the beginning and until the peak stress, the difference between the two models is very small, see second row in Fig. 6. Then, the stress rate becomes negative. During this second phase of the test (decreasing stress), the mechanical strain computed by the EC2 implicit model quickly decreases, because the transient creep strain is being recovered. On the contrary, the mechanical strain computed by the ETC model keeps on growing, though more and more slowly, because transient creep strain still develops in the material. The transient creep strain counterbalances the elastic unloading due to the stress decrease. During this phase, the behavior predicted by the ETC model better matches the measured behavior. This tends to confirm the fact that implicit models are not able to capture properly the actual unloading stiffness at elevated temperatures. At the end of the test, the stress is kept constant (Situation III)

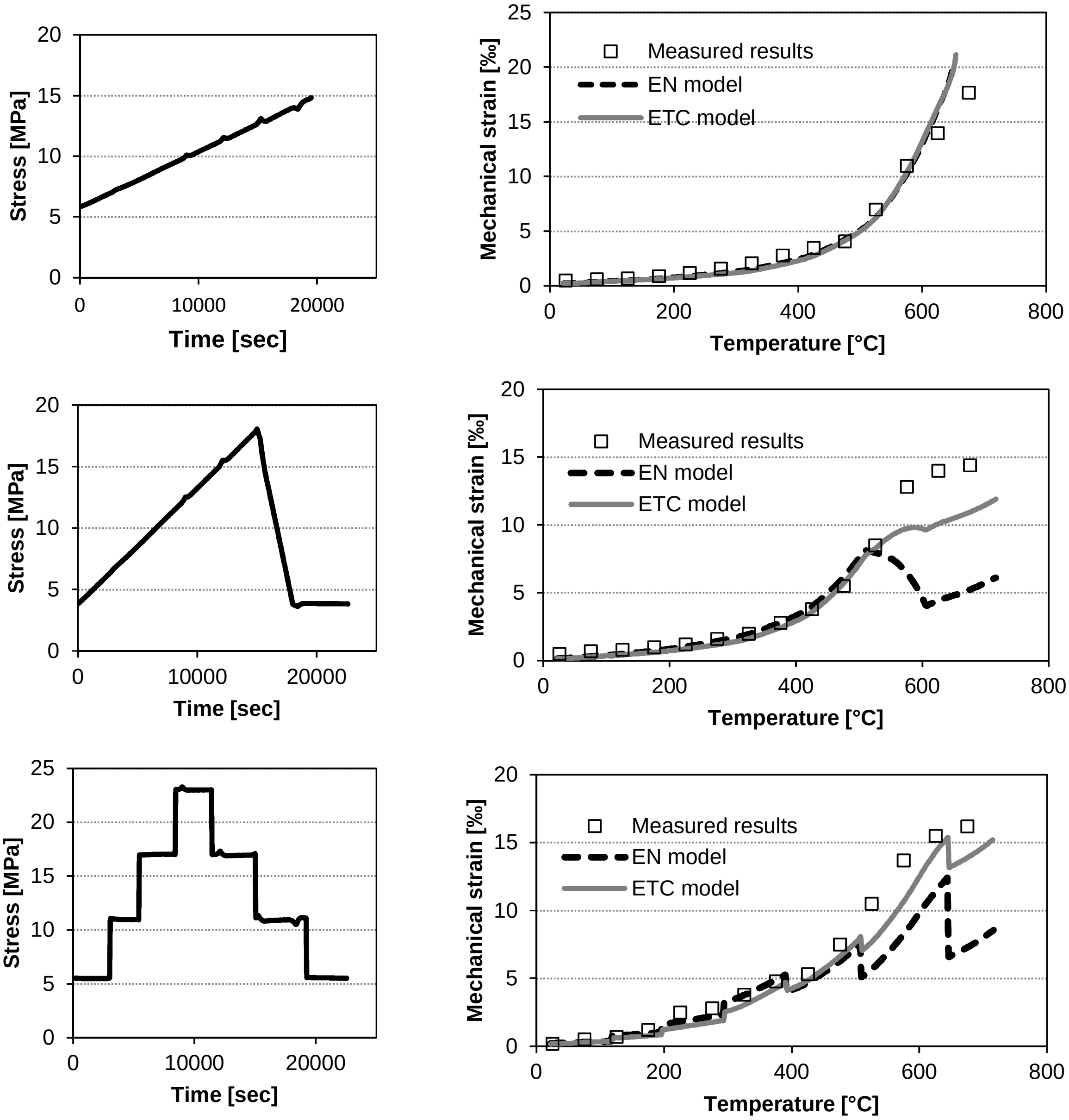

Figure 6. Mechanical strain-temperature relationships: comparison between measured and computed results.

and both models predict exactly the same variation of the mechanical strain. The ETC model is able to capture qualitatively the experimental response during the three phases of the test, as opposite to the EC2 model.

In the third test, the specimen is successively subjected to different constant stress levels while the temperature is increasing (Situation III). The transition between two stress levels is made by a ‘‘step’’, i.e. a quasi-instantaneous variation from one stress level to another, see third row in Fig. 6. At each stress step, the corresponding mechanical strain variations predicted by the two models are slightly different. Implicit models such as the EC2 models amplify the effect of a stress step on the mechanical strain variation. Indeed, the transient creep strain considered in implicit models is suddenly increased or decreased together with the elastic strain. On the contrary in explicit models, transient creep strain does not vary in such situations where the stress varies at constant temperature. It can be seen that the behavior predicted by the ETC model better matches the experimental behavior of the specimens, thanks to a better modeling of the material stiffness at constant high temperature.

6. COMPARISON WITH EXPERIMENTAL DATA ON A CONCRETE COLUMN SUBJECTED TO FIRE

An experimental fire test made in Japan on a centrally loaded concrete column [25] was simulated using the nonlinear finite element software SAFIR. A comparison between the numerical results considering different concrete models and the experimental data was performed. The column is 300 mm by 300 mm in cross section with a central hole of 100 mm diameter. The concrete compressive strength is 55 MPa. Four 16 mm longitudinal rebars are present with a cover of 40 mm. The column, submitted to a load of 677 kN, was exposed to Japanese standard fire temperature–time curve for 180 min. Then, the element was allowed to cool down. The deformation behavior can be observed in Fig. 7.

- The ENV model (1995) with recommended value of the peak stress strain (PSS) [3] leads to too large elongations, because of a highly underestimated transient creep strain. This model had been found to be far too stiff and has been removed when transforming the Eurocode from an ENV to an EN.
- The ETC model and the EC2 model lead to comparable results during approximately the first 140 min of heating. Beyond 140 min, the behavior predicted by the ETC model tends to differ from the behavior predicted by the EC2 model; the effect of the explicit consideration of transient creep strain on the structural behavior becomes notable. The ETC model matches better than the EC2 model the actual behavior of the structure.
- The difference between the behaviors predicted by the ETC and the EC2 models is particularly significant during the cooling phase. Measured data showed a very important decrease of the elongation, due to a progressive decrease of thermal strain coupled with a very limited recovery of mechanical strain. Indeed, mechanical strain is mostly composed of permanent strain. This behavior is well represented by the ETC model owing to the explicit consideration of transient creep. On the contrary, the EC2 model implicitly recovers the transient creep strain, leading to an underestimated final shortening of the column.

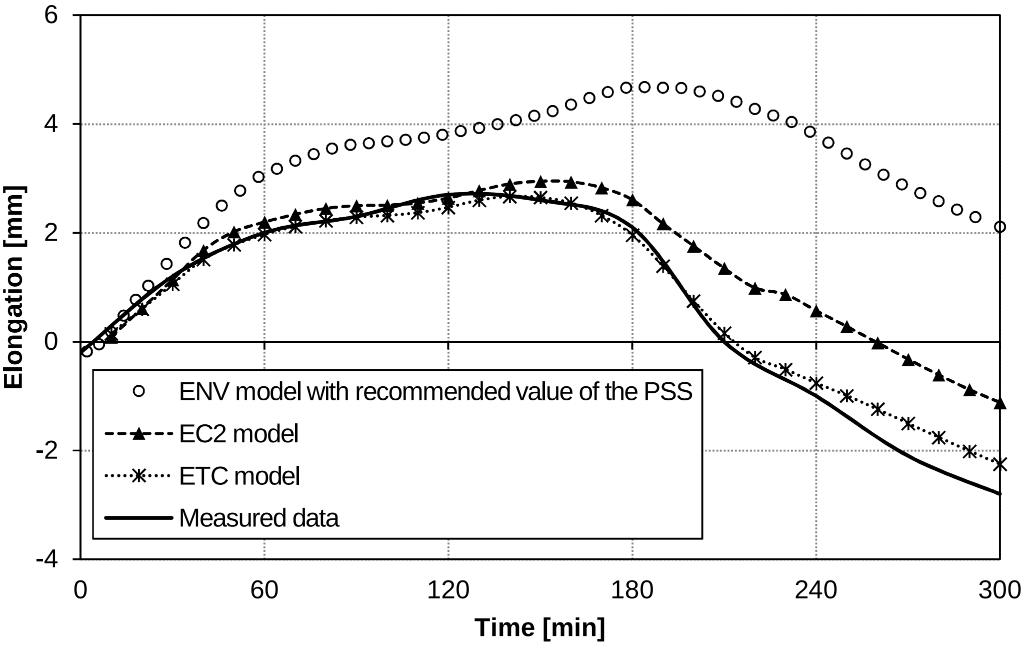

Figure 7. Comparison between numerical simulations and experimental test.

In a real building, restraint forces and moments typically appear in the structural elements subjected to fire due to the connection with the rest of the structure. The analysis of rein- forced concrete elements with axial and/or rotational restraint has not been performed in this study. It is expected that the influence of the transient creep strain in the explicit model compared to the implicit model will be even more pronounced for restrained fire-exposed structures than for simply supported fire-exposed structures. For instance, in an axially restrained concrete column subjected to fire, the restraint force first increases due to thermal expansion and then decreases when the mechanical strain in compression exceeds the thermal elongation strain. During the contraction phase of the column, when the load is progressively transferred from the fire-exposed column to the rest of the structure, the computation of the concrete material unloading stiffness is a key issue for the validity of the simulation of the structural behavior. The use of a concrete model that includes an explicit term for transient creep strain is thus necessary for statically indeterminate structures, also during the heating phase of the fire.

7. CONCLUSION

Concrete constitutive models that include implicitly the transient creep strain, such as the current Eurocode 2 model, have inherent limitations that prevent them from accurately representing the mechanical strains developed in concrete members subjected to fire. Especially, implicit models are not able to capture properly the actual unloading stiffness at elevated temperatures. The new formulation of the generic Eurocode 2 concrete model that contains an explicit term for consideration of the transient creep (ETC model) brings a supplementary accuracy without removing the generic characteristic of the EC2 model. The ETC model has the same formalism as the EC2 model and its implementation in finite-element software can be performed by an adaptation of the current EC2 model. The improvement may be significant as indicated by comparisons against experimental data performed at the material level and for a simple structural element. The utilization of an ETC model should be particularly recommended when modeling the cooling phase of a fire because it is able to capture the irreversibility of transient creep.

More experimental and numerical comparisons have to be performed in order to quantify the consequences of the explicit consideration of transient creep strain on the global behavior of concrete subjected to fire. Additional research work has been performed on restrained fire-exposed structural elements [26], as these elements are commonly found in real buildings, and the effect of the transient creep strain model proved to be more pronounced for restrained elements than for statically determinate elements.

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Nomenclature

ε_{tot}	Total strain
ε_{th}	Free thermal strain
ε_{m}	Mechanical strain
ε_{cr}	Basic creep strain
ε_{σ}	Instantaneous stress-related strain
ε_{tr}	Transient creep strain
ε_{el}	Elastic strain
ε_{p}	Plastic strain
T	Temperature
σ	Stress
f_c	Concrete compressive strength at elevated temperature
$\varepsilon_{c1,EC2}$	Peak stress strain (PSS) given in the EC2 [17]
$\varepsilon_{c1,\min}$	Minimum value of the PSS given in ENV [3]
$\phi(T)$	Function considered for the calculation of the transient creep
f_{ck}	Concrete compressive strength at ambient temperature
E_{ENV}	ENV elastic modulus
$E_{\mathrm{EC2}}^{\mathrm{implicit}}$	Tangent at 0 stress to the EC2 curve [17]
$t_i\,;\ t_{i+1}$	Time at the beginning and at the end of the considered step
$\varepsilon_{tot,i}\,;\varepsilon_{tot,i+1}$	Total strain at time t_i and at time t_{i+1}
$\varepsilon_{p,i}\,;\varepsilon_{p,i+1}$	Plastic strain at time t_i and at time t_{i+1}
$\varepsilon_{tr,i}\,;\varepsilon_{tr,i+1}$	Transient creep strain at time t_i and at time t_{i+1}
$\sigma_i\,;\sigma_{i+1}$	Stress at time t_i and at time t_{i+1}
$\Delta\varepsilon_{tot}$	Increment in total strain between t_i and t_{i+1}
$\Delta\phi$	Increment of the function ϕ from time t_i to time t_{i+1}
$\Delta\sigma$	Increment of stress from time t_i to time t_{i+1}
ξ	Variable considered for the calculation of the tangent modulus
n	Parameter in the ETC relationship
$\varepsilon_{c1,ETC}$	Peak stress strain for the ETC relationship
E_{ETC}	ETC initial stiffness (elastic modulus)