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Efficient Evaluation of the Geometrical Validity of Curvilinear Finite Elements

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The development of high-order numerical techniques on unstructured grids has been underway for many years. The accuracy of these methods strongly depends of the accuracy of the geometrical discretization, and thus depends on the availability of quality curvilinear meshes.

The usual way of building such curvilinear meshes is to first generate a straight sided mesh. Then, mesh entities that are classified on the curved boundaries of the domain are curved accordingly. Some internal mesh entities may be curved as well. If we assume that the straight sided mesh is composed of well shaped elements, curving elements introduces a kind of "shape distortion" that should be controlled so that the final curvilinear mesh is also composed of well shaped elements.

In this work we propose a method to analyze curvilinear meshes in terms of their elementary jacobians. The method does not deal with the actual generation of the high order mesh. Instead, it provides an efficient way to guarantee that a curvilinear element is geometrically valid, i.e., that its jacobian is strictly positive in all its reference domain. It also provides a way to measure the distortion of the curvilinear element. The key feature of the method is to adaptively expand the elementary jacobians in a polynomial basis, built using Bzier functions, that has both properties of boundedness and positivity.

The algorithm has been implemented in the open-source mesh generator Gmsh, and allows to control the geometrical validity of curvilinear meshes made of triangles, quadrangles, tetrahedra, hexahedra and prisms of any order. Figure **??** shows a typical curvilinear mesh made of planar second order triangles.

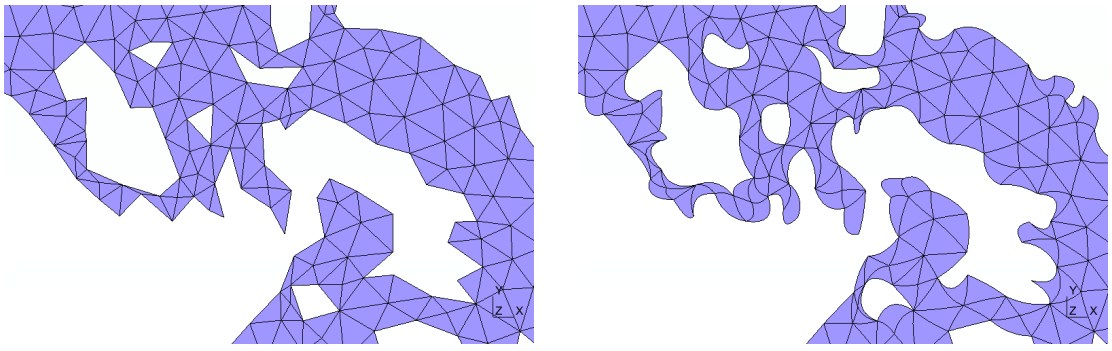

Figure 1: Initial and final stage of the curvilinear meshing procedure: straight-sided mesh (left) and curved mesh without invalid elements (right)