University of Liège
Aerospace & Mechanical Engineering

Numerical simulations of brittle and elasto-plastic fracture for thin structures subjected to dynamic loadings

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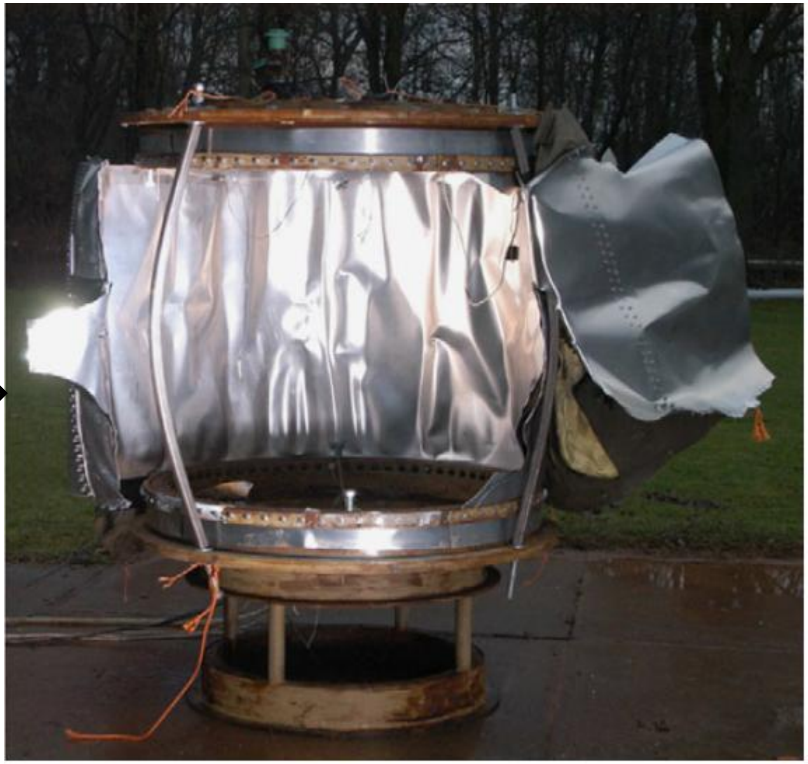
- A thin body is a structure with a dimension largely smaller than the other ones
  - This dimension is called the thickness

Introduction

- Improve the safety of pressurized thin bodies by understanding their fracture behavior

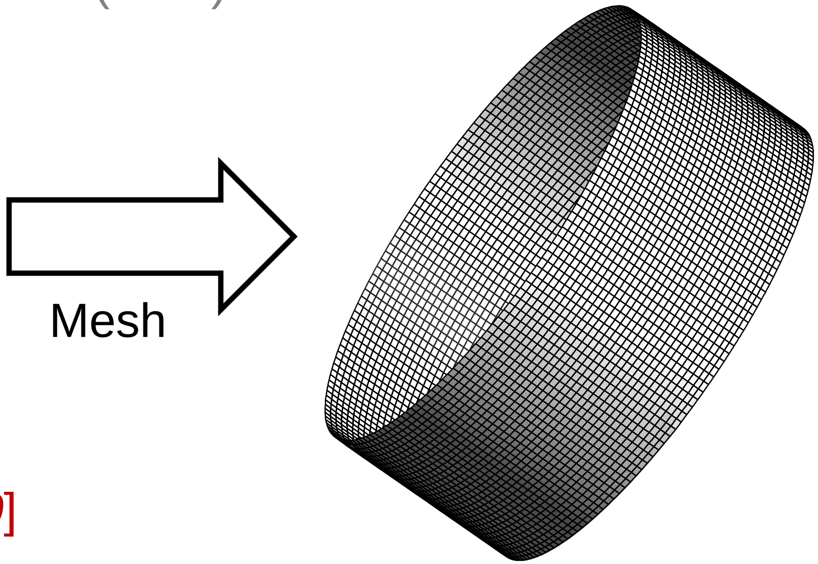
[*Larsson et al ijnme 2010*]

Blast



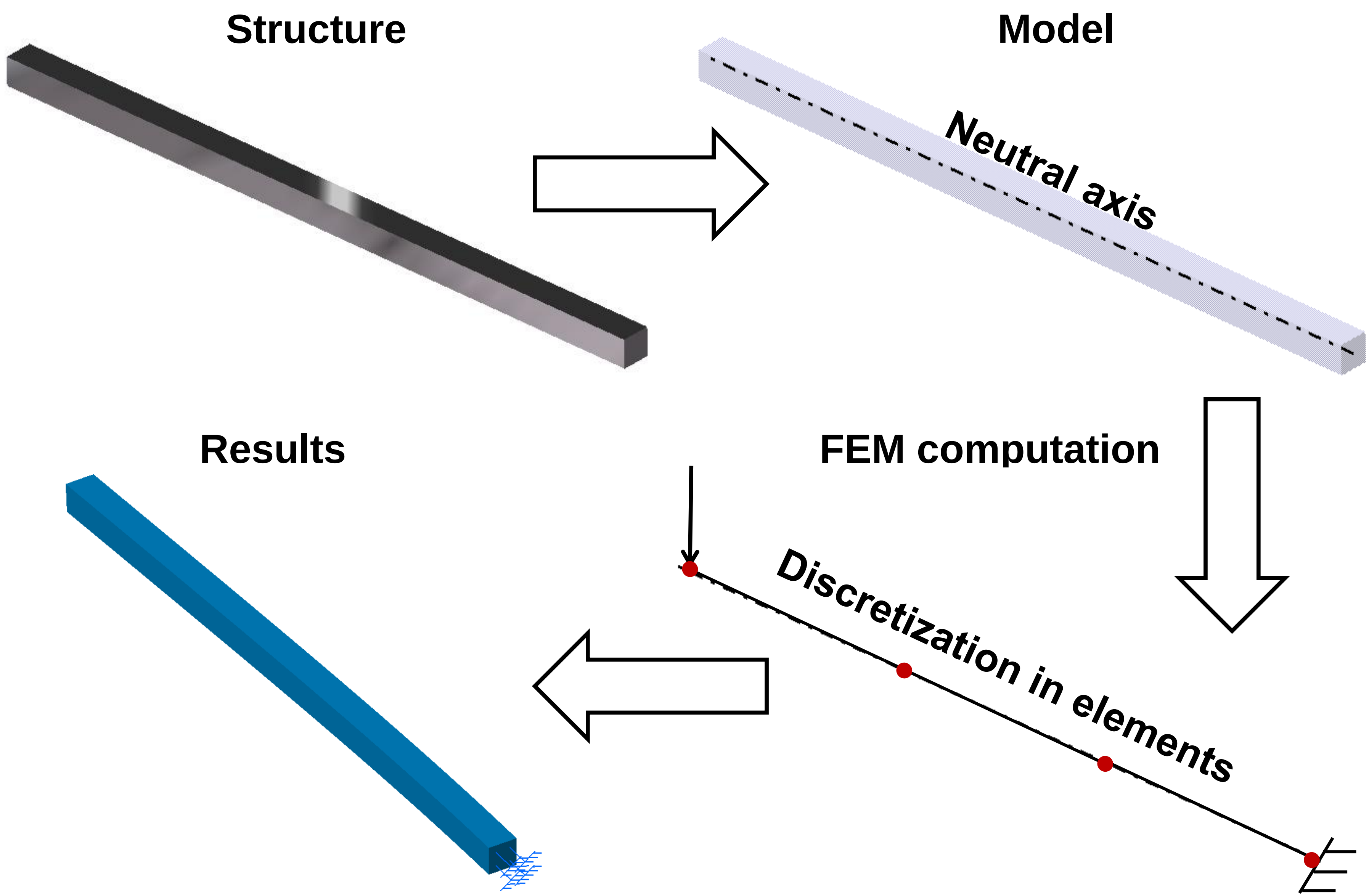
- Recourse to the finite element method allows cheaper designs
  - A numerical model is an idealization of reality based on mathematical equations
  - The finite element method (FEM) discretizes the structure in elements

[*Larsson et al ijnme 2010*]



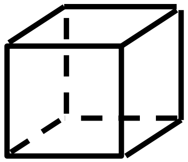
  - The finite element method is a powerful tool in mechanics

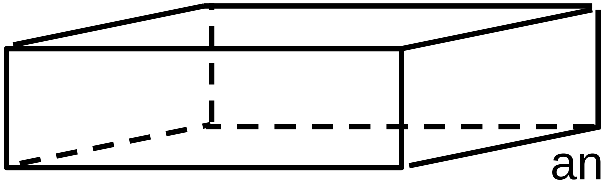
Introduction



- Beam or shell elements can advantageously be used to model thin bodies
  - The dimensions of an element have to be slightly the same in all directions

Better results

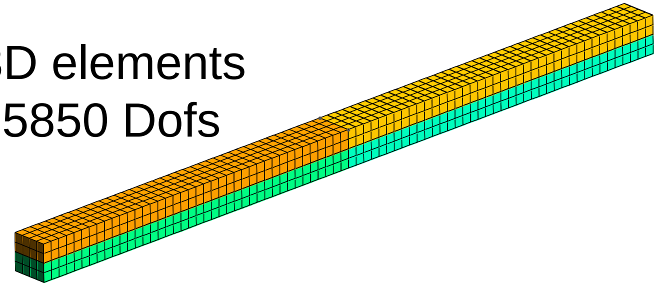




anisotropy

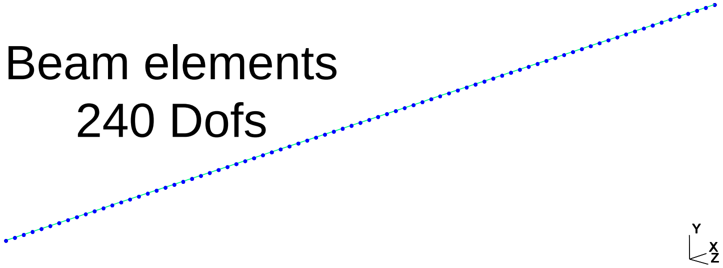
  - Classical 3D elements leads to a huge number of elements for thin bodies

3D elements
5850 Dofs

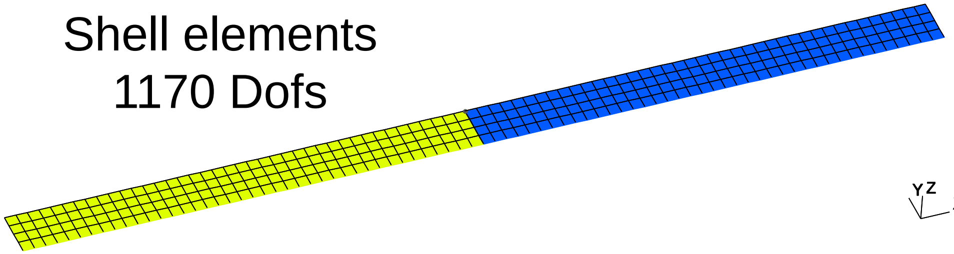


  - Beam and shell elements use a 1D or 2D element as basis and account separately for the thickness → drastically reduces the time of computation

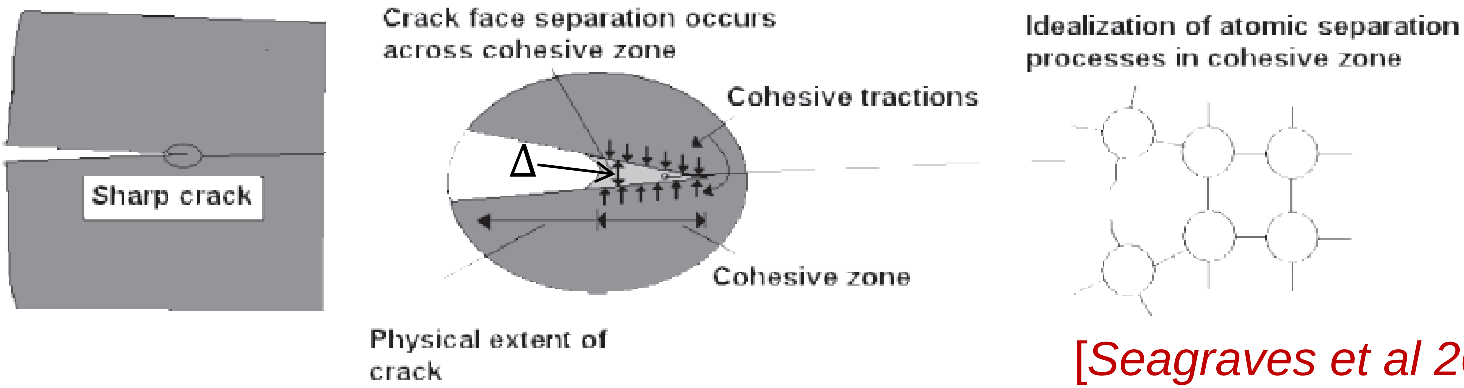
Beam elements
240 Dofs



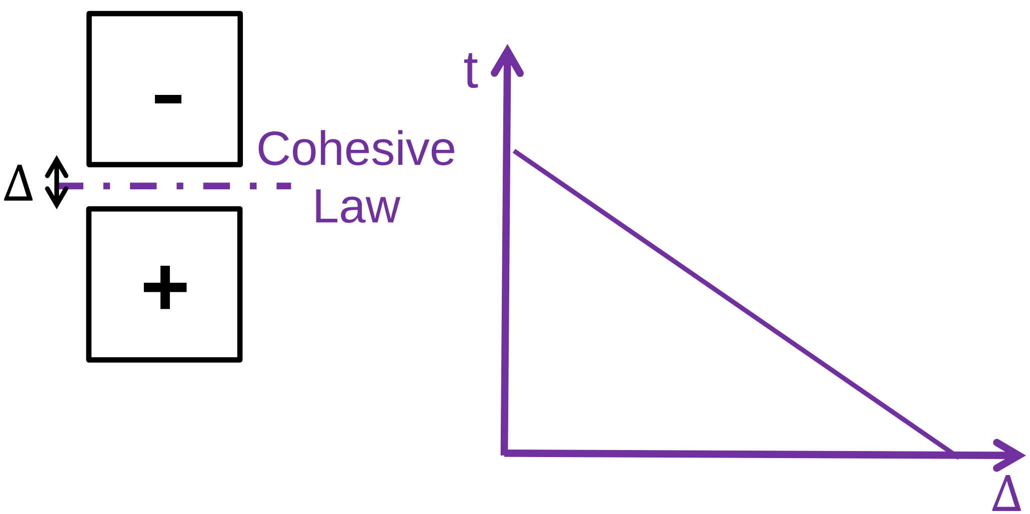
Shell elements
1170 Dofs



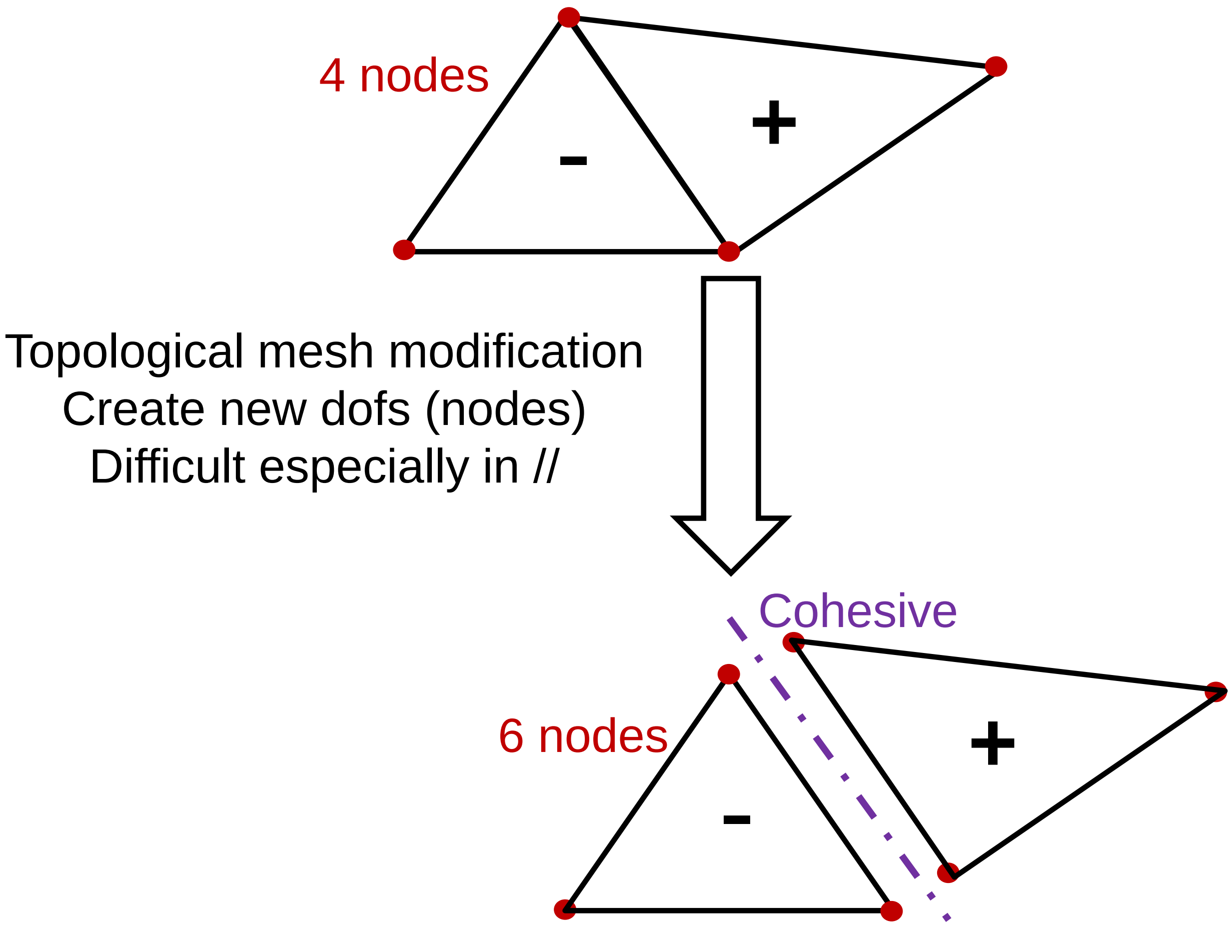
- Cohesive zone model is very appealing to model crack initiations in a numerical model
  - Model the separation of crack lips in brittle materials



[*Seagraves et al 2010*]

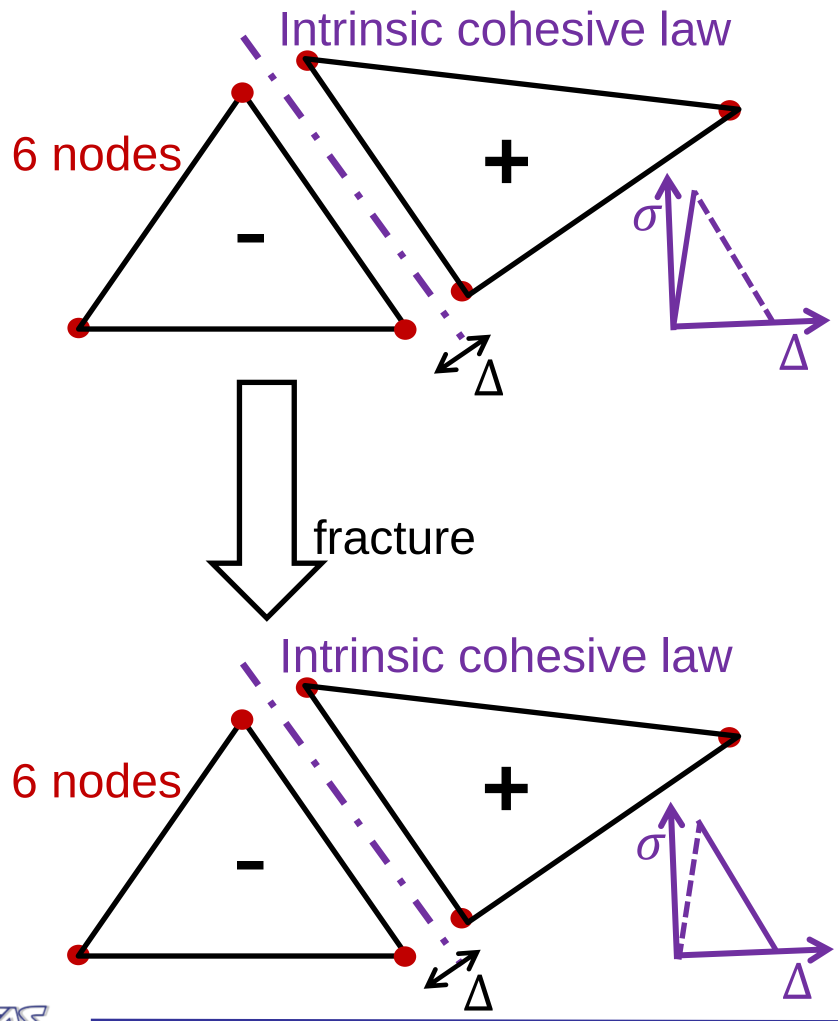


- The insertion of cohesive elements during the simulation is difficult to implement as it requires topological mesh modifications
  - FEM (continuous Galerkin)



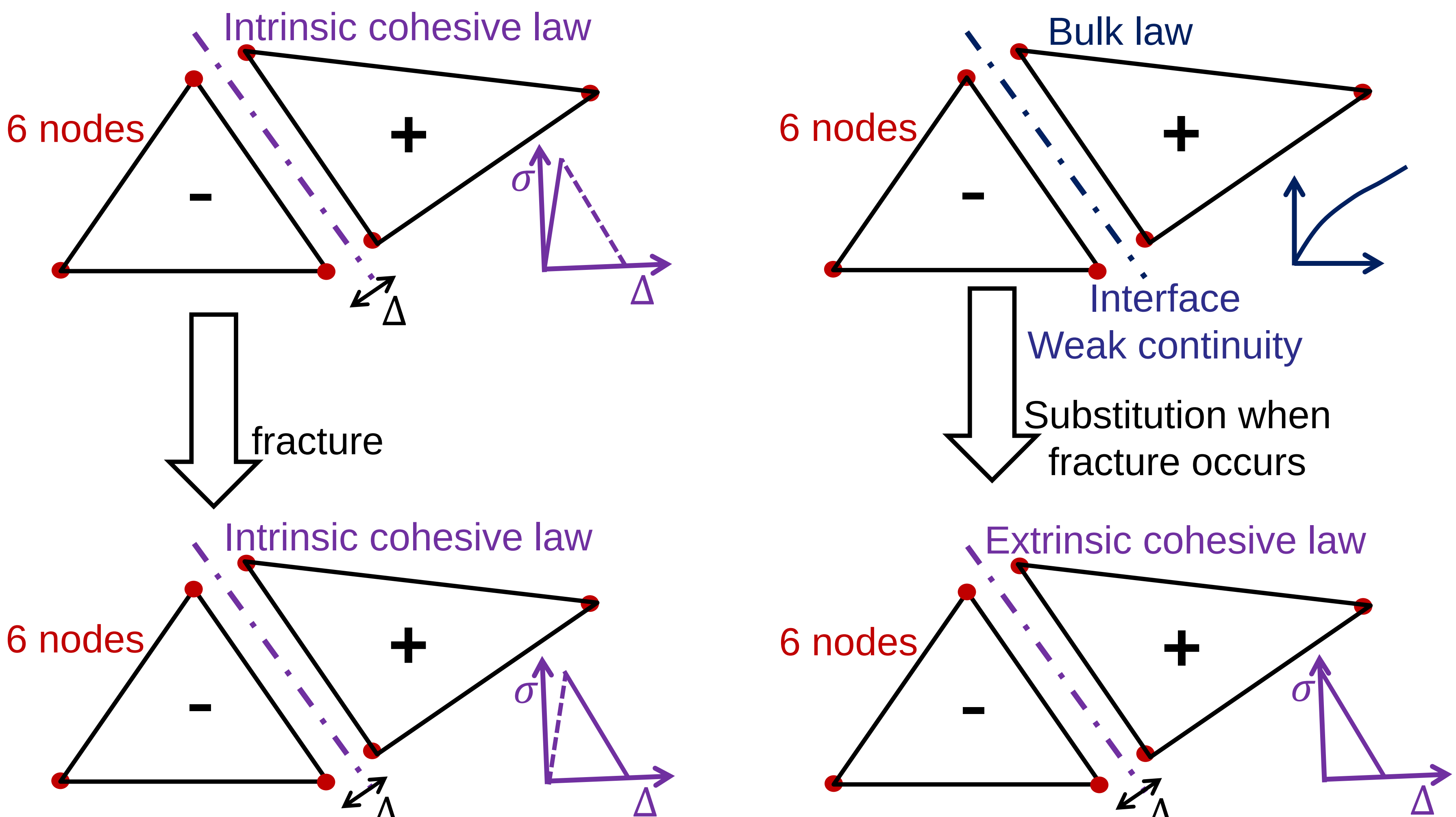
- A recourse to an intrinsic cohesive law is generally done with FEM

  – FEM (continuous Galerkin)



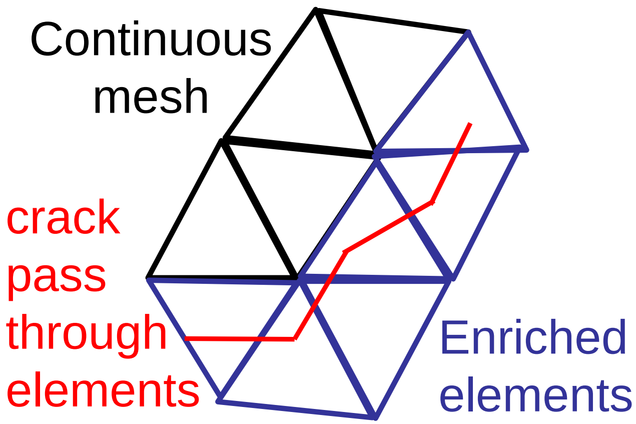
- Intrinsic cohesive law leads to numerical problems [*Seagraves et al 2010*]
  - Spurious stress wave propagation
  - Mesh dependency
  - Too fast crack propagation

- Use of extrinsic cohesive law is easier when coupled with DG
  - FEM (continuous Galerkin)
  - Discontinuous Galerkin



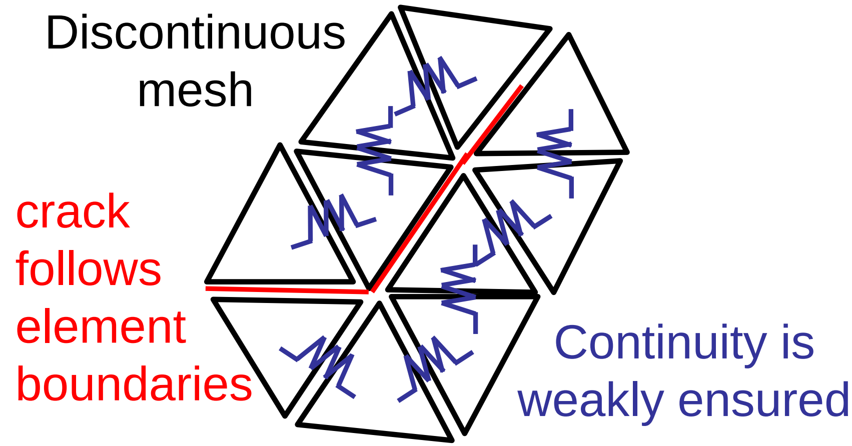
- Other methods exist but we focus on the discontinuous Galerkin method which has to be extended for thin bodies

  - XFEM



Commonly used for crack propagation

  - Discontinuous Galerkin



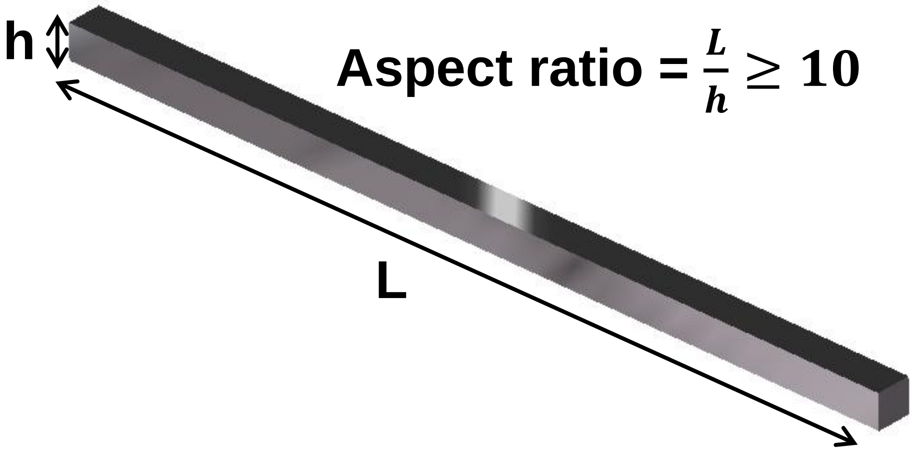
Recently developed for dynamic phenomena (crack propagation due to impact, fragmentation) but for 3D elements only

Plan

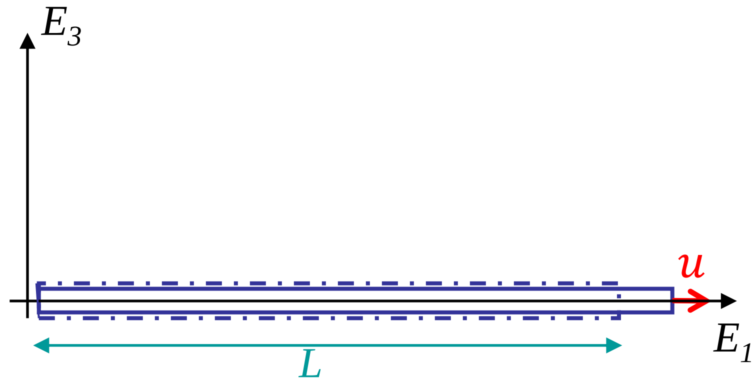
- Develop a discontinuous Galerkin method for thin bodies
  - Beam elements (1.5D case)
  - Shell elements (2.5D case)
- Discontinuous Galerkin / Extrinsic Cohesive law framework
  - Develop a suitable cohesive law for thin bodies
- Applications
  - Fragmentations, crack propagations under blast loadings

Full-DG formulation of Euler-Bernoulli beams

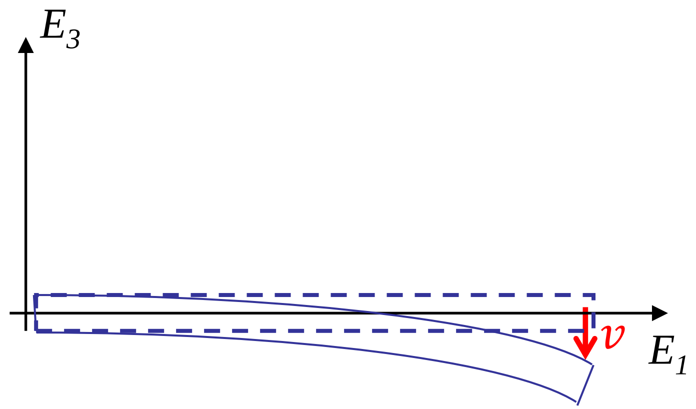
- Highlights
- Simple 1D thin structure
- Restrict the analysis to
  - Linear small strains
  - Straight rectangular beam (without initial deformation)
  - Out-of-plane shearing can be neglected
  - Plane stress state



- 2 (independent in small deformations) deformation modes (shearing is neglected)
  - Membrane mode

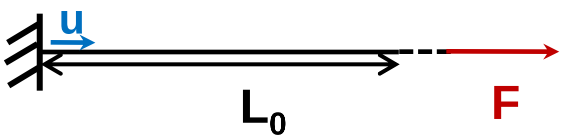


  - Bending mode



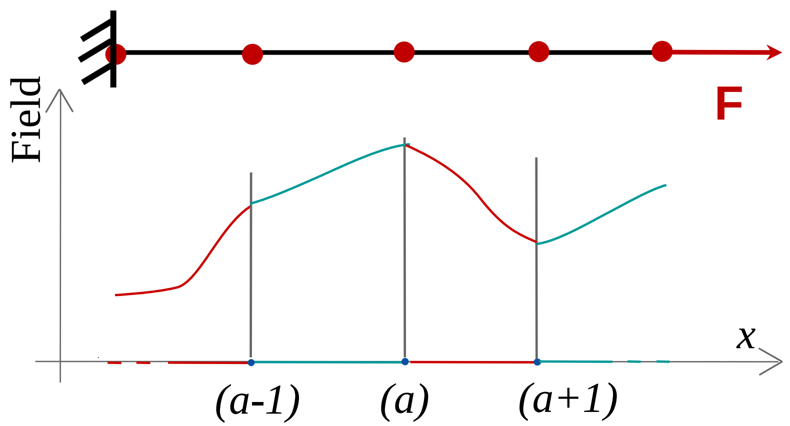
Full-DG formulation of Euler-Bernoulli beams

- Membrane mode
  - Strong form $(n^{11})_{,1} = 0$ with $n^{11} = \int_{-h/2}^{h/2} \sigma^{11} d\xi^3$



  - Weak form $\int_0^L (n^{11})_{,1} \delta u \, dx = 0$

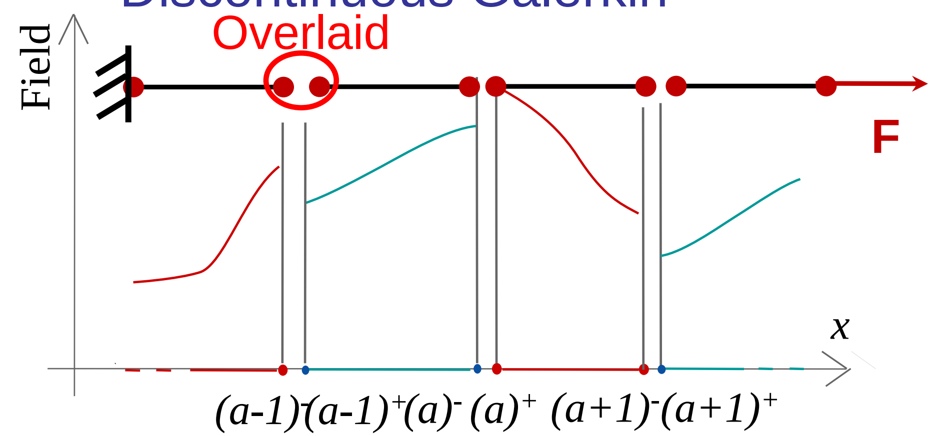
- FEM (Continuous Galerkin)



Integration by parts on the beam

$$\sum_e \int_{l_e} \underbrace{n^{11} \delta u_{,1} dx}_{\text{Bulk term}} = 0$$

- Discontinuous Galerkin

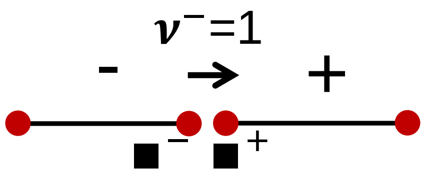


Integration by parts on each element

$$\sum_e \left(\int_{l_e} \underbrace{n^{11} \delta u_{,1} dx}_{\text{Bulk term}} - \underbrace{n^{11} \delta u]_{l_e}}_{\text{Interface term}} \right) = 0$$

- The interface terms are developed

  – Operators definition



Mean: $\langle \blacksquare \rangle = \frac{1}{2}\left(\blacksquare^{+} + \blacksquare^{-}\right)$

Jump: $[\![\blacksquare]\!] = \blacksquare^{+} - \blacksquare^{-}$

  – Using operators

$$-\sum_{e} n^{11}\delta u]_{l_e} = \sum_{s} [\![n^{11}\delta u]\!]_s$$

  – Using mathematical identity $[\![ab]\!] = \langle a \rangle [\![b]\!] + [\![a]\!] \langle b \rangle$

$$-\sum_{e} n^{11}\delta u]_{l_e} = \sum_{s} \left(\langle n^{11} \rangle [\![\delta u]\!] + [\![n^{11}]\!] \langle \delta u \rangle \right)_s$$

Full-DG formulation of Euler-Bernoulli beams

- The jump is replaced by a consistent numerical flux (no equality)

$$-\sum_e n^{11}\delta u]_{l_e} = \sum_s [\![n^{11}\delta u]\!]_s = \sum_s \left(\langle n^{11}\rangle [\![\delta u]\!] + \underset{\substack{\parallel \\ 0}}{[\![n^{11}]\!]}\langle \delta u\rangle\right)_s \overset{\neq}{\rightarrow} \sum_s \left(\langle n^{11}\rangle [\![\delta u]\!]\right)_s$$

0 for the exact continuous solution (consistency is preserved)

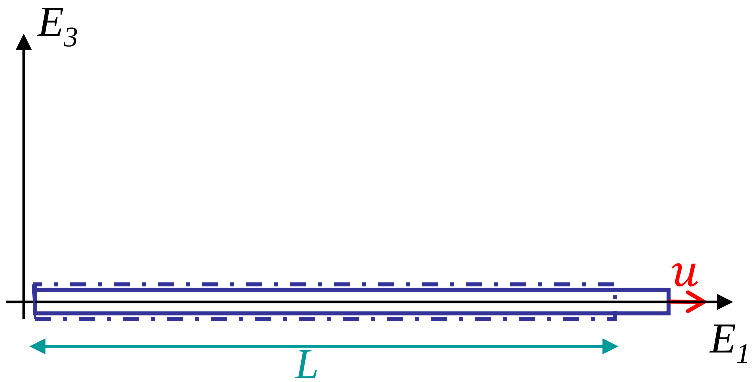
  - Main idea of DG

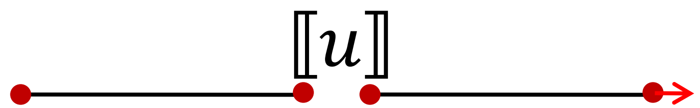
- The governing equation becomes

$$\sum_e \left(\int_{l_e} \underbrace{n^{11}\delta u_{,1} dx}_{\text{Bulk term}} - \underbrace{n^{11}\delta u]_{l_e}}_{\text{Interface term}} \right) \overset{\neq}{\rightarrow} \sum_e \int_{l_e} \underbrace{n^{11}\delta u_{,1} dx}_{\text{Bulk term}} + \sum_s \underbrace{\left(\langle n^{11}\rangle [\![\delta u]\!]\right)_s}_{\text{Consistency term}} = 0$$

  - ≠ pure penalty method (Intrinsic cohesive law) which does not include the consistency terms

- Discontinuous elements → displacement jumps have to be constrained
  - Membrane mode





  - Continuity is weakly ensured by symmetrization terms

$$\sum_s \left(\langle Eh\delta u_{,1}\rangle [\![u]\!]\right)_s = 0$$

||

0

for the exact continuous solution

→ consistency is preserved

Full-DG formulation of Euler-Bernoulli beams

- Method is stabilized by quadratic terms

$$\left.\begin{array}{l}\sum_s \left(\langle n^{11}\rangle [\![\delta u]\!]\right)_s \\ \sum_s \left(\langle E h \delta u_{,1}\rangle [\![u]\!]\right)_s\end{array}\right\} \rightarrow \sum_s \left(\underset{\underset{0}{\parallel}}{[\![u]\!]} \left\langle \frac{E h \beta_2}{h^s}\right\rangle [\![\delta u]\!]\right)_s = 0$$

for the exact continuous solution

→ consistency is preserved

  - $\beta_2 > 1$ dimensionless stability parameter (Practically stable if $\beta_2 \geq 10$)

  - h^s characteristic mesh size which ensures the dimensionless nature of β_2

Full-DG formulation of Euler-Bernoulli beams

- The final equation (membrane mode) is obtained by adding the terms

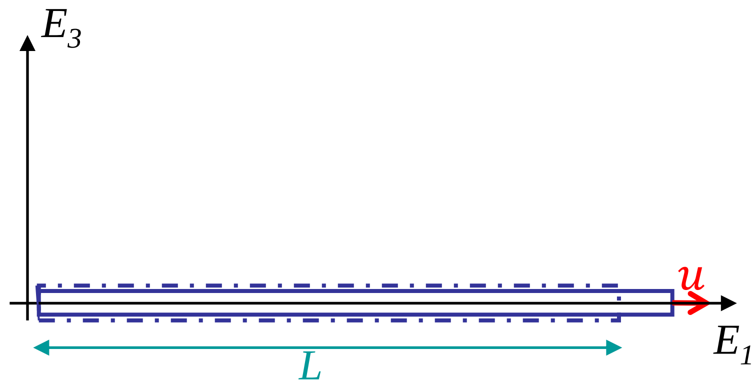
$$\sum_e \left(\int_{l_e} \underbrace{n^{11}\delta u_{,1} dx}_{\text{Bulk term}} - \underbrace{n^{11}\delta u]_{l_e}}_{\text{Interface term}} \right) \rightarrow \overset{\text{FEM (CG)}}{\boxed{\sum_e \int_{l_e} \underbrace{n^{11}\delta u_{,1} dx}_{\text{Bulk term}}}} + \sum_s \left(\underbrace{\langle n^{11} \rangle [\![\delta u]\!]}_{\text{Consistency term}} \right)_s +$$

$$\sum_s \left(\underbrace{\langle Eh\delta u_{,1} \rangle [\![u]\!]}_{\text{Symmetrization term}} \right)_s + \sum_s \left(\underbrace{[\![u]\!] \left\langle \frac{Eh\beta_2}{h^s} \right\rangle [\![\delta u]\!]}_{\text{Stability term}} \right)_s = 0$$

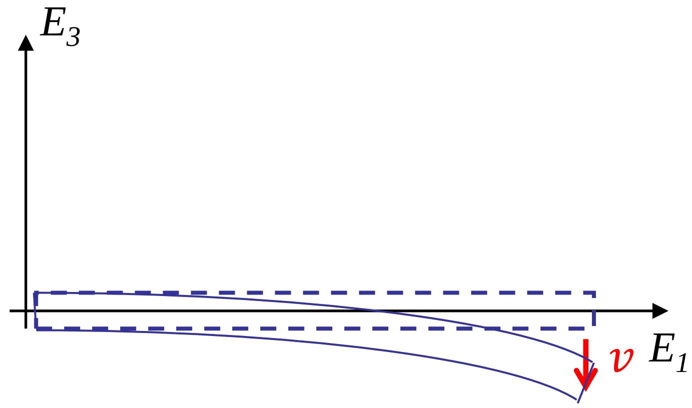
  - Consistent, (Weakly) continuous and stable
  - Same as FEM but with extra interface terms

Full-DG formulation of Euler-Bernoulli beams

- 2 (independent) deformation modes (shearing is neglected)
  - Membrane mode (OK)

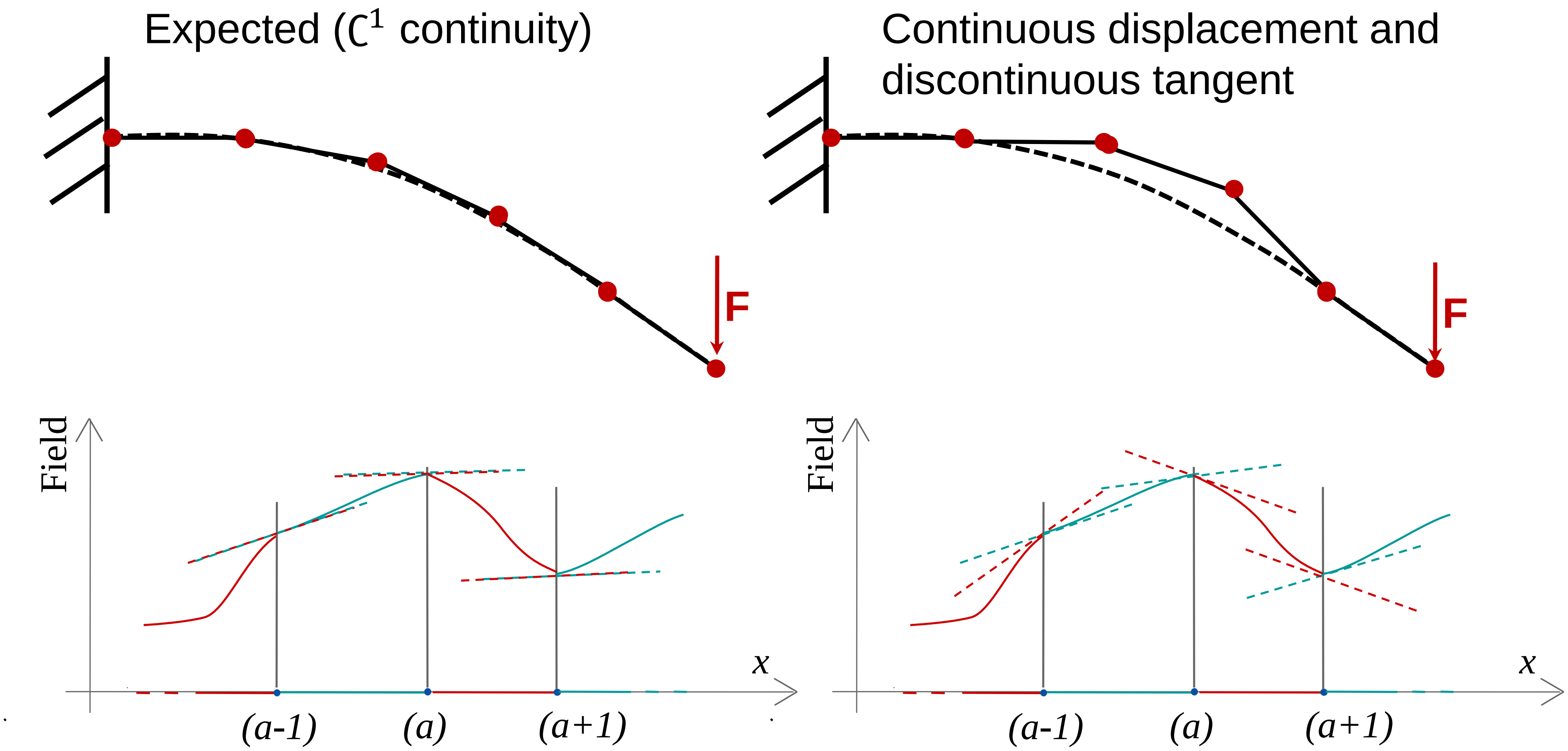


  - Bending mode



Full-DG formulation of Euler-Bernoulli beams

- The bending mode requires the C^1 continuity (*i.e.* the tangent continuity)
  - For FEM without rotational Dofs



Full-DG formulation of Euler-Bernoulli beams

- Several techniques exist to ensure the tangent continuity using FEM
  - C^1 Shape functions (beams only)
  - Recourse to rotational degrees of freedom (2-field formulation)
  - Lagrange multipliers (add degrees of freedom)
  - …

Full-DG formulation of Euler-Bernoulli beams

- The discontinuous Galerkin method can be advantageously used to ensure the tangent continuity
  - Ensured weakly by interface terms
  - C^0/DG method (elements are continuous)
  - One-field formulation (displacements are the only unknowns)
  - First DG methods for thin bodies formulation [*Engel et al cmame 2002*]

Full-DG formulation of Euler-Bernoulli beams

- The form of the DG formulation is similar to the one obtained for the membrane problem

  - Strong form $(m^{11})_{,1} = 0$ with $m^{11} = \int_{-h/2}^{h/2} \sigma^{11}\xi^3 d\xi^3$
  - Weak form $\int_L (m^{11})_{,1}\delta(-v_{,1})dx = 0$
  - Shearing is neglected
  - External forces and inertial parts are omitted
  - FEM (Continuous Galekin)
  - Discontinuous Galerkin

$$\sum_e \int_{l_e} \underbrace{m^{11}\delta(-v_{,11})dx}_{\text{Bulk term}} = 0$$

$$\sum_e \left(\int_{l_e} \underbrace{m^{11}\delta(-v_{,11})dx}_{\text{Bulk term}} - \underbrace{m^{11}\delta(-v_{,1})\big]_{l_e}}_{\text{Interface term}} \right) = 0$$

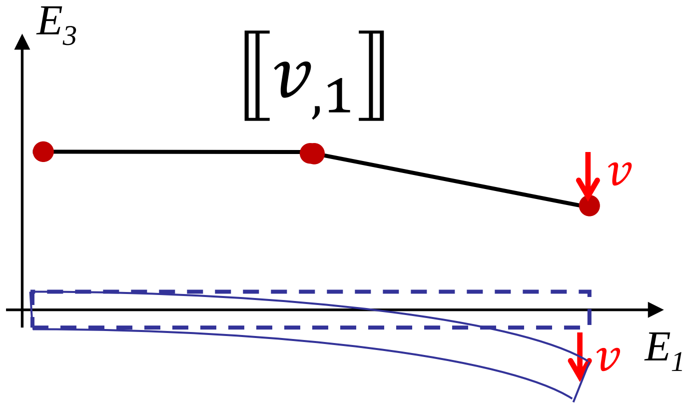
Full-DG formulation of Euler-Bernoulli beams

- 3 interfaces terms are considered following the framework made for the membrane mode
  - Consistent terms

$$-\sum_e m^{11}\delta(-v_{,1})\big]_{l_e} = \sum_s [\![m^{11}\delta(-v_{,1})]\!]_s \rightarrow \sum_s \left(\langle m^{11} \rangle [\![\delta(-v_{,1})]\!] \right)_s$$

  - Symmetrization terms

$$\sum_s \left(\left\langle \frac{Eh^3}{12} \delta(-v_{,11}) \right\rangle [\![-v_{,1}]\!] \right)_s = 0$$



  - Stability terms

$$\sum_s \left([\![-v_{,1}]\!] \left\langle \frac{Eh^3\beta_1}{12h^s} \right\rangle [\![\delta(-v_{,1})]\!] \right)_s = 0$$

$\beta_1 > 1$ dimensionless stability parameter

Full-DG formulation of Euler-Bernoulli beams

- Bending equation

$$\sum_e \left(\int_{l_e} \underbrace{m^{11}\delta(-v_{,11})dx}_{\text{Bulk term}} - \underbrace{m^{11}\delta(-v_{,1})\big]_{l_e}}_{\text{Interface term}} \right) \rightarrow \sum_e \int_{l_e} \underbrace{m^{11}\delta(-v_{,11})dx}_{\text{Bulk term}} +$$

$$\sum_s \left(\underbrace{\langle m^{11} \rangle [\![\delta(-v_{,1})]\!]}_{\text{Consistency term}} \right)_s +$$

$$\sum_s \left(\underbrace{\left\langle \left(\frac{Eh^3}{12}\right)\delta(-v_{,11}) \right\rangle [\![-v_{,1}]\!]}_{\text{Symmetrization term}} \right)_s +$$

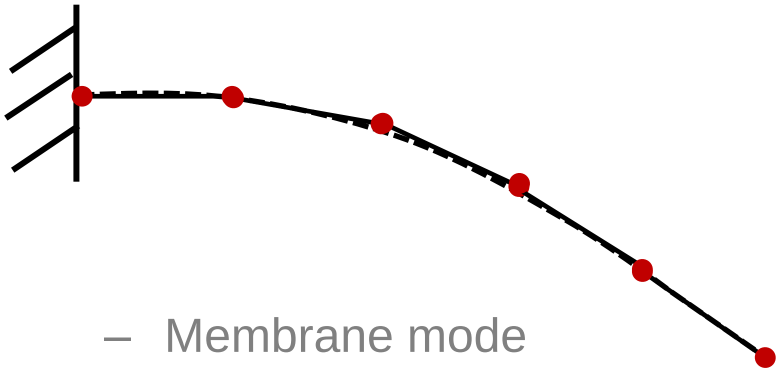
$$\sum_s \left(\underbrace{[\![-v_{,1}]\!] \left\langle \frac{Eh^3\beta_1}{12h^s} \right\rangle [\![\delta(-v_{,1})]\!]}_{\text{Stability term}} \right)_s = 0$$

  - Consistent, stable and weakly continuous thanks to interface terms
  - Same as FEM with extra interface terms

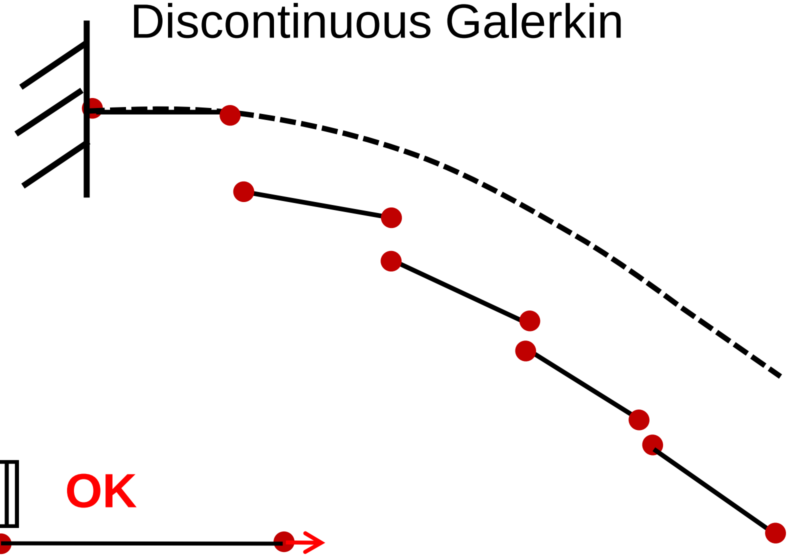
Full-DG formulation of Euler-Bernoulli beams

- Out-of-plane continuity is not ensured

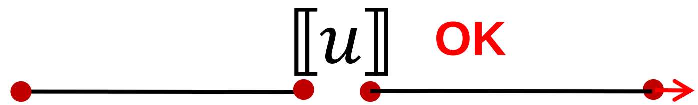
Expected



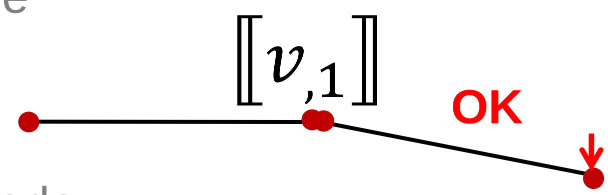
Discontinuous Galerkin



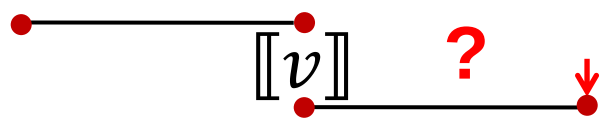
  - Membrane mode



  - Bending mode



  - “Shearing” mode



- Out-of-plane continuity is ensured by introducing an interface term in δv

  - Account (temporarily) for negligible shearing in the simplified bending equation

$$\boxed{\int_L \left[(m^{11})_{,1}\delta(-v_{,1})\right.} - l^1\delta(-v_{,1})\big]dx = 0 \quad \text{with} \quad l^1 = \int_{-h/2}^{h/2} \sigma^{31} d\xi^3 \approx 0$$

Simplified bending equation

  - Unusual integration by parts on $\delta(-v_{,1})$for the shearing term

Term in $\delta v_{,1}$ to constrain $[\![v_{,1}]\!]$

$$\int_L \left[(m^{11})_{,1}\delta(-v_{,1}) - l^1\delta(-v_{,1})\right]dx = \sum_e \left(\int_{l_e} \underbrace{m^{11}\delta(-v_{,11})dx}_{\text{Bulk term}} - \underbrace{\left[m^{11}\delta(-v_{,1})\right]_{l_e}}_{\text{Interface term}} - \int_{l_e} \underbrace{(l^1)_{,1}\delta(-v)dx}_{\text{Bulk term}} + \underbrace{\left[l^1\delta(-v)\right]_{l_e}}_{\text{Interface term}} \right) = 0$$

Term in δv → We can ensure weakly this continuity using DG

Full-DG formulation of Euler-Bernoulli beams

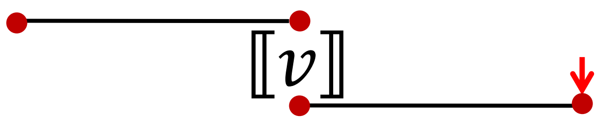
- 3 interface terms are derived from $l^1\delta(-v)]_{l_e}$ exactly as for the membrane and bending modes

  - Consistency terms

$$\sum_e l^1\delta(-v)]_e = -\sum_s [\![l^1\delta(-v)]\!]_s \rightarrow -\sum_s \left(\langle l^1\rangle [\![\delta(-v)]\!]\right)_s$$

  - Symmetrization terms

$$\sum_s \left(\left\langle \frac{Eh}{2(1+\nu)} \delta(-v_{,1}) \right\rangle [\![-v]\!] \right)_s = 0$$



  - Stability terms

$$\sum_s \left([\![-v]\!] \left\langle \frac{Eh\beta_3}{2(1+\nu)} \right\rangle [\![\delta(-v)]\!] \right)_s = 0$$

$\beta_3 > 0$ dimensionless stability parameter

Full-DG formulation of Euler-Bernoulli beams

- Only the stabilization terms remain as the shearing is neglected (Euler-Bernoulli assumption)

  - Consistency terms

$$\sum_e l^1 \delta(-v)]_e = -\sum_s [\![l^1 \delta(-v)]\!]_s \rightarrow -\sum_s \cancel{(\langle l^1 \rangle [\![\delta(-v)]\!])_s} \approx 0$$

NEGLECTED

  - Symmetrization terms

$$\sum_s \left(\cancel{\left\langle \frac{Eh}{2(1+\nu)} \delta(-v_{,1}) \right\rangle} [\![-v]\!] \right)_s = 0$$

ENSURES CONTINUITY BUT LEAD TO UNSYMMETRIC FORMULATION → IS NOT CONSIDERED

  - Stability terms

$$\sum_s \left([\![-v]\!] \left\langle \frac{Eh\beta_3}{2(1+\nu)h^s} \right\rangle [\![\delta(-v)]\!] \right)_s = 0$$

ENSURES STABILTY AND CONTINUITY

- The final full-DG equation is obtained by adding the different contributions (membrane + bending) [*Becker et al , ijnme 2011*]

$$\sum_e \int_{l_e} \underbrace{n^{11}\delta u_{,1} dx}_{\text{Bulk term}} + \sum_e \int_{l_e} \underbrace{m^{11}\delta(-v_{,11}) dx}_{\text{Bulk term}} +$$

FEM (CG) equation

$$\sum_s \left(\underbrace{\langle n^{11}\rangle [\![\delta u]\!]}_{\text{Consistency term}} \right)_s + \sum_s \left(\underbrace{\langle Eh\delta u_{,1}\rangle [\![u]\!]}_{\text{Symmetrization term}} \right)_s + \sum_s \left(\underbrace{[\![u]\!] \left\langle \frac{Eh\beta_2}{h^s} \right\rangle [\![\delta u]\!]}_{\text{Stability term}} \right)_s$$

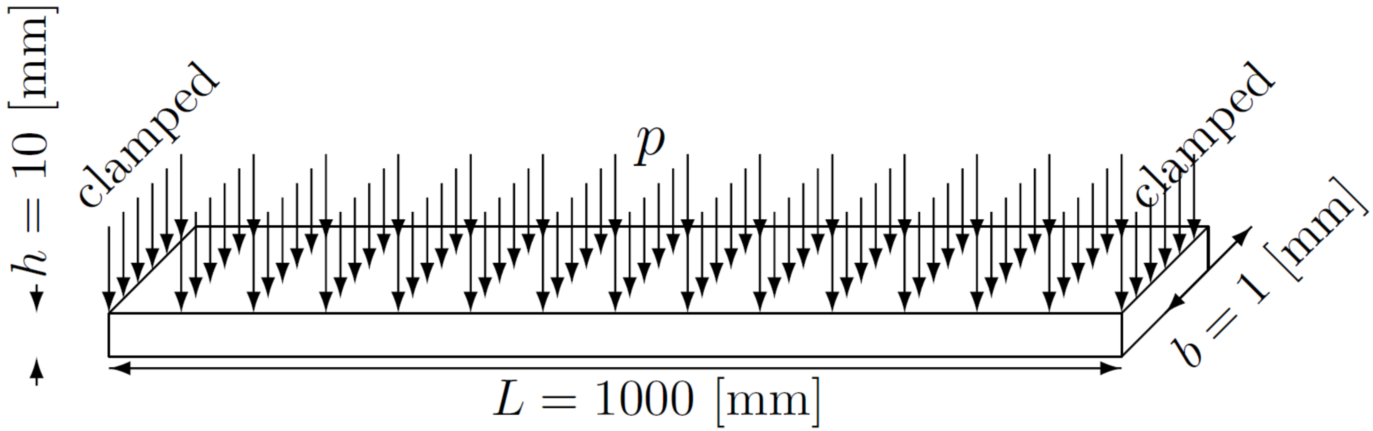
$$\sum_s \left(\underbrace{\langle m^{11}\rangle [\![\delta(-v_{,1})]\!]}_{\text{Consistency term}} \right)_s + \sum_s \left(\underbrace{\left\langle \left(\frac{Eh^3}{12}\right)\delta(-v_{,11}) \right\rangle [\![-v_{,1}]\!]}_{\text{Symmetrization term}} \right)_s + \sum_s \left(\underbrace{[\![-v_{,1}]\!] \left\langle \frac{Eh^3\beta_1}{12h^s} \right\rangle [\![\delta(-v_{,1})]\!]}_{\text{Stability term}} \right)_s$$

$$+ \sum_s \left(\underbrace{[\![-v]\!] \left\langle \frac{Eh\beta_3}{2(1+\nu)h^s} \right\rangle [\![\delta(-v)]\!]}_{\text{Stability term}} \right)_s$$

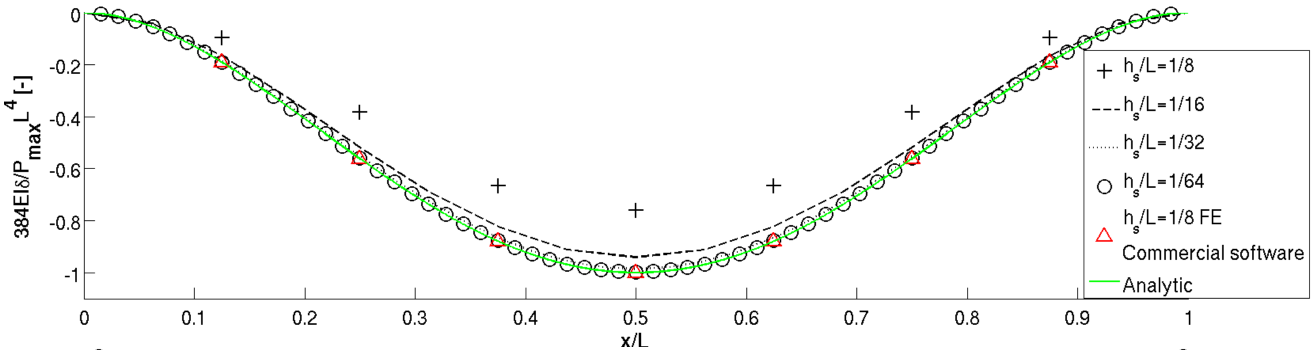
$$= 0$$

Full-DG formulation of Euler-Bernoulli beams

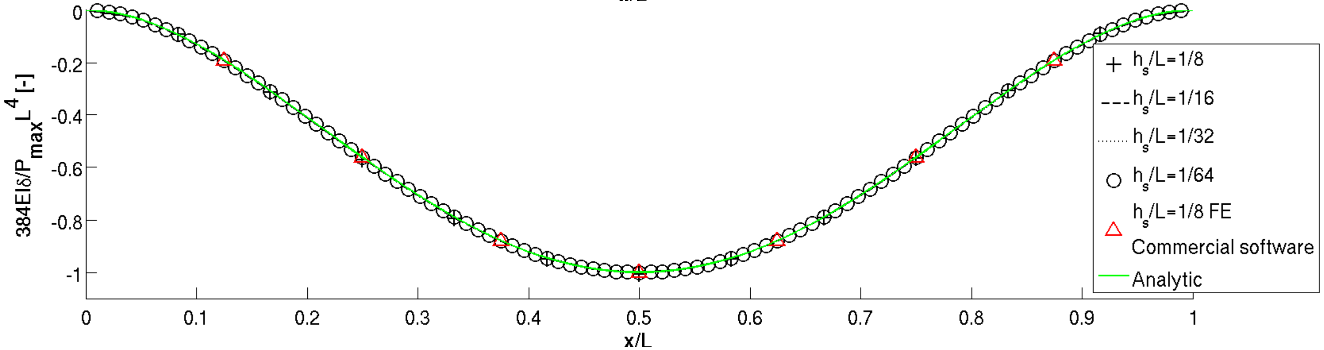
- The analytical solution is matched with discontinuous elements



2nd order elements

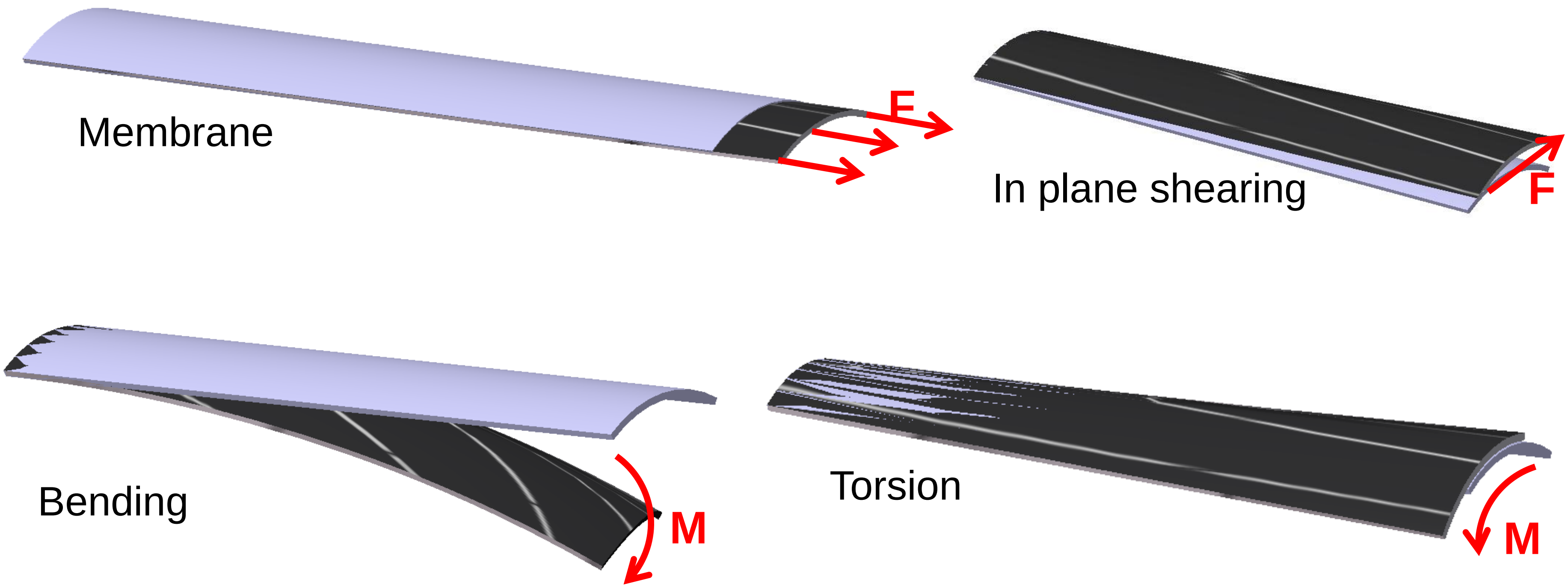


3rd order elements

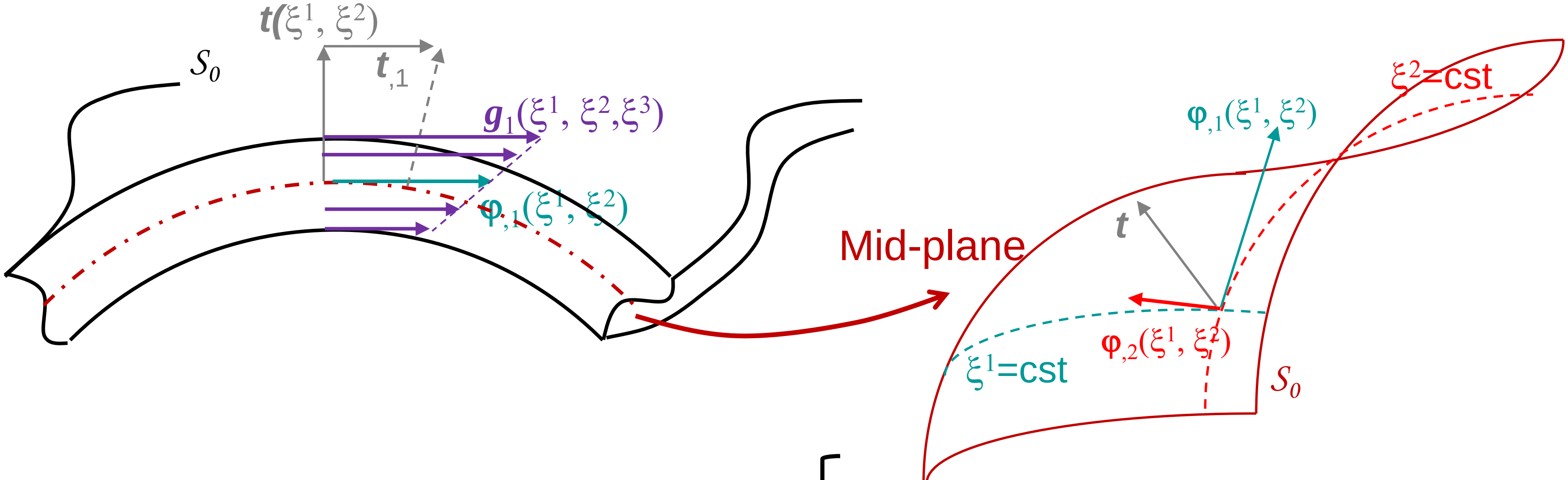


Full-DG formulation of Kirchhoff-Love shells

- Structure whose thickness is << other dimensions
- Initial curvature (otherwise it is a plate) ⇔ bending/membrane coupling
- Modes
  - Out-of-plane shearing is neglected (Kirchhoff-Love theory)



- The kinematics of the shell is formulated in a basis linked to the shell
  - Convected basis $(\boldsymbol{g}_1, \boldsymbol{g}_2, \boldsymbol{g}_3)$



$$\boldsymbol{g}_\alpha = \boldsymbol{\varphi}_{,\alpha} + \xi^3 \lambda_h \boldsymbol{t}_{,\alpha}$$
$$\boldsymbol{g}_3 = \lambda_h \boldsymbol{t}$$

where $\begin{cases} \lambda_h \text{ accounts for a thickness variation} \\ \boldsymbol{t} = \boldsymbol{t}(\xi^1, \xi^2, \cancel{\xi^3}) \text{ by assumption} \end{cases}$

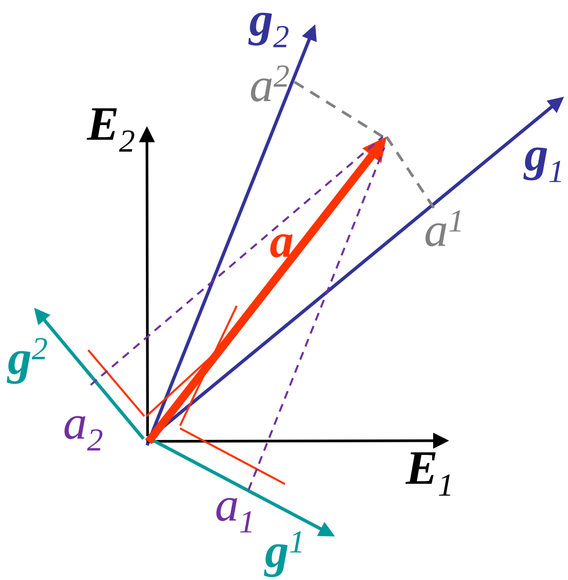
  - The convected basis is not orthonormal
  - Curvature of the shell is characterized by

$$\lambda_\alpha^\beta = \boldsymbol{t}_{,\alpha} \cdot \boldsymbol{\varphi}^{,\beta}$$

Full-DG formulation of Kirchhoff-Love shells

- As the convected basis is not orthonormal, a conjugate (or dual) basis is defined to decompose vectors or matrices

$$\boldsymbol{g}_I \cdot \boldsymbol{g}^J = \delta_{IJ}$$



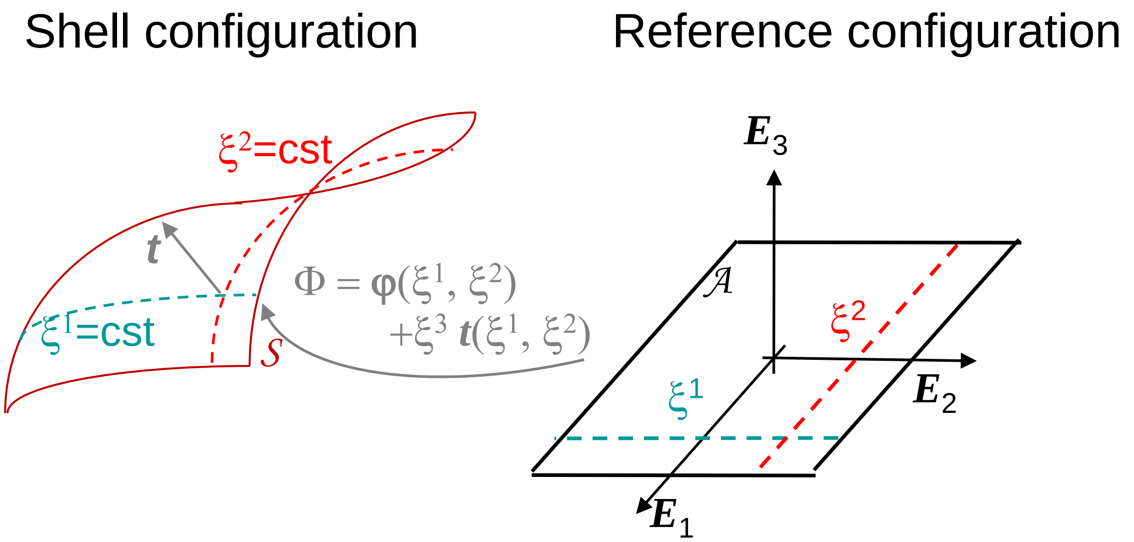
The vector $\boldsymbol{a}$ can be formulated in both bases:

$$\boldsymbol{a} = a^1 \boldsymbol{g}_1 + a^2 \boldsymbol{g}_2$$
$$\boldsymbol{a} = a_1 \boldsymbol{g}^1 + a_2 \boldsymbol{g}^2$$

And (for example)

$$\boldsymbol{a} \cdot \boldsymbol{g}^1 = (a^1 \boldsymbol{g}_1 + a^2 \boldsymbol{g}_2) \cdot \boldsymbol{g}^1 = a^1$$

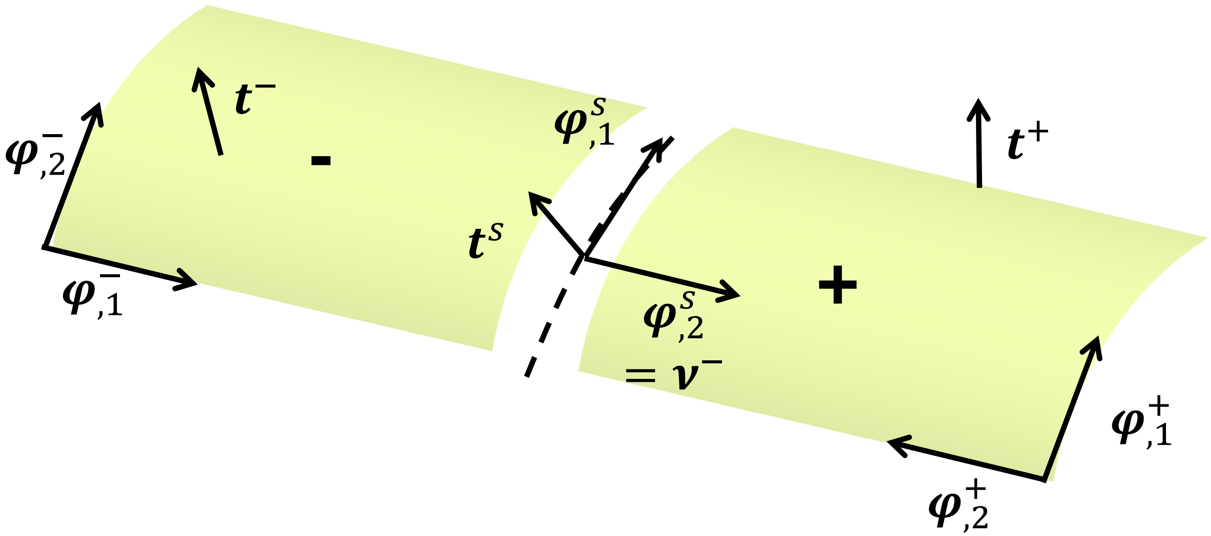
- The equations are formulated in the reference frame
  - The Jacobian describes the change between the configurations



$$j = \det(\nabla \boldsymbol{\Phi}) = (\boldsymbol{g}_1 \wedge \boldsymbol{g}_2) \cdot \boldsymbol{g}_3$$

$$\bar{j} = \lambda_h(\boldsymbol{\varphi}_{,1} \wedge \boldsymbol{\varphi}_{,2}) \cdot \boldsymbol{t}$$

- The normal at the interface is chosen as the outward normal to the minus element (convention)



  – Normal components

$$\nu_\alpha^- = \boldsymbol{\varphi}_{,2}^s \cdot \boldsymbol{\varphi}_{,\alpha}^s$$

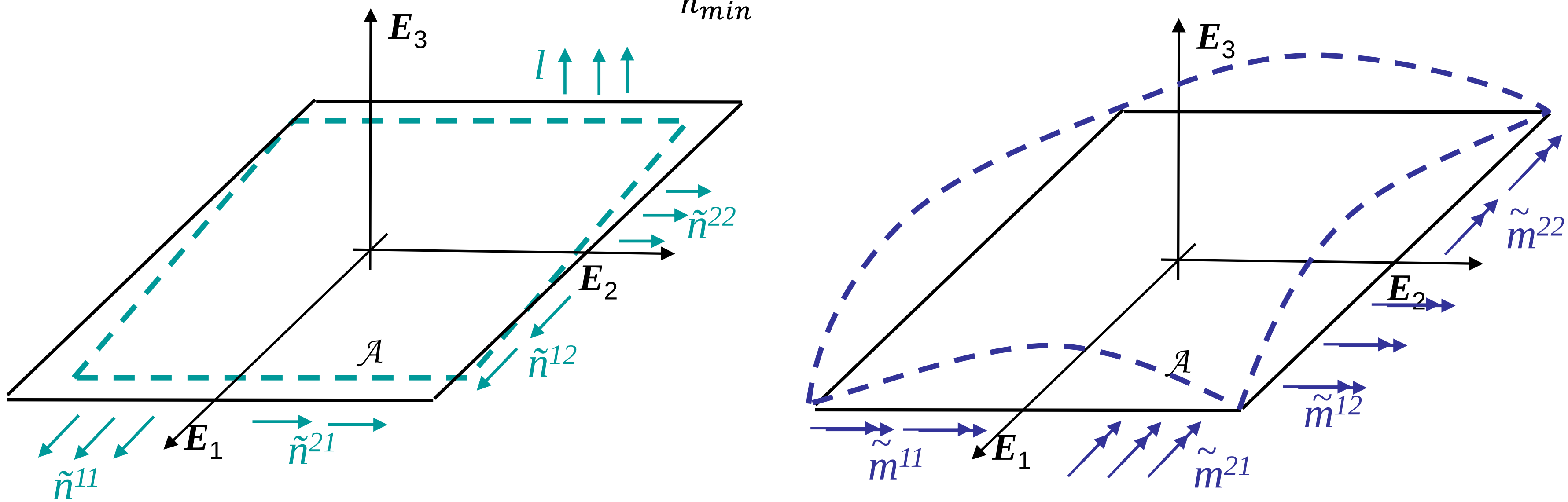
- The stress tensor $\boldsymbol{\sigma}$ is integrated on the thickness in the convected basis
  - Reduced stresses

coupling

$$\boldsymbol{n}^{\alpha} = \frac{1}{\bar{j}}\int_{h_{min}}^{h_{max}} j\boldsymbol{\sigma}\cdot\boldsymbol{g}^{\alpha}d\xi^{3} = \boxed{\left(\tilde{n}^{\alpha\beta} + \lambda_{\mu}^{\beta}\tilde{m}^{\alpha\mu}\right)}\boldsymbol{\varphi}_{,\beta}$$

$$\tilde{\boldsymbol{m}}^{\alpha} = \frac{1}{\bar{j}}\int_{h_{min}}^{h_{max}} j\xi^{3}\boldsymbol{\sigma}\cdot\boldsymbol{g}^{\alpha}d\xi^{3}$$

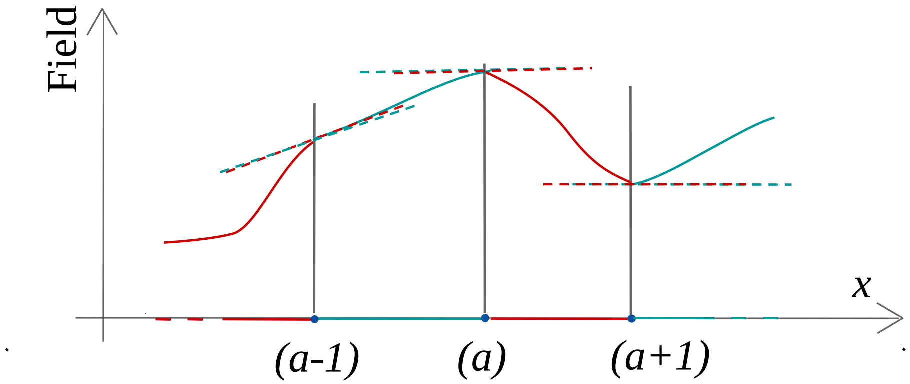
$$\boldsymbol{l} = \frac{1}{\bar{j}}\int_{h_{min}}^{h_{max}} j\boldsymbol{\sigma}\cdot\boldsymbol{g}^{3}d\xi^{3} \approx 0$$



Full-DG formulation of Kirchhoff-Love shells

- The (Simplified) equations of the problem are formulated in terms of the reduced stresses
  - Strong form $\frac{1}{\bar{j}}\left(\bar{j}\boldsymbol{n}^{\alpha}\right)_{,\alpha} + \frac{1}{\bar{j}}\left(\bar{j}\tilde{\boldsymbol{m}}^{\alpha}\right) - \boldsymbol{l} = 0$
  - Weak form $\int_A \left[\left(\bar{j}\boldsymbol{n}^{\alpha}\right)_{,\alpha} \cdot \delta\boldsymbol{\varphi} + \left(\bar{j}\tilde{\boldsymbol{m}}^{\alpha}\right)_{,\alpha} \cdot \lambda_h \delta\boldsymbol{t} - \bar{j}\boldsymbol{l} \cdot \lambda_h \delta\boldsymbol{t}\right] dA = 0$
  - Highlights of the full DG concept
  - External forces and inertial terms are omitted (same as FEM)

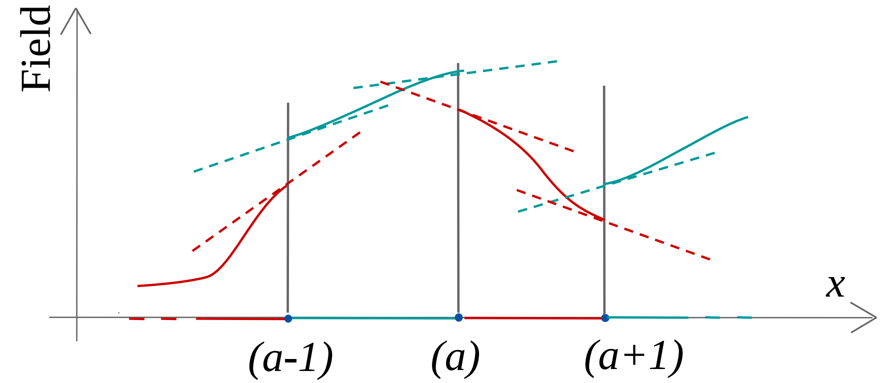
- FEM (Continuous Galerkin)



Integration by parts on the structure

$$\sum_e \int_{A_e} \left[\bar{j}\boldsymbol{n}^\alpha \cdot \delta\boldsymbol{\varphi}_{,\alpha} + \bar{j}\tilde{\boldsymbol{m}}^\alpha \cdot \lambda_h \delta \boldsymbol{t}_{,\alpha} - \bar{j}\boldsymbol{l}\cdot\lambda_h\delta\boldsymbol{t}\right] dA = 0$$

- Discontinuous Galerkin



Integration by parts on each element (unusual on $\boldsymbol{l}$)

$$\sum_e \left\{ \int_{A_e} \left[\left(\bar{j}\boldsymbol{n}^\alpha\right)_{,\alpha}\cdot\delta\boldsymbol{\varphi} + \left(\bar{j}\tilde{\boldsymbol{m}}^\alpha\right)_{,\alpha}\cdot\lambda_h\delta\boldsymbol{t} - \left(\bar{j}\boldsymbol{l}\right)_{,\alpha}\cdot\int_\alpha \lambda_h\delta\boldsymbol{t}\,d\alpha'\right]dA \right.$$

$$\left. - \int_{\partial A_e} \left[\bar{j}\boldsymbol{n}^\alpha\cdot\delta\boldsymbol{\varphi}\nu_\alpha^- + \bar{j}\tilde{\boldsymbol{m}}^\alpha\cdot\lambda_h\delta\boldsymbol{t}\nu_\alpha^- - \bar{j}l\cdot\int_\alpha \lambda_h\delta t\,d\alpha'\;\nu_\alpha^- \right]dA \right\} = 0$$

Additional interface terms exactly as for beams

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Full-DG formulation of Kirchhoff-Love shells

- The 3 interface terms are replaced by consistent numerical fluxes
  - Average fluxes are considered (exactly as for beams)

Interface terms = A sum of jumps → Consistent terms

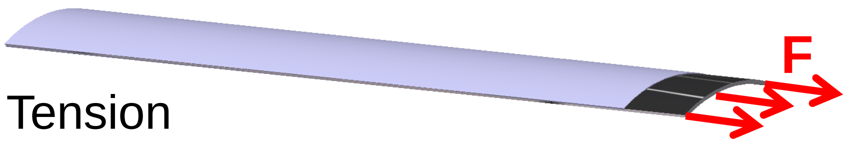
$$-\sum_e \int_{\partial A_e} \bar{j}\boldsymbol{n}^{\alpha} \cdot \delta\boldsymbol{\varphi} \nu_{\alpha}^{-} \, dA = \sum_s \int_s [\![\bar{j}\boldsymbol{n}^{\alpha} \cdot \delta\boldsymbol{\varphi} \nu_{\alpha}^{-}]\!]_s d\delta A_e \rightarrow \sum_s \int_s \langle \bar{j}\boldsymbol{n}^{\alpha} \rangle \cdot [\![\delta\boldsymbol{\varphi}]\!] \nu_{\alpha}^{-} \, d\partial A_e$$

$$-\sum_e \int_{\partial A_e} \bar{j}\tilde{\boldsymbol{m}}^{\alpha} \cdot \lambda_h \delta\boldsymbol{t} \nu_{\alpha}^{-} \, dA = \sum_s \int_s [\![\bar{j}\tilde{\boldsymbol{m}}^{\alpha} \cdot \lambda_h \delta\boldsymbol{t} \nu_{\alpha}^{-}]\!]_s d\delta A_e \rightarrow \sum_s \int_s \langle \bar{j}\tilde{\boldsymbol{m}}^{\alpha} \rangle \cdot [\![\lambda_h \delta\boldsymbol{t}]\!] \nu_{\alpha}^{-} d\partial A_e$$

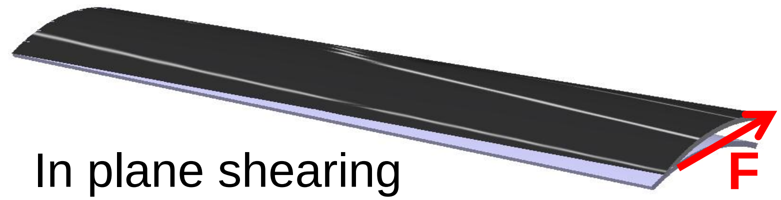
$$\sum_e \int_{\partial A_e} \bar{j}\boldsymbol{l} \cdot \int_{\alpha} \lambda_h \delta\boldsymbol{t} d\alpha' \;\; \nu_{\alpha}^{-} \, dA = -\sum_s \int_s \left[\!\!\left[\bar{j}\boldsymbol{l} \cdot \int_{\alpha} \lambda_h \delta\boldsymbol{t} d\alpha' \;\; \nu_{\alpha}^{-} \right]\!\!\right]_s d\partial A_e$$

$$\rightarrow -\sum_s \int_s \langle \bar{j}\boldsymbol{l} \rangle \cdot \left[\!\!\left[\int_{\alpha} \lambda_h \delta\boldsymbol{t} d\alpha' \right]\!\!\right] \nu_{\alpha}^{-} d\partial A_e \approx 0$$

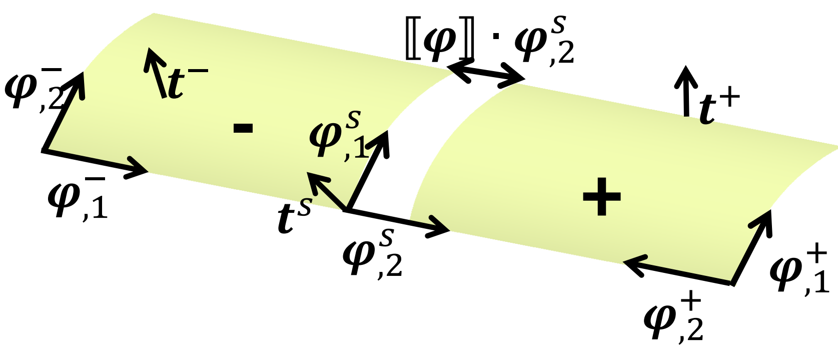
- 3 Symmetrization terms are introduced to ensure (weakly) the continuity
  - The in-plane displacement jump is constrained by symmetrizing the consistency terms on $\boldsymbol{n}^{\alpha}$

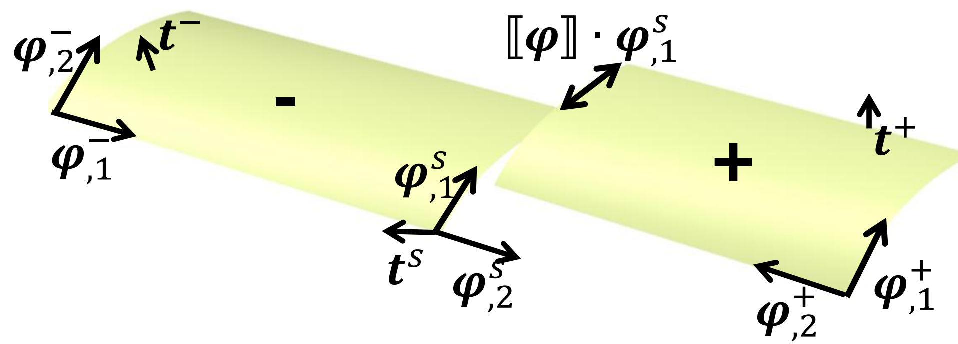


Tension
(+ coupled bending)



In plane shearing
(+ coupled torsion)



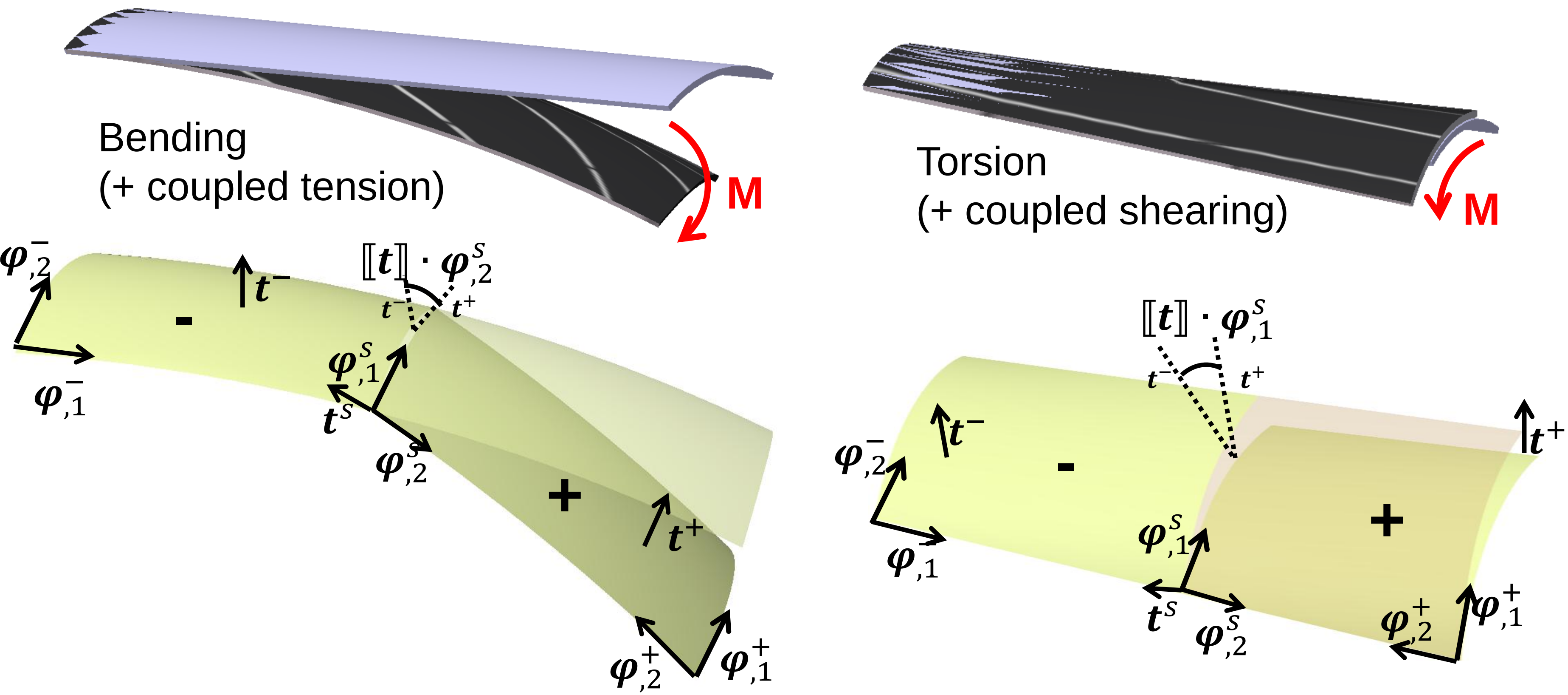


Consistency term $\langle \boldsymbol{a} \rangle \cdot [\![\delta \boldsymbol{b}]\!]$ **→ Symmetrization term** $\langle \boldsymbol{b} \rangle \cdot [\![\delta \boldsymbol{a}]\!]$

$$\sum_{s} \int_{s} [\![\boldsymbol{\varphi}]\!] \cdot \langle \delta(\bar{j}\boldsymbol{n}^{\alpha}) \rangle \nu_{\alpha}^{-} d\partial A_{e} = 0$$

  - Leads to a symmetric formulation only in the linear case

- 3 Symmetrization terms are introduced to ensure (weakly) the continuity
  - The rotational jump is constrained by symmetrizing the consistency terms on $\tilde{\boldsymbol{m}}^{\alpha}$

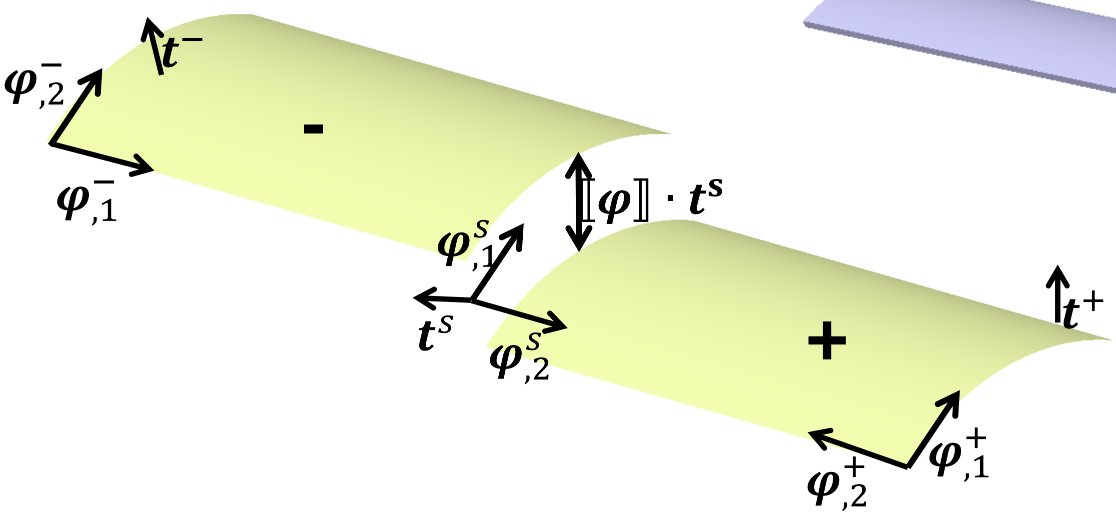


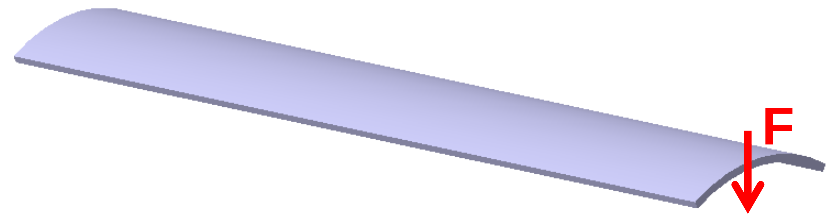
Consistency term $\langle \boldsymbol{a} \rangle \cdot [\![\delta \boldsymbol{b}]\!]$ → Symmetrization term $\langle \boldsymbol{b} \rangle \cdot [\![\delta \boldsymbol{a}]\!]$

$$\sum_{s} \int_{s} [\![\boldsymbol{t}]\!] \cdot \langle (\bar{j} \lambda_h \tilde{\boldsymbol{m}}^{\alpha}) \rangle \nu_{\alpha}^{-} d\partial A_e = 0$$

Full-DG formulation of Kirchhoff-Love shells

- 3 Symmetrization terms are introduced to ensure (weakly) the continuity
  - The out-of-plane displacement jump is constrained by symmetrizing the consistency terms on $\boldsymbol{l}$





Consistency term $\langle \boldsymbol{a} \rangle \cdot [\![\delta \boldsymbol{b}]\!]$ → **Symmetrization term** $\langle \boldsymbol{b} \rangle \cdot [\![\delta \boldsymbol{a}]\!]$

$$\sum_s \int_s \left[\!\!\left[\int_\alpha \lambda_h \boldsymbol{t}\, d\alpha' \right]\!\!\right] \cdot \left\langle \delta(\bar{j}\boldsymbol{l}) \right\rangle \nu_\alpha^- \, d\partial A_e = 0$$

$$\wr$$

$\lambda_h [\![\boldsymbol{\varphi}]\!] \cdot \boldsymbol{t}\boldsymbol{\varphi}^{,\alpha}$ Primitive approximation

- 3 Stabilization terms have to be introduced to ensure the stability of the method
  - Quadratic terms are formulated from consistent and symmetrization terms in $\boldsymbol{n}^{\alpha}$

Form of stabilization terms $[\![\boldsymbol{a}]\!] \cdot \boldsymbol{\varphi}_{,\gamma} \nu_{\delta}^{-} \left\langle \frac{\beta}{h^s} Invariant\ stiff \right\rangle [\![\delta \boldsymbol{a}]\!] \cdot \boldsymbol{\varphi}_{,\beta} \nu_{\alpha}^{-}$

$$\left.\begin{matrix} \sum_s \int_s \langle \bar{j} \boldsymbol{n}^{\alpha} \rangle \cdot [\![\delta \boldsymbol{\varphi}]\!] \nu_{\alpha}^{-}\, d\partial A_e \\ \sum_s \int_s [\![\boldsymbol{\varphi}]\!] \cdot \langle \delta (\bar{j} \boldsymbol{n}^{\alpha}) \rangle \nu_{\alpha}^{-} d\partial A_e \end{matrix}\right\} \rightarrow \sum_s \left[\int_s [\![\boldsymbol{\varphi}]\!] \cdot \boldsymbol{\varphi}_{,\gamma} \nu_{\delta}^{-} \left\langle \frac{\beta_2 \mathcal{H}_n^{\alpha\beta\gamma\delta} \bar{j}_0}{h^s} \right\rangle [\![\delta \boldsymbol{\varphi}]\!] \cdot \boldsymbol{\varphi}_{,\beta}\, \nu_{\alpha}^{-} d\partial A_e = 0 \right.$$

- 3 Stabilization terms have to be introduced to ensure the stability of the method
  - Quadratic terms are formulated from consistent and symmetrization terms in $\tilde{\boldsymbol{m}}^{\alpha}$

Form of stabilization terms $[\![\boldsymbol{a}]\!] \cdot \boldsymbol{\varphi}_{,\gamma} \nu_{\delta}^{-} \left\langle \frac{\beta}{h^s} Invariant\ stiff \right\rangle [\![\delta \boldsymbol{a}]\!] \cdot \boldsymbol{\varphi}_{,\beta} \nu_{\alpha}^{-}$

$$\left.\begin{array}{l} \sum_s \int_s \langle \bar{j} \tilde{\boldsymbol{m}}^{\alpha} \rangle \cdot [\![\lambda_h \delta \boldsymbol{t}]\!] \nu_{\alpha}^{-} d\partial A_e \\ \sum_s \int_s [\![\boldsymbol{t}]\!] \cdot \langle (\bar{j} \lambda_h \tilde{\boldsymbol{m}}^{\alpha}) \rangle \nu_{\alpha}^{-} d\partial A_e \end{array}\right\} \rightarrow \sum_s \int_s [\![\boldsymbol{t}]\!] \cdot \boldsymbol{\varphi}_{,\gamma} \nu_{\delta}^{-} \left\langle \frac{\beta_1 \mathcal{H}_m^{\alpha\beta\gamma\delta} \bar{j}_0}{h^s} \right\rangle [\![\delta \boldsymbol{t}]\!] \cdot \boldsymbol{\varphi}_{,\beta} \nu_{\alpha}^{-} d\partial A_e = 0$$

- 3 Stabilization terms have to be introduced to ensure the stability of the method
  - Quadratic terms are formulated from consistent and symmetrization terms in $\boldsymbol{l}$

Form of stabilization terms $[\![\boldsymbol{a}]\!] \cdot \boldsymbol{t}\nu_{\beta}^{-} \left\langle \frac{\beta}{h^s} Invariant\ stiff \right\rangle [\![\delta\boldsymbol{a}]\!] \cdot \boldsymbol{t}\nu_{\alpha}^{-}$

$$\left.\begin{array}{l} \sum_{s} \int_{s} \langle \bar{j}\boldsymbol{l} \rangle \cdot \left[\!\!\left[\int_{\alpha} \lambda_h \delta\boldsymbol{t}\, d\alpha' \right]\!\!\right] \nu_{\alpha}^{-} d\partial A_e \\ \sum_{s} \int_{s} \lambda_h [\![\boldsymbol{\varphi}]\!] \cdot \boldsymbol{t}\, \boldsymbol{\varphi}_{,}{}^{\alpha} \langle \delta(\bar{j}\boldsymbol{l}) \rangle \nu_{\alpha}^{-} d\partial A_e \end{array}\right\} \rightarrow \sum_{s} \int_{s} [\![\boldsymbol{\varphi}]\!] \cdot \boldsymbol{t}\nu_{\beta}^{-} \left\langle \frac{\beta_3 \mathcal{H}_s^{\alpha\beta} \bar{j}_0}{h^s} \right\rangle [\![\delta\boldsymbol{\varphi}]\!] \cdot \boldsymbol{t}\nu_{\alpha}^{-} d\partial A_e = 0$$

- The terms of stabilization in $\boldsymbol{l}$ ensure also weakly the out-of-plane continuity

  - The shearing is neglected (Kirchhoff-Love assumption) → $\boldsymbol{l} \approx 0$

  - Consistency terms

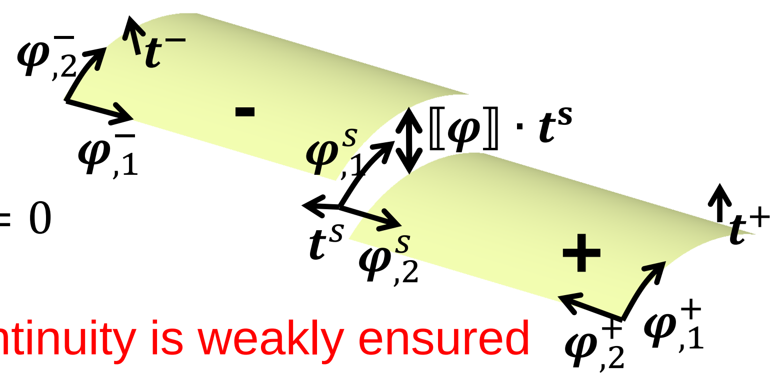
$$\int_s \langle \bar{j}\boldsymbol{l} \rangle \cdot \left[\!\left[\int_\alpha \lambda_h \delta \boldsymbol{t} d\alpha' \right]\!\right] \nu_\alpha^- d\partial A_e \approx 0$$

  - Symmetrization terms (if considered → unsymmetrical formulation)

$$\int_s \left[\!\left[\int_\alpha \lambda_h \boldsymbol{t} d\alpha' \right]\!\right] \cdot \langle \delta(\bar{j}\boldsymbol{l}) \rangle \nu_\alpha^- d\partial A_e \approx 0$$

  - Stabilization terms

$$\int_s [\![\boldsymbol{\varphi}]\!] \cdot \boldsymbol{t} \, \nu_\beta^- \left\langle \frac{\beta_3 \mathcal{H}_s^{\alpha\beta} \bar{j}_0}{h^s} \right\rangle [\![\delta\boldsymbol{\varphi}]\!] \cdot \boldsymbol{t} \nu_\alpha^- d\partial A_e = 0$$



Out-of-plane displacement jump is constrained → continuity is weakly ensured

- The equation of the full-DG formulation is obtained by adding the different contributions [*Becker et al cmame2011, Becker et al ijnme2012*]

$$\boxed{\sum_e \int_{A_e} \left[\left(\bar{j}\boldsymbol{n}^\alpha\right)_{,\alpha} \cdot \delta\boldsymbol{\varphi} + \left(\bar{j}\tilde{\boldsymbol{m}}^\alpha\right)_{,\alpha} \cdot \lambda_h \delta\boldsymbol{t} \right] dA +}$$

FEM (CG) equation

$$\sum_s \int_s \left[\langle \bar{j}\boldsymbol{n}^\alpha \rangle \cdot [\![\delta\boldsymbol{\varphi}]\!] + [\![\boldsymbol{\varphi}]\!] \cdot \langle \delta(\bar{j}\boldsymbol{n}^\alpha) \rangle + [\![\boldsymbol{\varphi}]\!] \cdot \boldsymbol{\varphi}_{,\gamma} \nu_\delta^- \left\langle \frac{\beta_2 \mathcal{H}_n^{\alpha\beta\gamma\delta} \bar{j}_0}{h^s} \right\rangle [\![\delta\boldsymbol{\varphi}]\!] \cdot \boldsymbol{\varphi}_{,\beta} \right] \nu_\alpha^- d\partial A_e +$$

$$\sum_s \int_s \left[\langle \bar{j}\tilde{\boldsymbol{m}}^\alpha \rangle \cdot [\![\lambda_h \delta\boldsymbol{t}]\!] + [\![\boldsymbol{t}]\!] \cdot \langle (\bar{j}\lambda_h \tilde{\boldsymbol{m}}^\alpha) \rangle + [\![\boldsymbol{t}]\!] \cdot \boldsymbol{\varphi}_{,\gamma} \nu_\delta^- \left\langle \frac{\beta_1 \mathcal{H}_m^{\alpha\beta\gamma\delta} \bar{j}_0}{h^s} \right\rangle [\![\delta\boldsymbol{t}]\!] \cdot \boldsymbol{\varphi}_{,\beta} \right] \nu_\alpha^- d\partial A_e +$$

$$\sum_s \int_s [\![\boldsymbol{\varphi}]\!] \cdot \boldsymbol{t} \nu_\beta^- \left\langle \frac{\beta_3 \mathcal{H}_s^{\alpha\beta} \bar{j}_0}{h^s} \right\rangle [\![\delta\boldsymbol{\varphi}]\!] \cdot \boldsymbol{t} \, \nu_\alpha^- d\partial A_e = 0$$

Consistency terms

Symmetrization terms

Stabilization terms

  - Similar form as the beam case (2 Bulk, 2 consistency, 2 symmetrization and 3 stabilization terms)

- The C^0/DG formulation [*Noels et al cmame2008, Noels ijnme2009*] is found if continuous elements are used ($[\![\boldsymbol{\varphi}]\!] = [\![\delta\boldsymbol{\varphi}]\!] = 0$)

$$\sum_e \int_{A_e} \left[\left(\bar{j}\boldsymbol{n}^\alpha\right)_{,\alpha} \cdot \delta\boldsymbol{\varphi} + \left(\bar{j}\tilde{\boldsymbol{m}}^\alpha\right)_{,\alpha} \cdot \lambda_h \delta\boldsymbol{t} \right] dA \; +$$

$$\cancel{\sum_s \int_s \left[\left\langle \bar{j}\boldsymbol{n}^\alpha \right\rangle \cdot [\![\delta\boldsymbol{\varphi}]\!] + [\![\boldsymbol{\varphi}]\!] \cdot \left\langle \delta\left(\bar{j}\boldsymbol{n}^\alpha\right)\right\rangle + [\![\boldsymbol{\varphi}]\!] \cdot \boldsymbol{\varphi}_{,\gamma} \nu^-_\delta \left\langle \frac{\beta_2 \mathcal{H}_n^{\alpha\beta\gamma\delta} \bar{j}_0}{h^s} \right\rangle [\![\delta\boldsymbol{\varphi}]\!] \cdot \boldsymbol{\varphi}_{,\beta} \right] \nu^-_\alpha d\partial A_e +}$$

$$\sum_s \int_s \left[\left\langle \bar{j}\tilde{\boldsymbol{m}}^\alpha \right\rangle \cdot [\![\lambda_h \delta\boldsymbol{t}]\!] + [\![\boldsymbol{t}]\!] \cdot \left\langle \left(\bar{j}\lambda_h \tilde{\boldsymbol{m}}^\alpha\right)\right\rangle + [\![\boldsymbol{t}]\!] \cdot \boldsymbol{\varphi}_{,\gamma} \nu^-_\delta \left\langle \frac{\beta_1 \mathcal{H}_m^{\alpha\beta\gamma\delta} \bar{j}_0}{h^s} \right\rangle [\![\delta\boldsymbol{t}]\!] \cdot \boldsymbol{\varphi}_{,\beta} \right] \nu^-_\alpha d\partial A_e +$$

$$\cancel{\sum_s \int_s [\![\boldsymbol{\varphi}]\!] \cdot \boldsymbol{t} \nu^-_\beta \left\langle \frac{\beta_3 \mathcal{H}_s^{\alpha\beta} \bar{j}_0}{h^s} \right\rangle [\![\delta\boldsymbol{\varphi}]\!] \cdot \boldsymbol{t} \; \nu^-_\alpha d\partial A_e} = 0$$

Consistency terms

Symmetrization terms

Stabilization terms

Full-DG formulation of Kirchhoff-Love shells

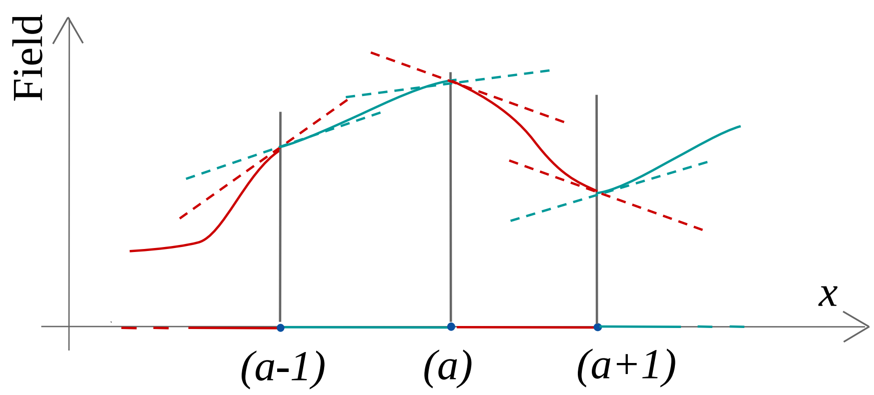
- The C^0/DG formulation [*Noels et al cmame2008, Noels ijnme2009*] is found if continuous elements are used ($[\![\boldsymbol{\varphi}]\!] = [\![\delta\boldsymbol{\varphi}]\!] = 0$)

$$\sum_e \int_{A_e} \left[\left(\bar{j}\boldsymbol{n}^\alpha\right)_{,\alpha} \cdot \delta\boldsymbol{\varphi} + \left(\bar{j}\tilde{\boldsymbol{m}}^\alpha\right)_{,\alpha} \cdot \lambda_h \delta\boldsymbol{t} \right] dA \; +$$

$$\sum_s \int_s \left[\boxed{\left\langle \bar{j}\tilde{\boldsymbol{m}}^\alpha \right\rangle \cdot [\![\lambda_h \delta\boldsymbol{t}]\!]} + \boxed{[\![\boldsymbol{t}]\!] \cdot \left\langle \left(\bar{j}\lambda_h \tilde{\boldsymbol{m}}^\alpha\right) \right\rangle} + \boxed{[\![\boldsymbol{t}]\!] \cdot \boldsymbol{\varphi}_{,\gamma} \nu_{\bar{\delta}} \left\langle \frac{\beta_1 \mathcal{H}_m^{\alpha\beta\gamma\delta} \bar{j}_0}{h^s} \right\rangle [\![\delta\boldsymbol{t}]\!] \cdot \boldsymbol{\varphi}_{,\beta}} \right] \nu_{\bar{\alpha}} \, d\partial A_e = 0$$

Consistency terms — Symmetrization terms — Stabilization terms

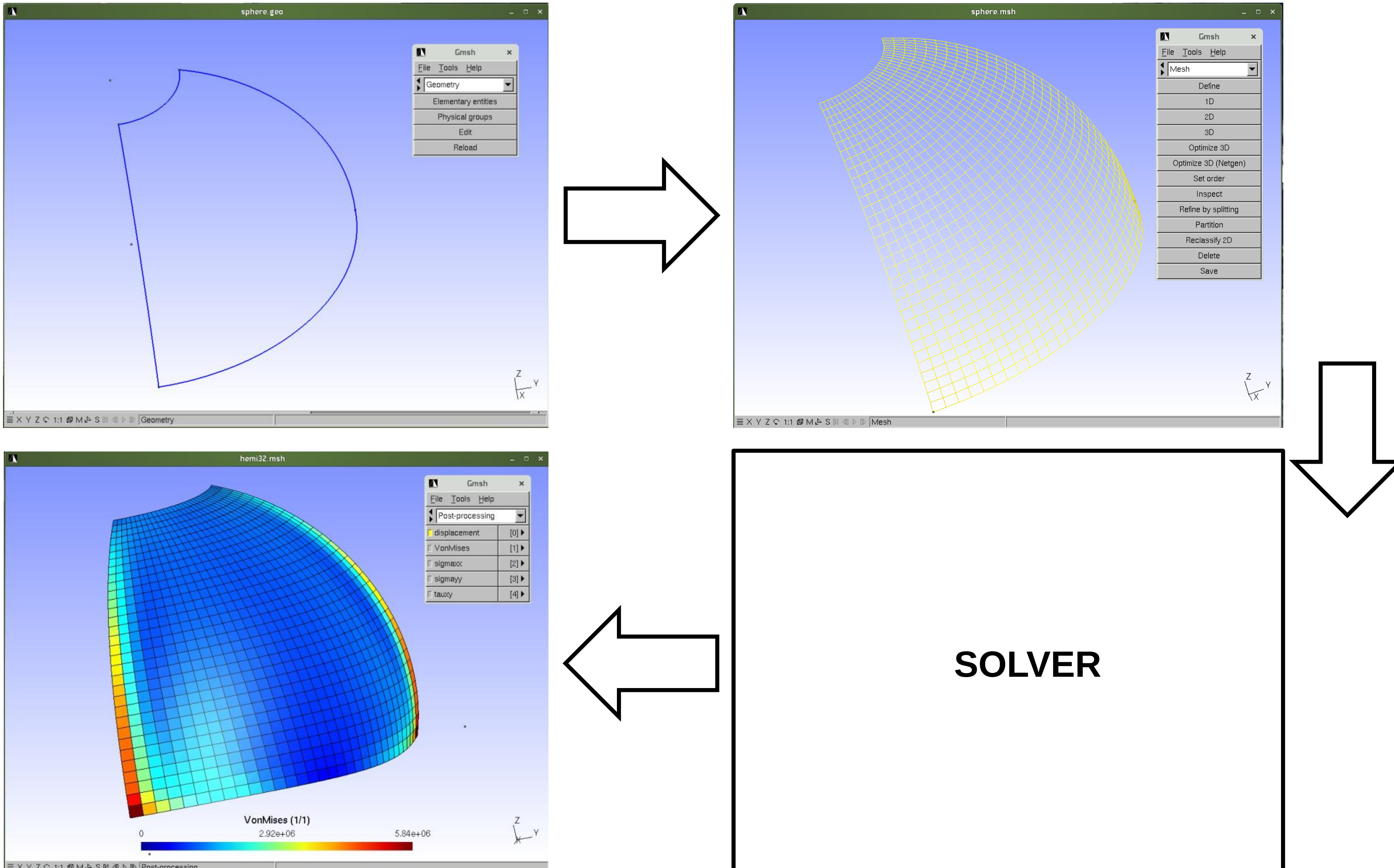
- Elements are continuous but the tangent continuity is ensured by DG



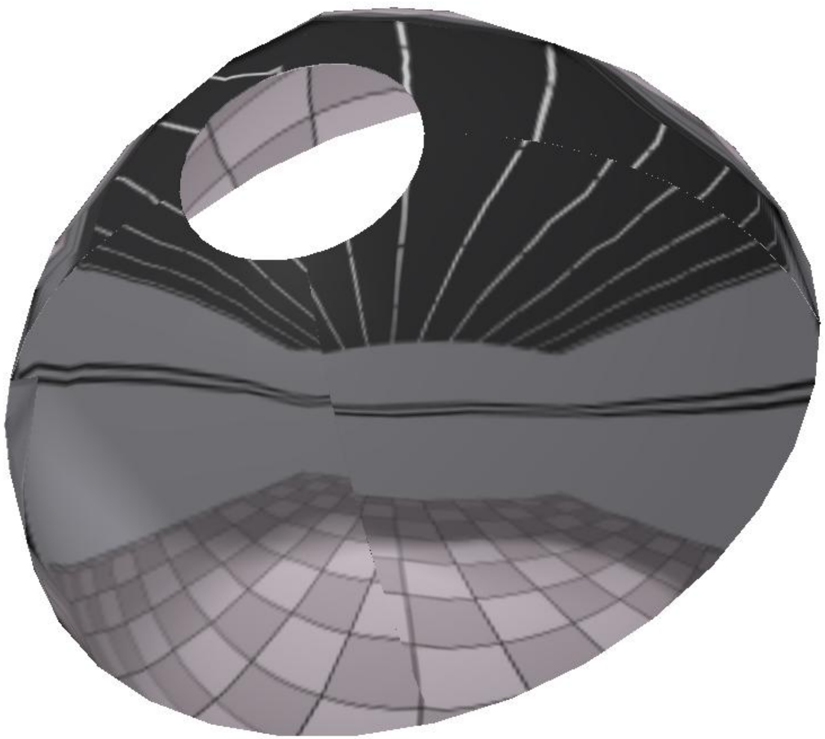
- The implementation is based on Gmsh
  - 3D finite element grid generator with a built-in CAD engine and a post-processor
  - Developed by C. Geuzaine (Ulg) and J.-F. Remacles (Ucl) [*Geuzaine et al ijnme2009*]
  - Industrially used (Cenaero, EDF, …)

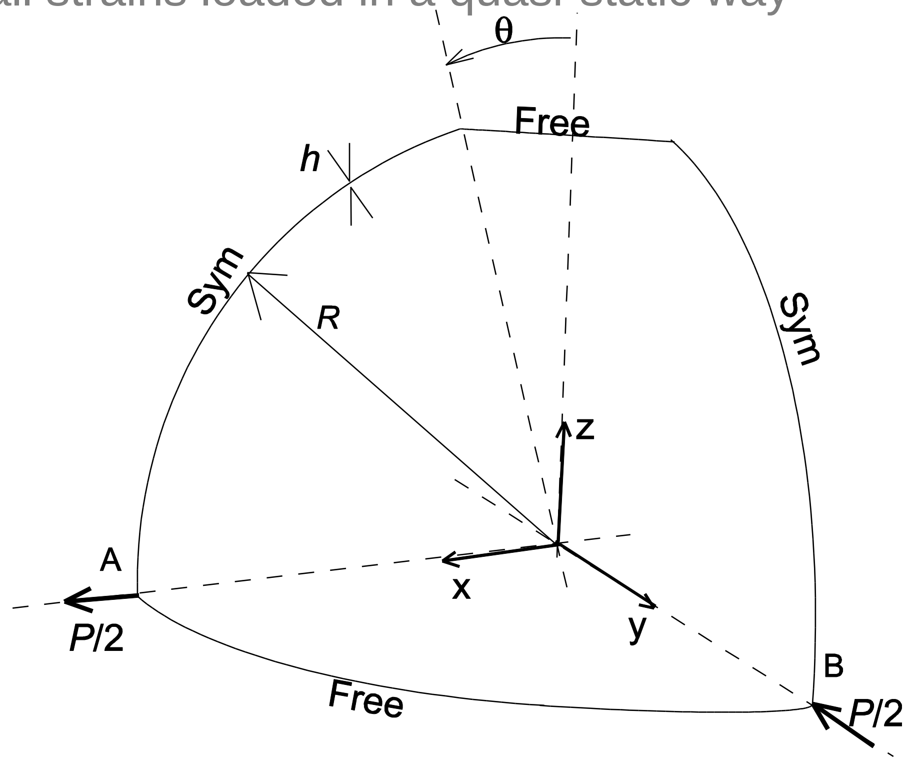
Full-DG formulation of Kirchhoff-Love shells

- Elements & post-processing C++ classes of Gmsh are used in the solver



- 2 benchmarks to prove the ability of the full-DG formulation to model continuous mechanics
  - Elastic open hemisphere with small strains loaded in a quasi-static way





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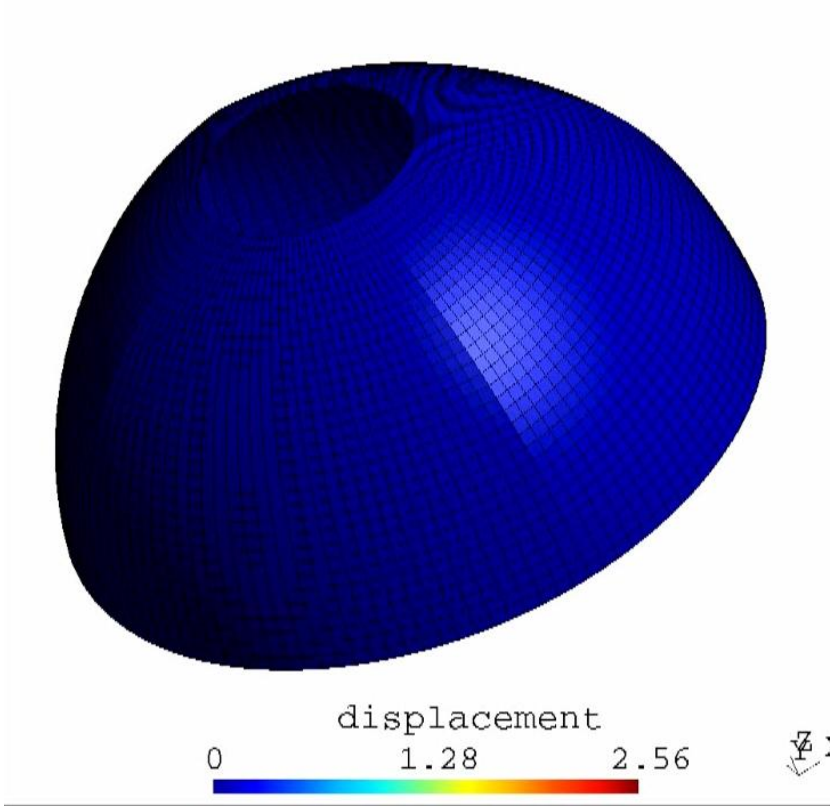
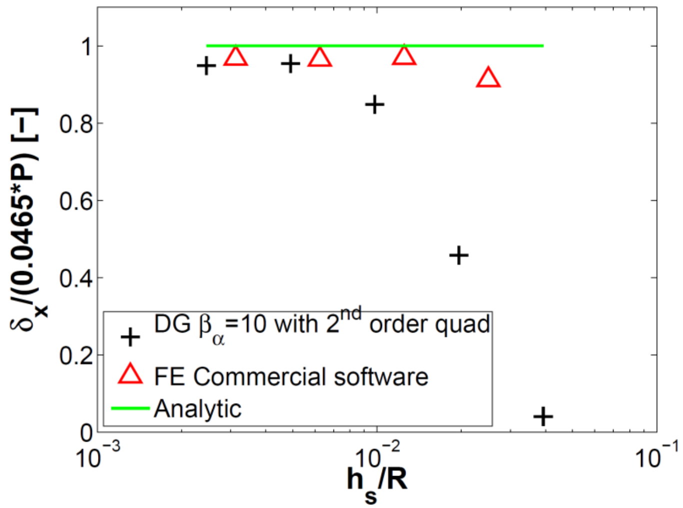
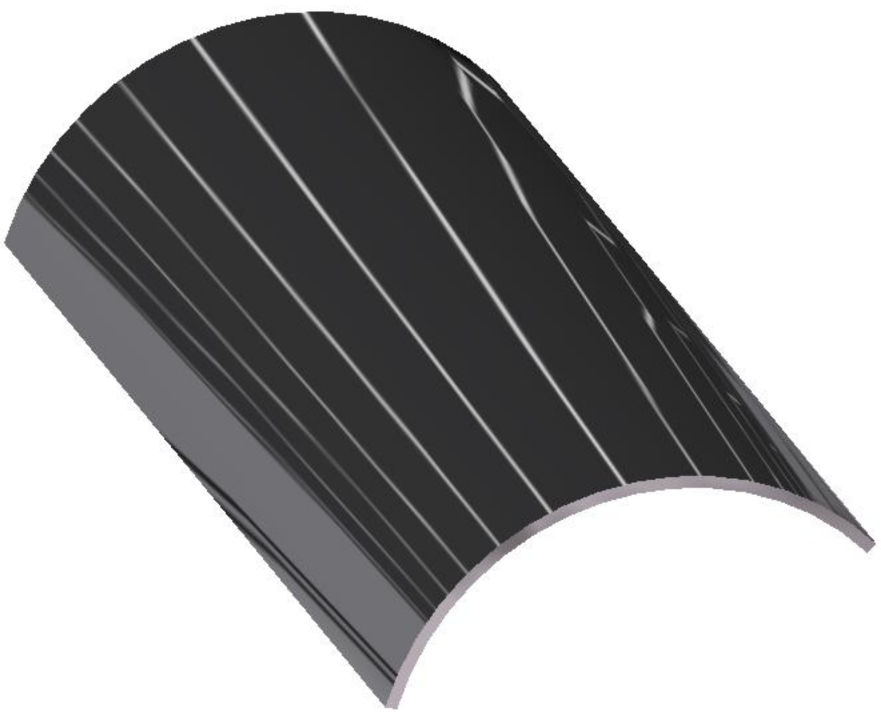
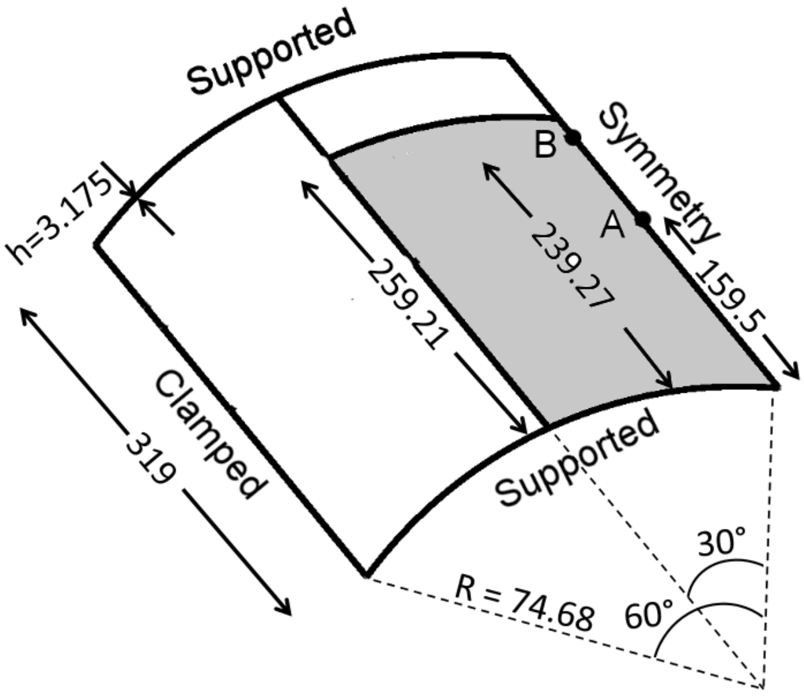




  - The method converges to the analytical solution with the mesh refinement

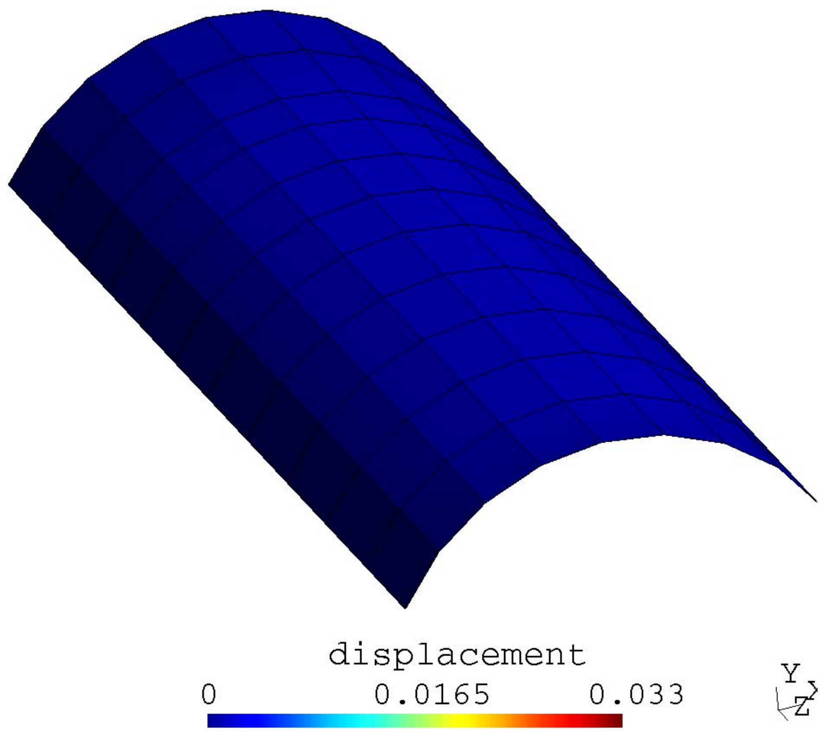
- 2 benchmarks to prove the ability of the full-DG formulation to model continuous mechanics
  - J_2-linear hardening (elasto-plastic large deformations) panel loaded dynamically (explicit Hulbert-Chung scheme)

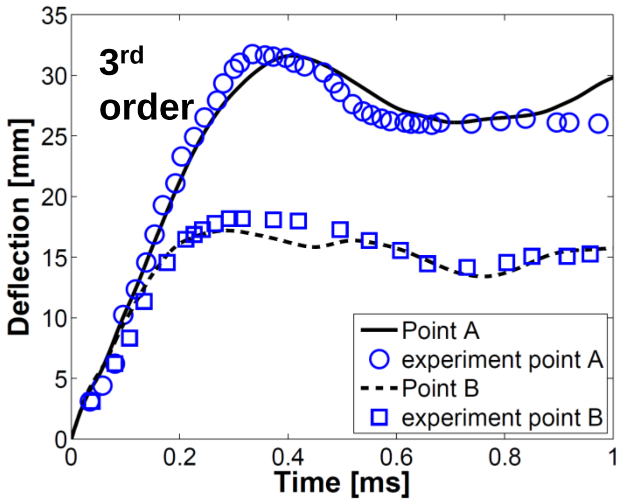




- 2 benchmarks to prove the ability of the full-DG formulation to model continuous mechanics

  – J_2-linear hardening (elasto-plastic large deformations) panel loaded dynamically (explicit Hulbert-Chung scheme)





  – The results match experimental data

Full-DG formulation of Kirchhoff-Love shells

- The full-DG method provides accurate results but is more costly than C^0/DG (memory, computational time) as it considers more degrees of freedom
  - Number of dofs (for the same mesh)

Benchmark	**C^0/DG**	**Full-DG**
Open hemisphere	867	1728
Cylindrical panel	1683	3456

  - The number of dofs is more or less twice larger for the full-DG formulation

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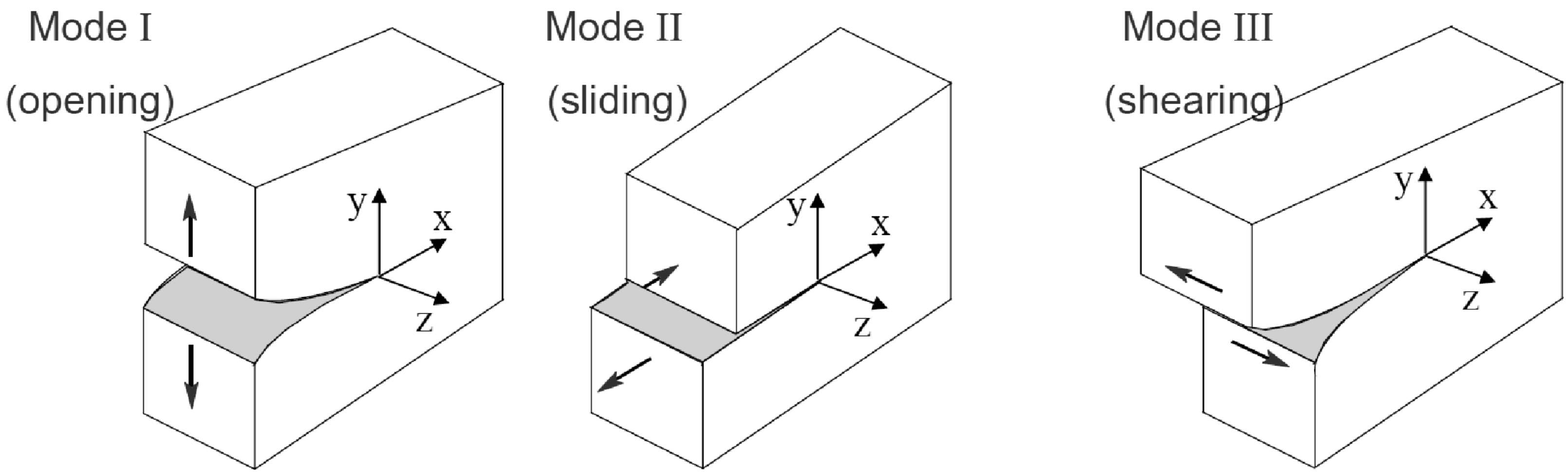
- The full-DG method can be advantageously used for
  - Parallel computation for explicit scheme [*Becker et al, ijnme2012*]
  - Fracture applications (same number of Dofs as FEM/ICL)

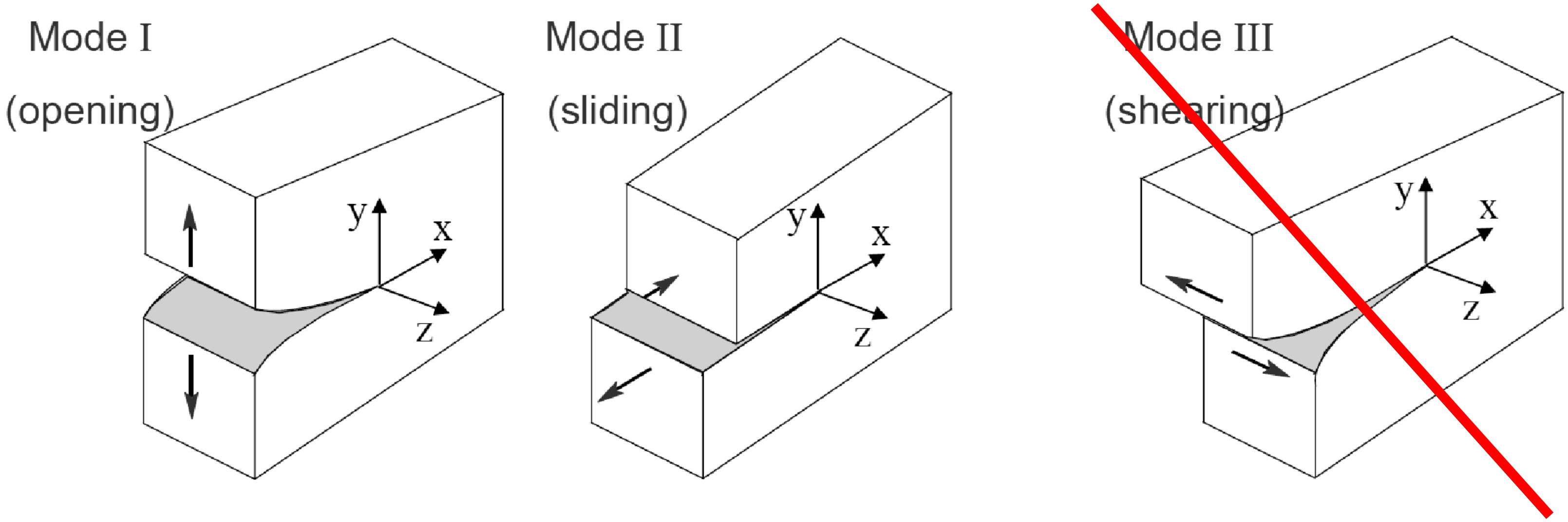
Plan

- Develop a discontinuous Galerkin method for thin bodies
  - Beam elements (1.5D case)
  - Shell elements (2.5D case)
- Discontinuous Galerkin / Extrinsic Cohesive law framework
  - Develop a suitable cohesive law for thin bodies
- Applications
  - Fragmentations, crack propagations under blast loadings

- There are 3 fracture modes in fracture mechanics



- Only modes I and II can be modeled by Kirchhoff-Love theory
  - Kirchhoff-Love → out-of-plane shearing is neglected

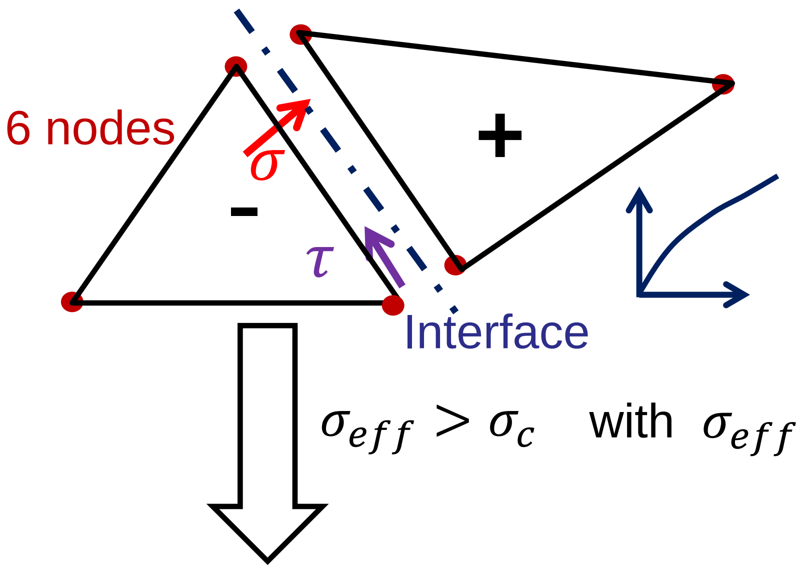


  - Model restricted to problems with negligible 3D effects at the crack tip

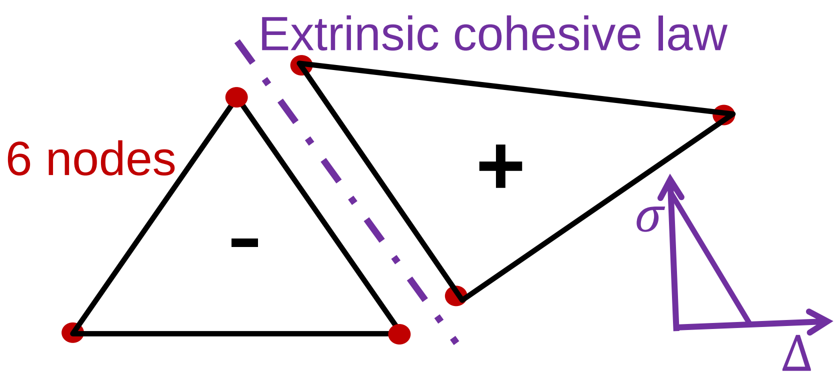
Full-DG/ECL framework

- Fracture criterion based on an effective stress

  – Camacho & Ortiz Fracture criterion [*Camacho et al ijss1996*]

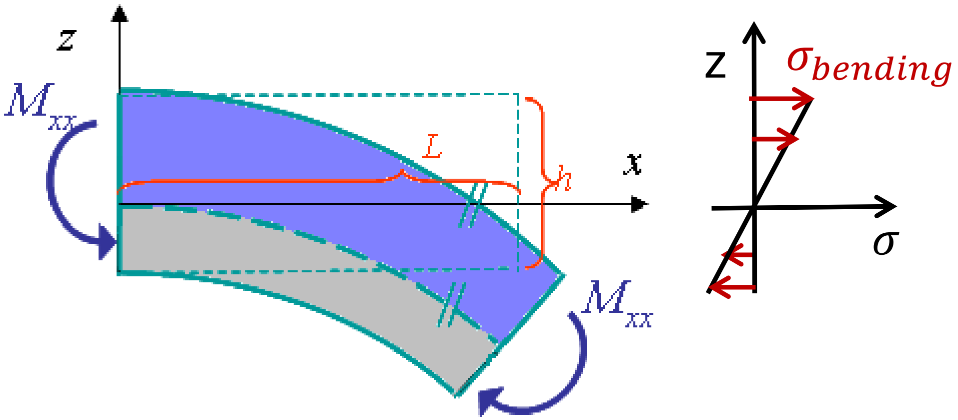


$$\sigma_{eff} > \sigma_c \quad \text{with} \quad \sigma_{eff} = \begin{cases} \sqrt{\sigma^2 + \beta^{-2}\tau^2} \text{ if } \sigma \geq 0 & \text{Traction} \\ \frac{1}{\beta} \ll |\tau| - \mu_c|\sigma| \text{ if } \sigma < 0 & \text{Compression} \end{cases}$$

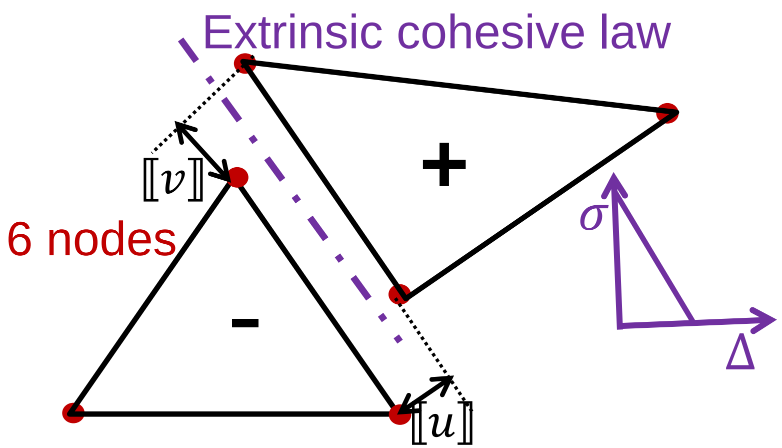


σ_c, β and μ_c are material parameters

- The effective stress is evaluated at the external fibers
  - The bending stress varies along the thickness
  - The fracture criterion is evaluated where the stress is maximum

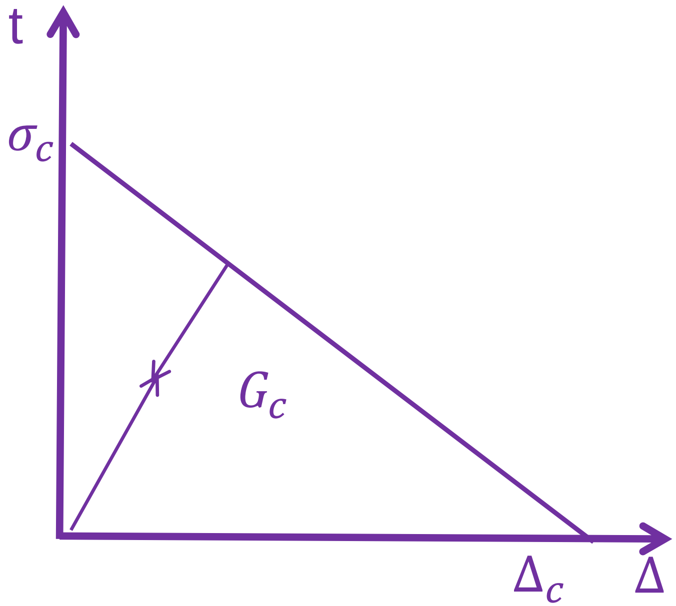


- The cohesive law is formulated in terms of an effective opening
  - Camacho & Ortiz Fracture criterion [*Camacho et al ijss1996*]

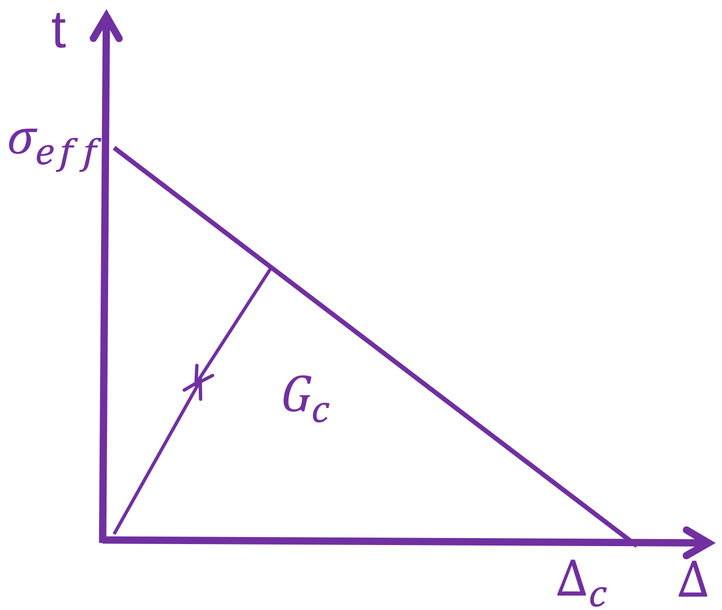


$$\Delta = \sqrt{[\![u]\!] + \beta^2 [\![v]\!]}$$

- The area under the cohesive law has to be equal to the fracture energy G_c
  - G_c is a material parameter

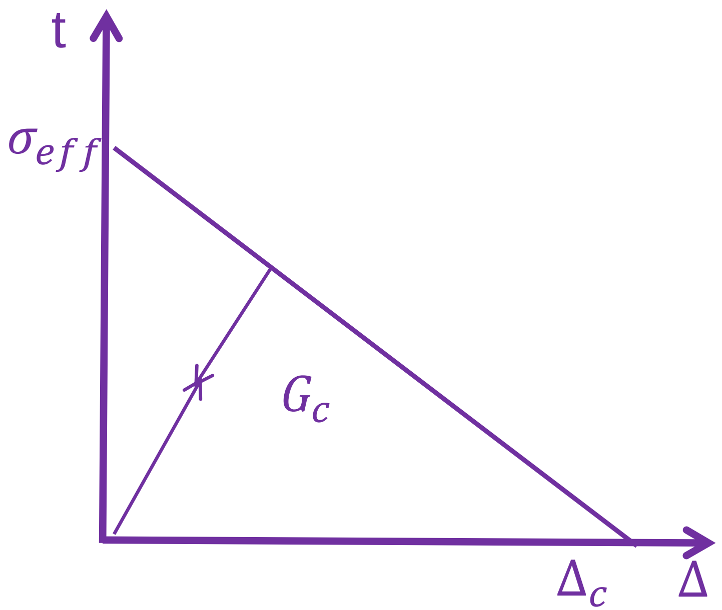


- The maximal stress of the cohesive law is equal to σ_{eff}
  - Ensure the continuity of stresses



  - Otherwise numerical problems [*Papoulia et al ijnme2003*]

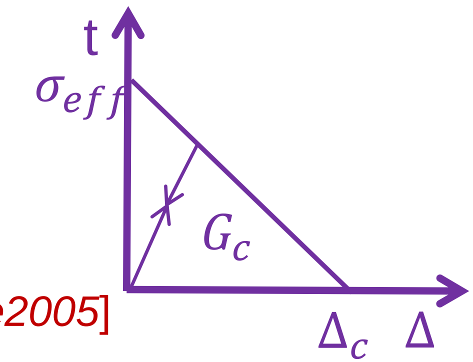
- The shape of the cohesive law is linearly decreasing
  - Little influence of the shape for brittle materials



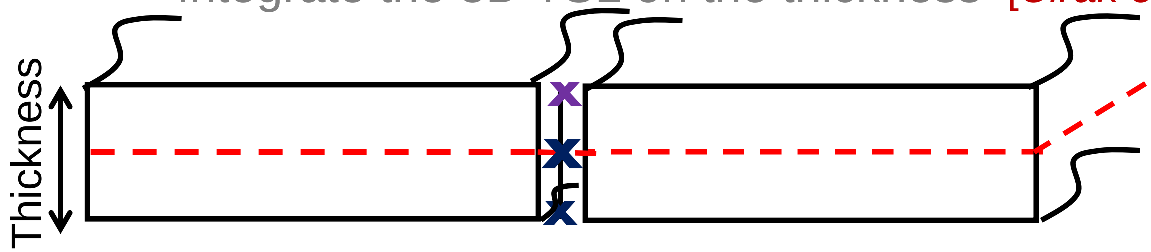
  - Δ_c is equal to $2G_c/\sigma_c$

Full-DG/ECL framework

- The through the thickness crack propagation is not straightforward with shell elements

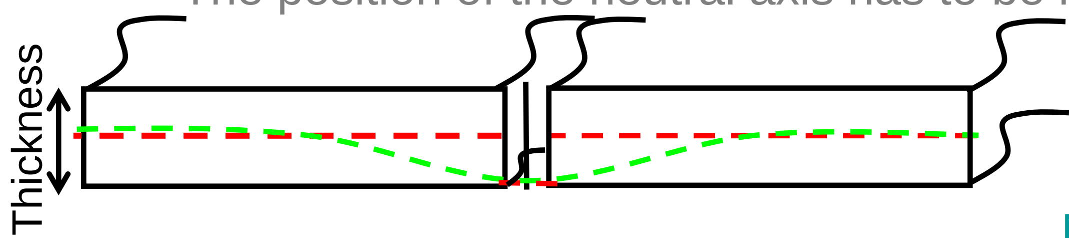


  - No elements on thickness
  - Integrate the 3D TSL on the thickness [*Cirak et al cmame2005*]

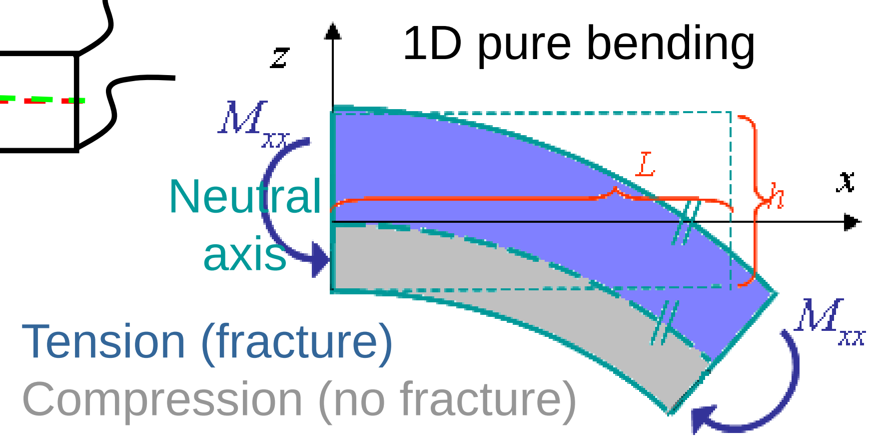


Fracture criterion is met
→ cohesive law
Unreached fracture
→ bulk law

  - The position of the neutral axis has to be recomputed to propagate the crack



Discontinuity
Continuity (Computation ?)



- The cohesive law can be formulated in terms of reduced stresses

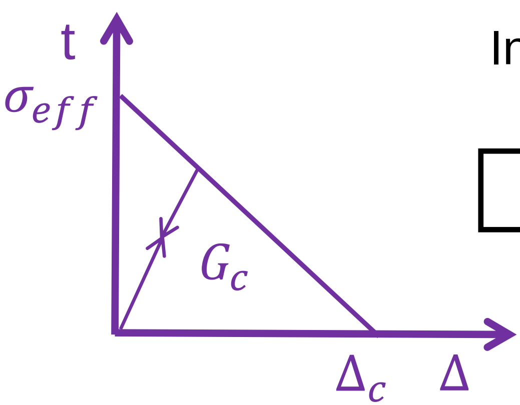
  – Same as shell equations

Bulk law
Stress tensor $\boldsymbol{\sigma}$

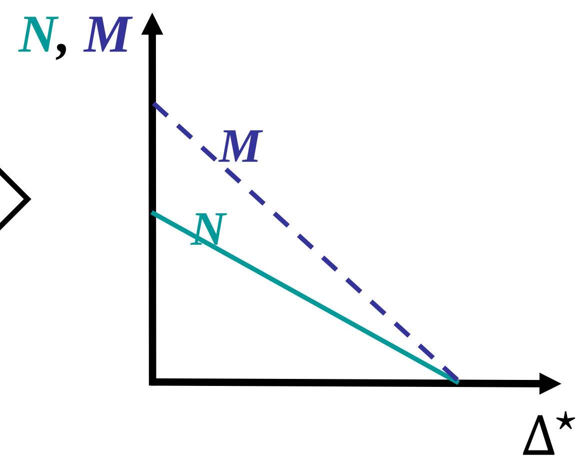
Integration on thickness

$$\boldsymbol{n}^{\alpha} = \frac{1}{\bar{j}} \int_{h_{min}}^{h_{max}} j\boldsymbol{\sigma} \cdot \boldsymbol{g}^{\alpha} d\xi^{3}$$

$$\tilde{\boldsymbol{m}}^{\alpha} = \frac{1}{\bar{j}} \int_{h_{min}}^{h_{max}} j\xi^{3}\boldsymbol{\sigma} \cdot \boldsymbol{g}^{\alpha} d\xi^{3}$$

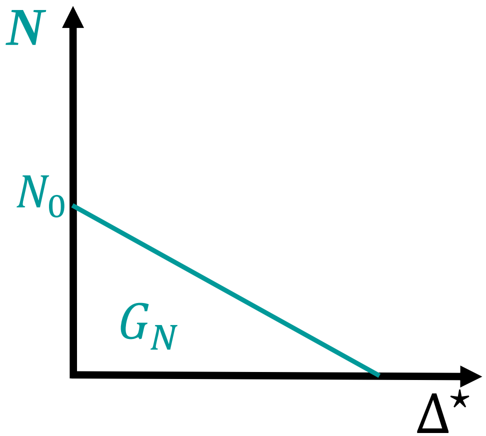


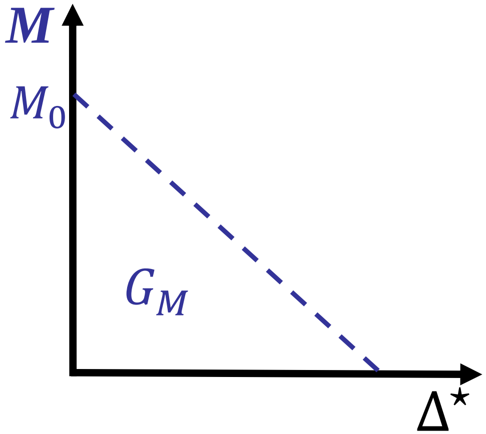
Integration on thickness



  – Similar concept suggested by Zavattieri [*Zavattieri jam2006*]

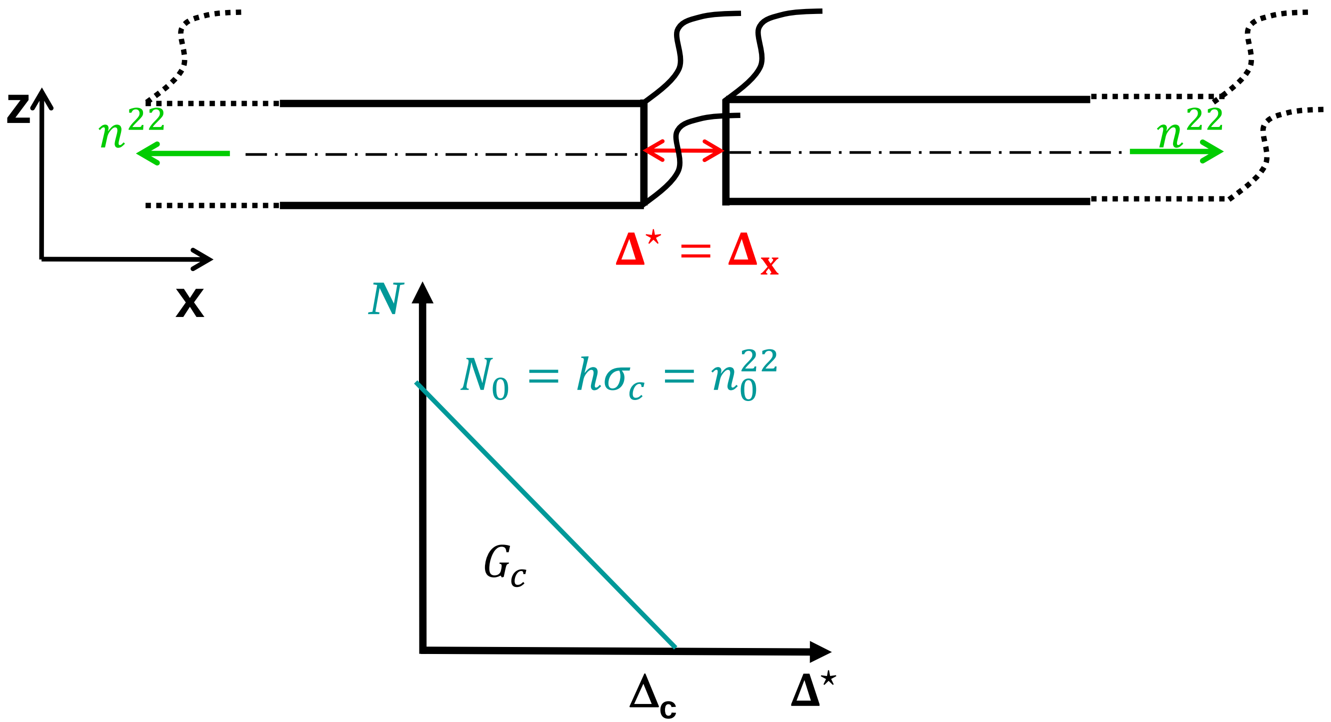
- Define $\Delta^{\star}$ and $N(\Delta^{\star}), M(\Delta^{\star})$ to dissipate an energy equal to hG_c during the fracture process [*Becker et al ijnme2012, Becker et al ijf2012*]
  - Integration on thickness





$$G_N + G_M = hG_c$$

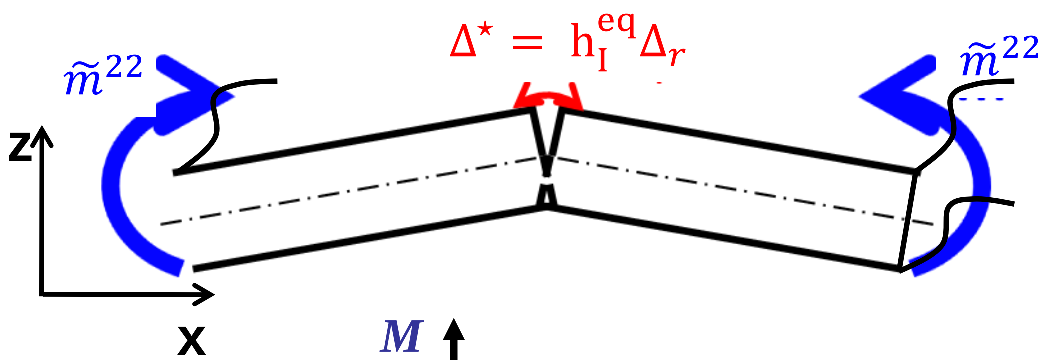
- The law $N(\Delta^{\star})$ is defined to release an energy equal to hG_c in pure tension
  - Pure mode I



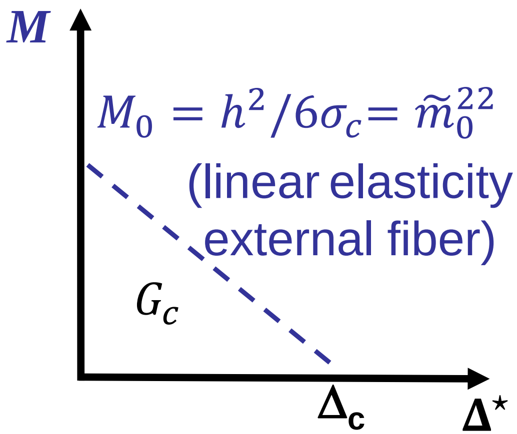
$$\int_0^{\Delta_c} N(\Delta_x)d\Delta_x = \frac{N_0\Delta_c}{2} = \frac{2h\sigma_c G_c}{2\sigma_c} = hG_c$$

Full-DG/ECL framework

- The law $M(\Delta^\star)$ is defined to release an energy equal to hG_c in pure bending
  - Pure mode I



$$\frac{\tilde{m}_0^{22}}{h_I^{eq}} = h\sigma_c$$

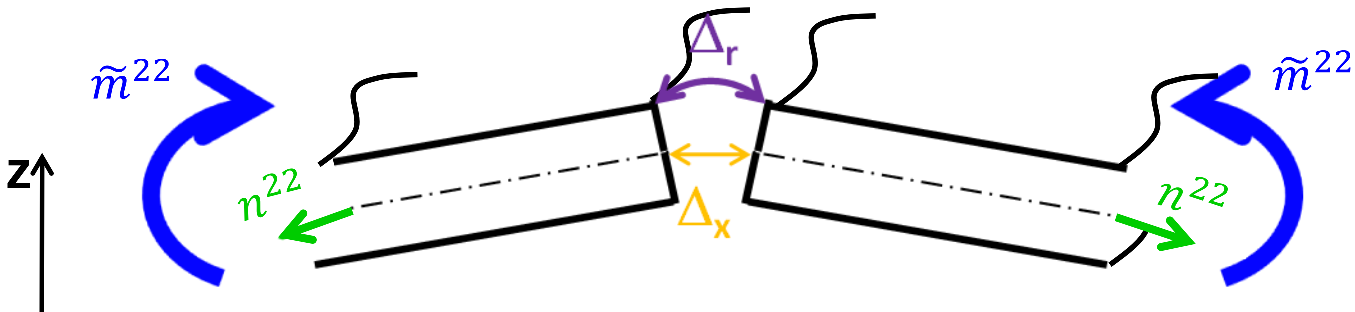


$$h_I^{eq} = \frac{h}{6}$$

$$\int_0^{\Delta_{rc}} M(\Delta^\star)d\Delta_r = \int_0^{\Delta_c} \pm\frac{6}{h}M_0\left(1-\frac{\Delta^\star}{\Delta_c}\right)d\Delta^\star = \frac{6}{h}\frac{h^2\sigma_c}{6}\frac{\Delta_c}{2} = hG_c$$

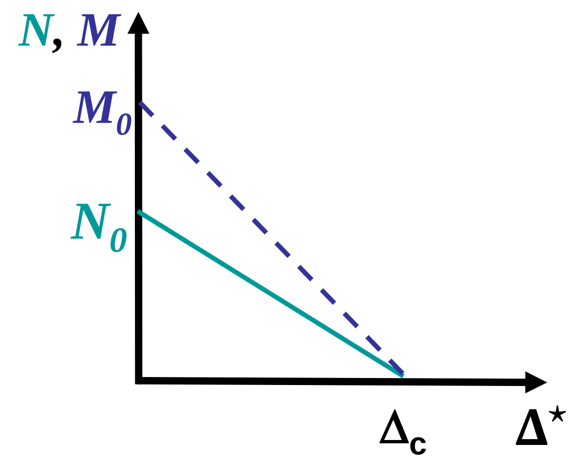
- Using the superposition principle the energy released for any couple N,M is equal to hG_c [*Becker et al ijnme2011*]

  - Pure mode I



$$\Delta_I^{\star} = (1-\eta_I)\Delta_x + \eta_I\, h_I^{eq}\, \Delta_r$$

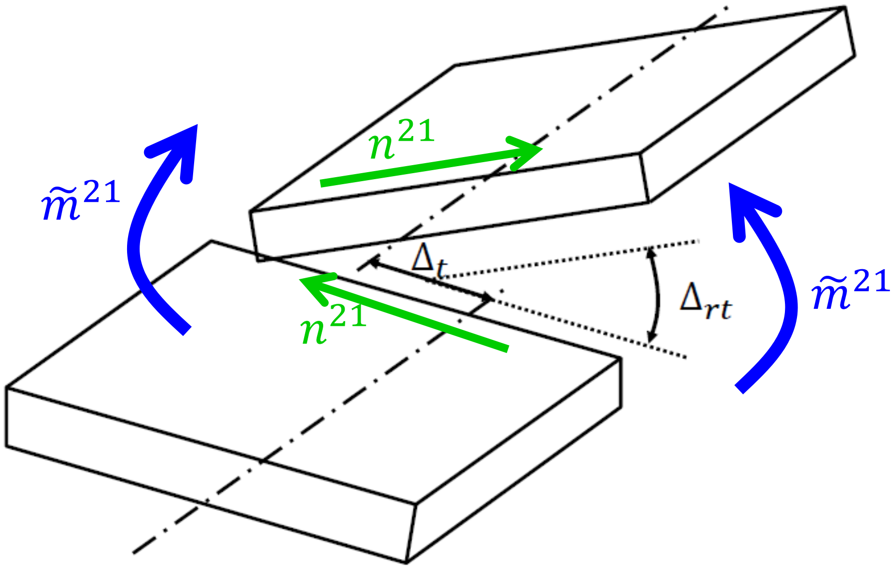
$$h_I^{eq} = \frac{M_0}{h\sigma_c - N_0}$$

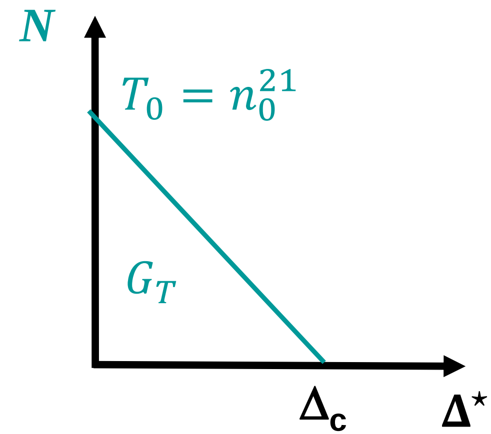


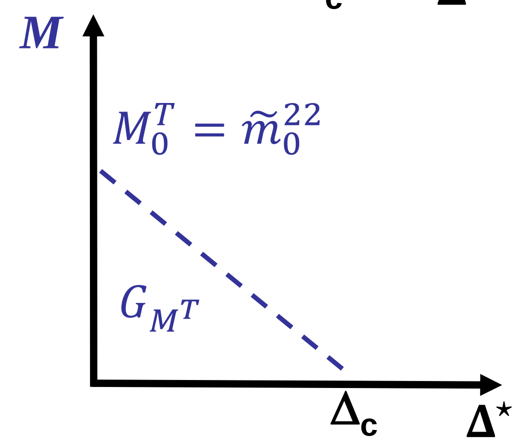
  - Coupling parameter

$$\eta_I = \frac{\left|1/h_I^{eq} M_0\right|}{N_0 + \left|1/h_I^{eq} M_0\right|} = \frac{\mathrm{h}\sigma_c - N_0}{h\sigma_c}$$

- The cohesive model for mode I can be extended to mode II







$$G_T + G_{M^T} = h\beta G_c$$

$$\Delta_{II}^\star = (1-\eta_{II})\Delta_t + \eta_{II} h_{II}^{eq} \Delta_{rt}$$

$$h_{II}^{eq} = \frac{M_0^T}{h\beta\sigma_c - T_0}$$

- Coupling parameter

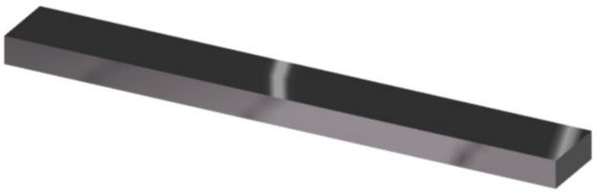
$$\eta_{II} = \frac{\left|1/h_{II}^{eq} M_0^T\right|}{T_0 + \left|1/h_{II}^{eq} M_0^T\right|} = \frac{h\beta\sigma_c - T_0}{h\beta\sigma_c}$$

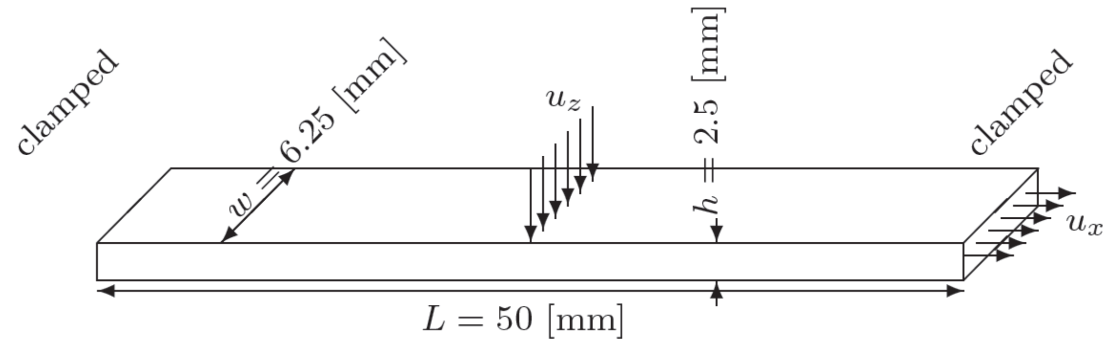
- Combination of mode I and II is performed following Camacho & Ortiz [*Camacho et al ijss1996*]
  - Usually perform in the literature
  - Define an effective opening $\Delta^\star = \sqrt{\ll \Delta_I^\star \gg^2 + \beta^2 \, {\Delta_{II}^\star}^2}$
  - Fracture initiation $\sigma_{eff} = \begin{cases} \sqrt{\sigma_I^2 + \beta^{-2}\tau_{II}^2} \text{ if } \sigma_I \geq 0 \\ \frac{1}{\beta} \ll |\tau_{II}| - \mu_c|\sigma_I| \text{ if } \sigma_I < 0 \end{cases} = \sigma_c$
  - The equivalent thicknesses become

$$h_I^{eq} = \frac{M_0}{h\sigma_I - N_0}$$

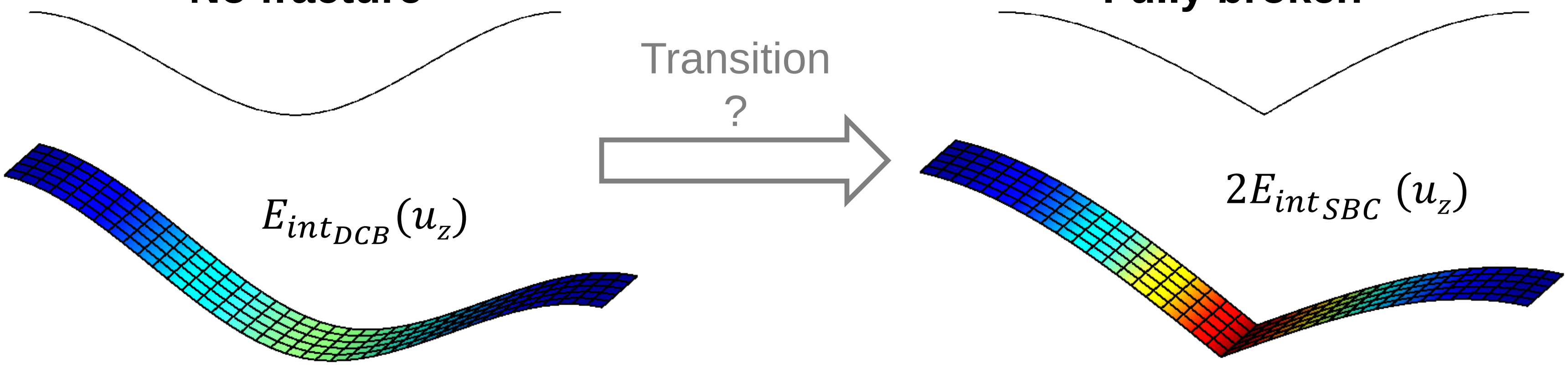
$$h_{II}^{eq} = \frac{M_0^T}{h\tau_{II} - T_0}$$

- The transition between uncracked to fully cracked body depends on ΔE_{int}
  - Double clamped elastic beam loaded in a quasi-static way





No fracture **Fully broken**

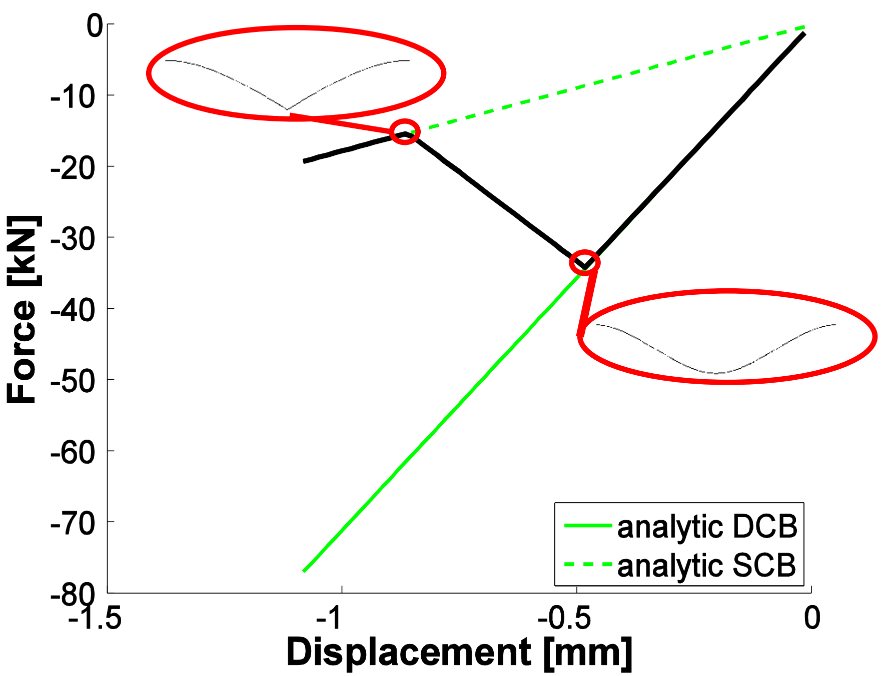


$$\Delta E_{int}(u_z) = E_{int_{DCB}}(u_z) - 2E_{int_{SBC}}(u_z)$$

- The framework can model stable/unstable crack propagation
  - Geometry effect (no pre-strain)

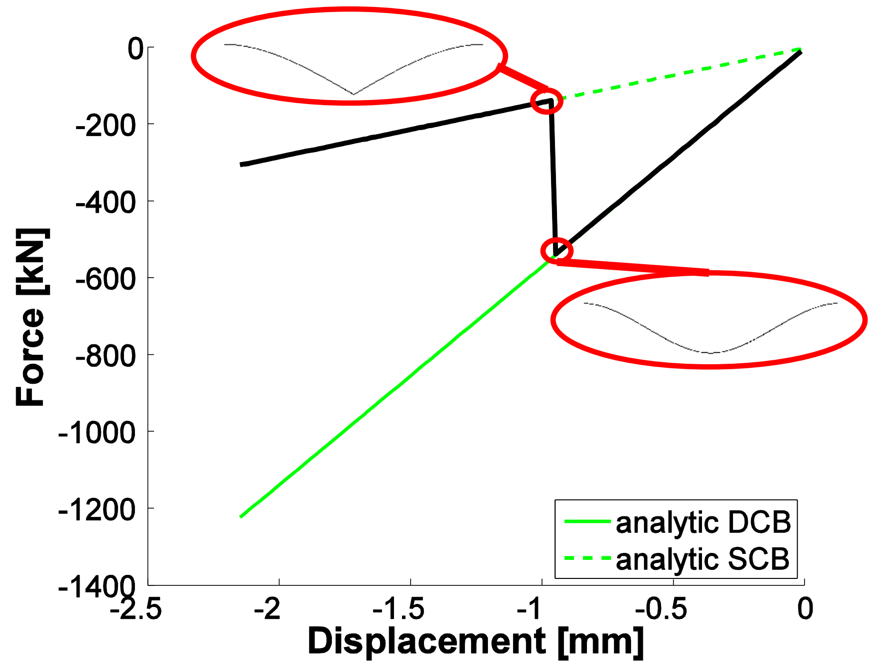
Stable transition

$\Delta E_{int}(u_z) < hGc$

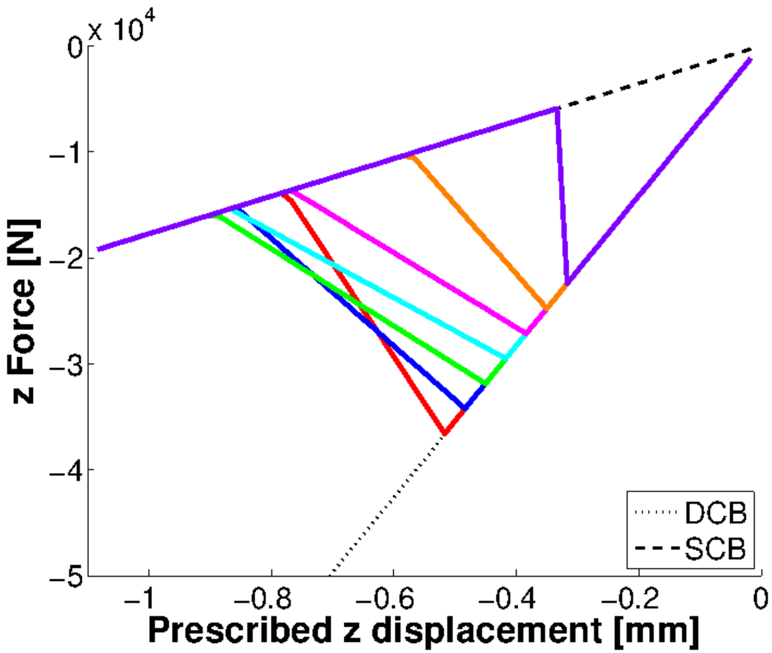


Unstable transition

$\Delta E_{int}(u_z) > hGc$



- The energy released during fracture is always equal to hG_c
  - Pre-strain effect



Pre-strain

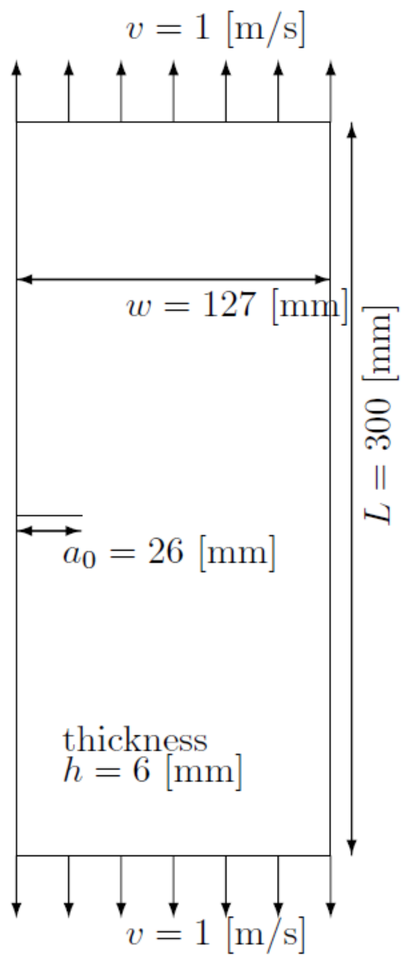
hG_c = 22.00

$u_{x,pres}$	η_I	ΔE_{int}	$E_{released}$
$-2e^{-5}$	1,0692	14.82	21.98
0.	1	12.33	21.98
$2e^{-5}$	0.93	11.39	21.98
$4e^{-5}$	0.86	11.99	21.98
$6e^{-5}$	0.79	14.11	21.98
$8e^{-5}$	0.72	17.76	21.99
$10e^{-4}$	0.66	22.95	--

> 22.00

Unstable

- A benchmark with a dynamic crack propagation
  - A single edge notched elastic plate dynamically loaded

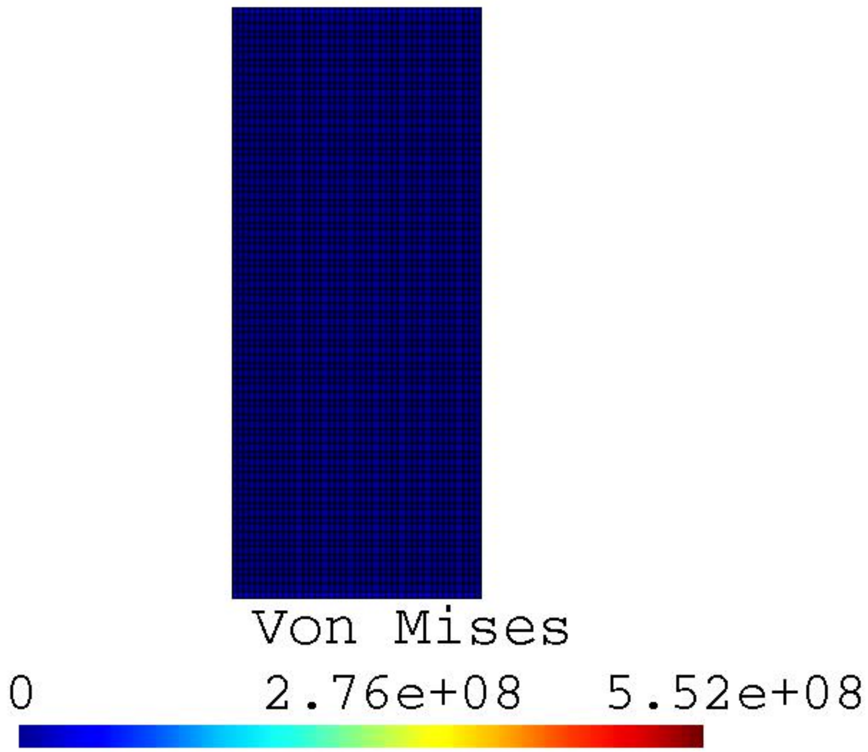


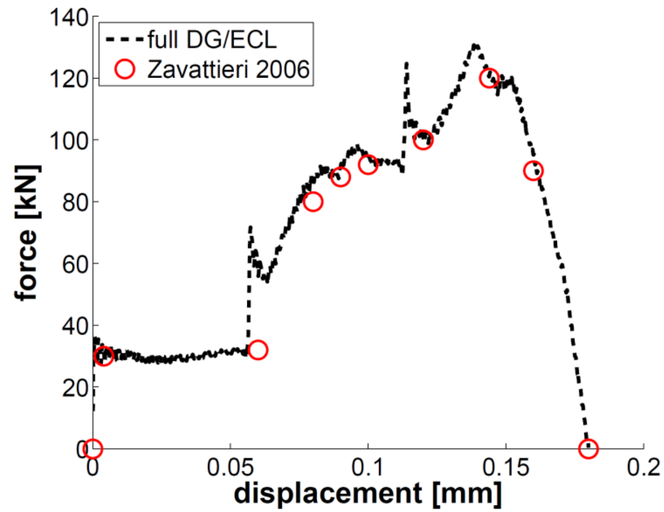
Full-DG/ECL framework

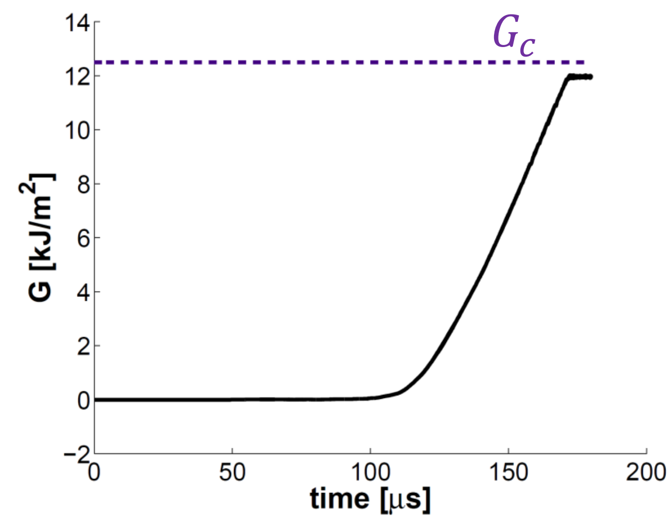
- The energy released in a dynamic crack propagation is equal to hG_c

  – Compare results to the literature

  [*Zavattieri jam2006*]

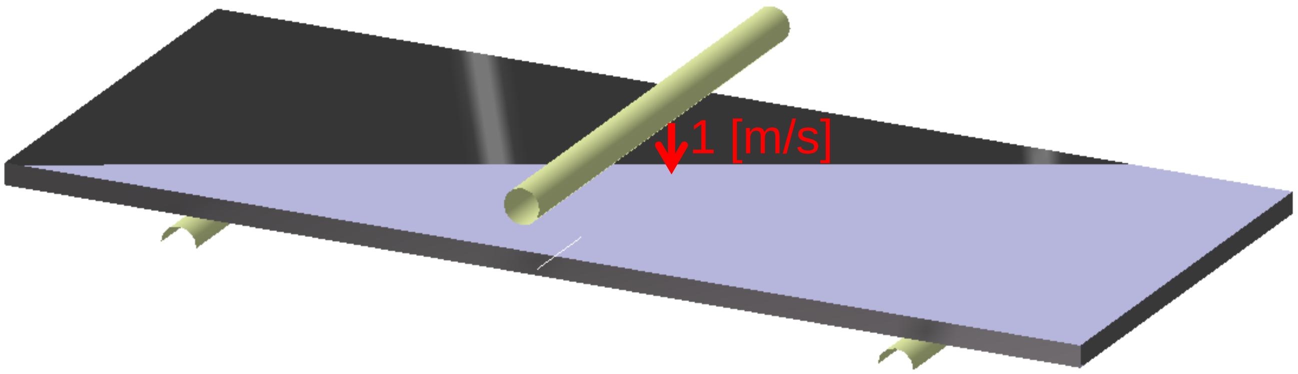


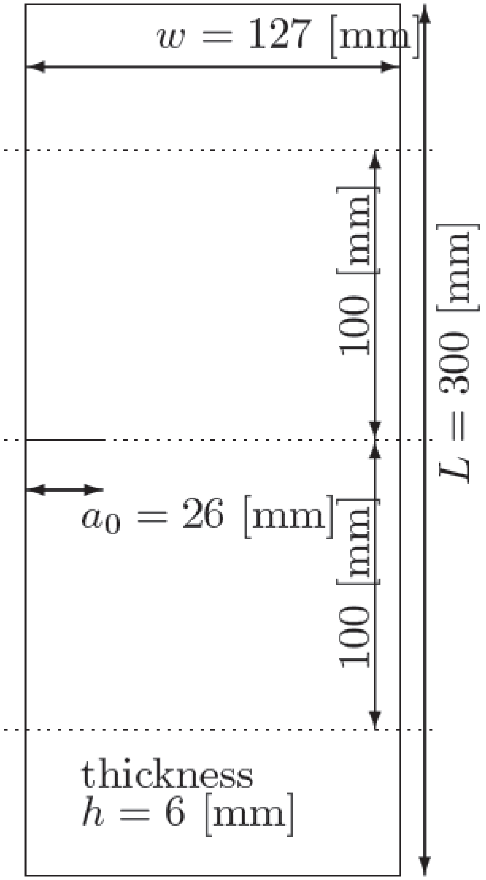




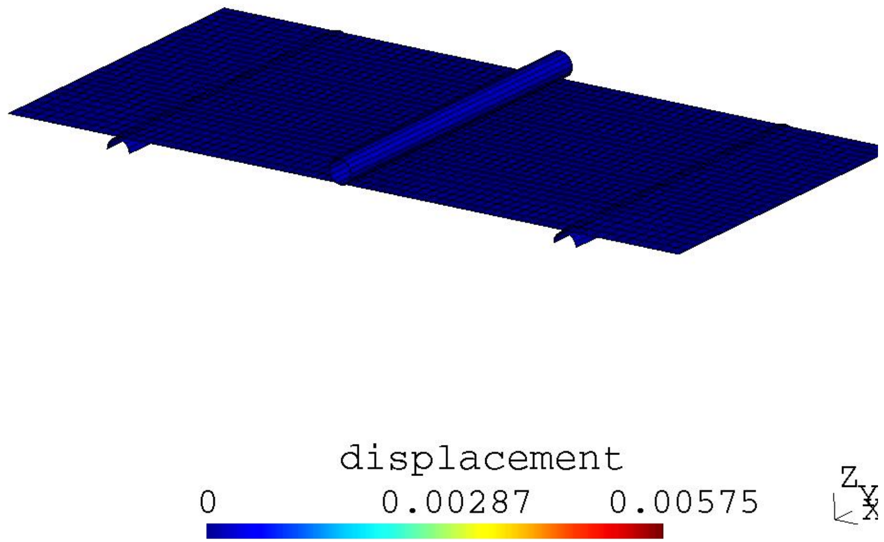
- A benchmark involving contact

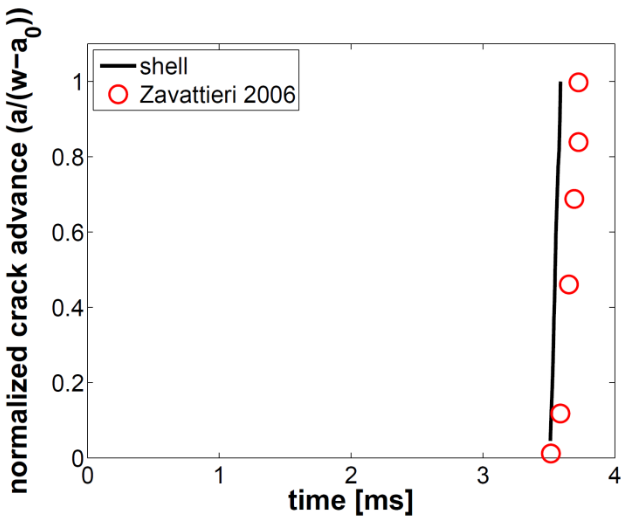
  – A single edge notched elastic plate impacted by a rigid cylinder



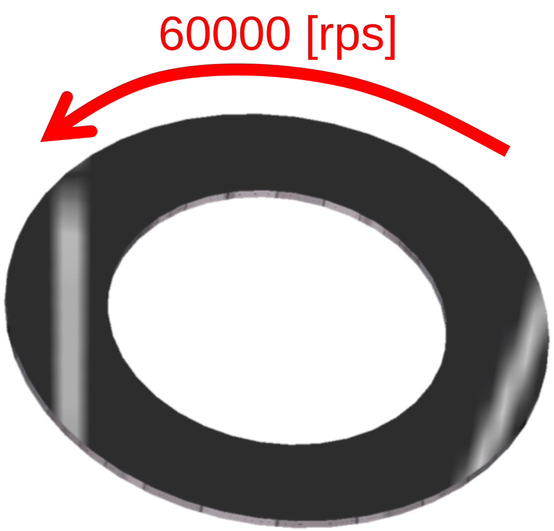


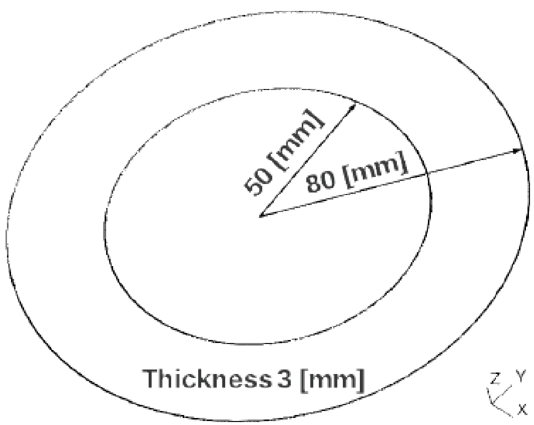
- The crack propagates correctly even if there is (rigid) contact
  - Results are compared to the literature [*Zavattieri jam2006*]

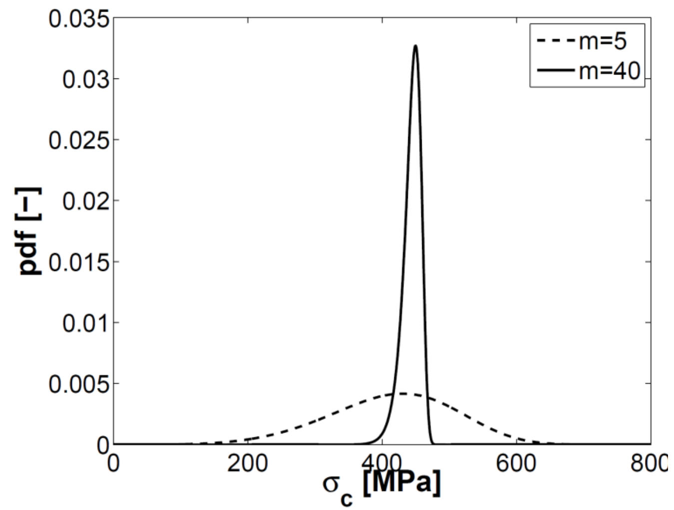




- A benchmark to investigate the fragmentation
  - Elastic plate ring loaded by a centrifugal force

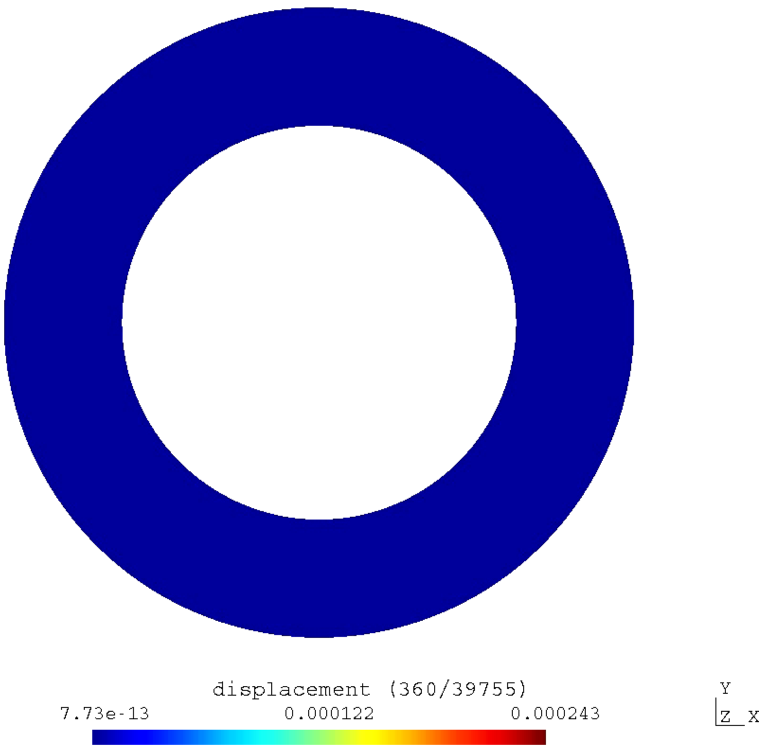






Full-DG/ECL framework

- Fragmentation phenomena can also be studied by the full-DG/ECL framework
  - Results are compared with the literature [*Zhou et al ijnme2004*]

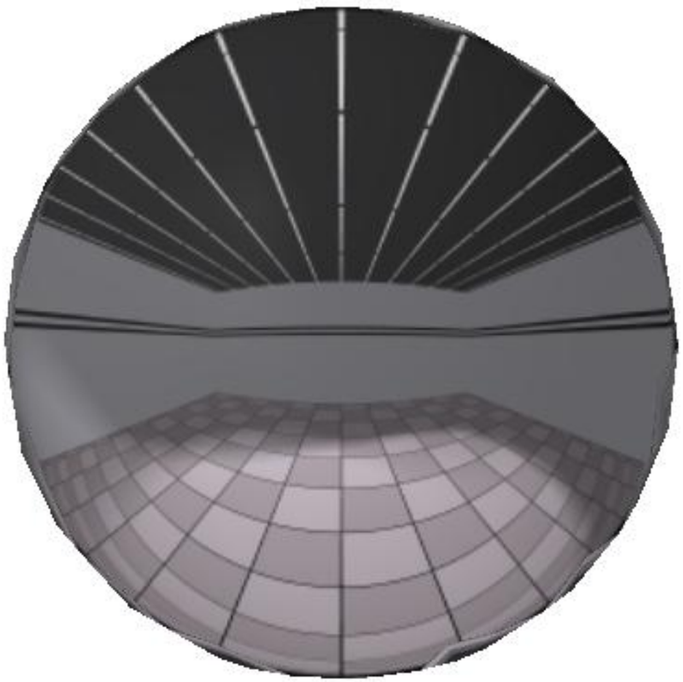


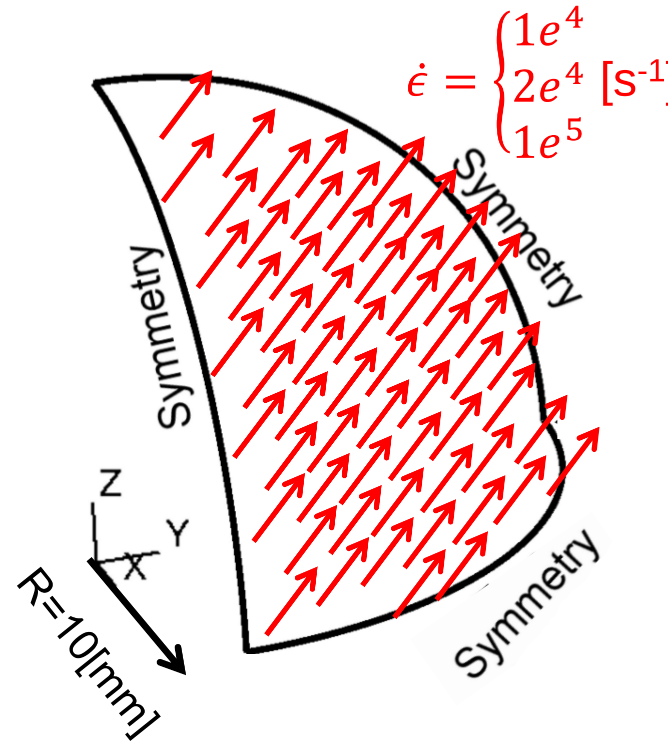
[*Zhou et al ijnme2004*]

Plan

- Develop a discontinuous Galerkin method for thin bodies
  - Beam elements (1.5D case)
  - Shell elements (2.5D case)
- Discontinuous Galerkin / Extrinsic Cohesive law framework
  - Develop a suitable cohesive law for thin bodies
- Applications
  - Fragmentations, crack propagations under blast loadings

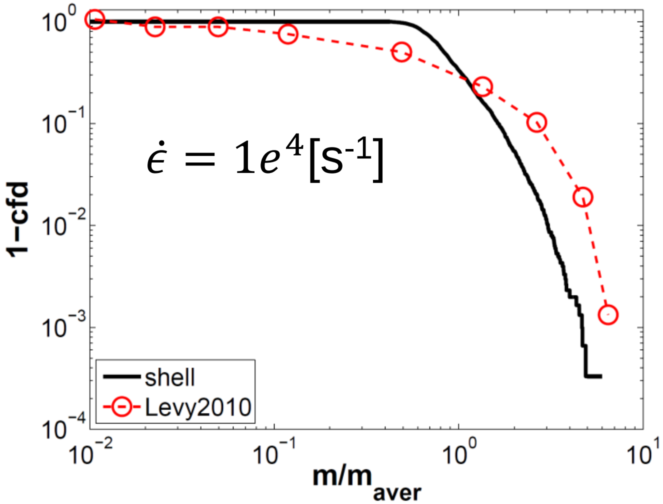
- Application to the dynamic fragmentation of a sphere
  - Elastic sphere under radial uniform expansion

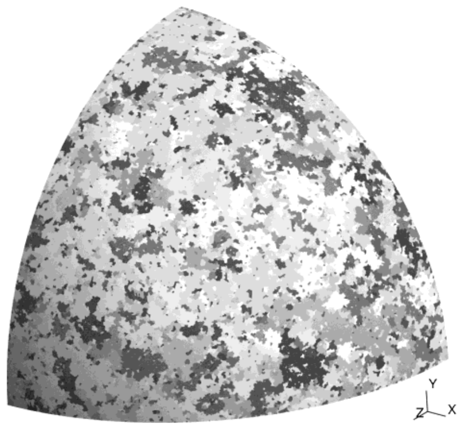




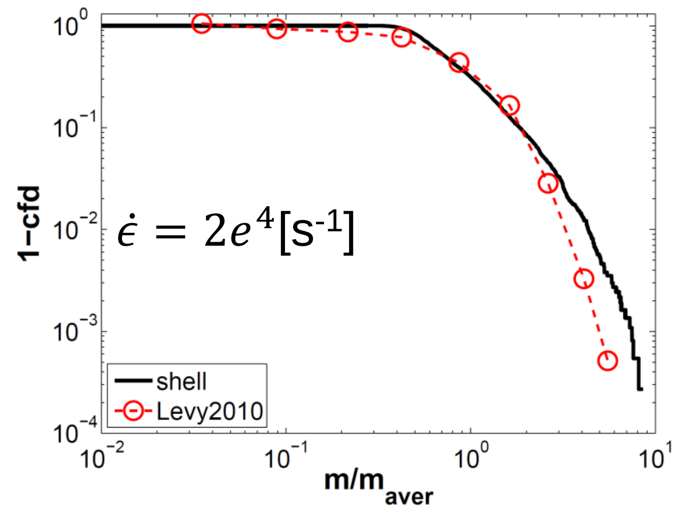
Applications of the DG/ECL framework

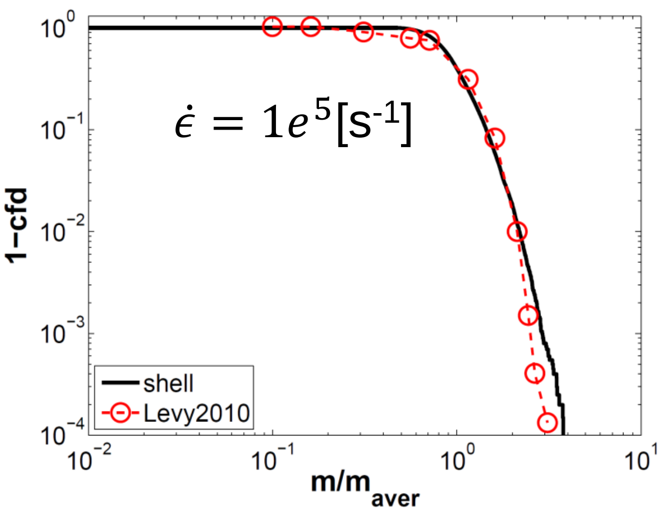
- The distribution of fragments and the number of fragments are in agreement with the literature [*Levy EPFL2010*]
  - Levy uses 3D elements

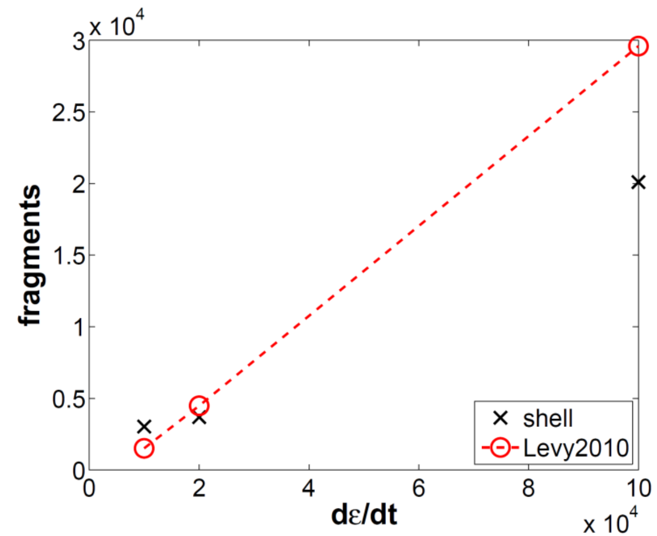




$\dot{\epsilon} = 1e^4$[s^{-1}]
2 588 265 Dofs
±48h on 32 cpus
(±17h for $\dot{\epsilon} = 1e^5$[s^{-1}])

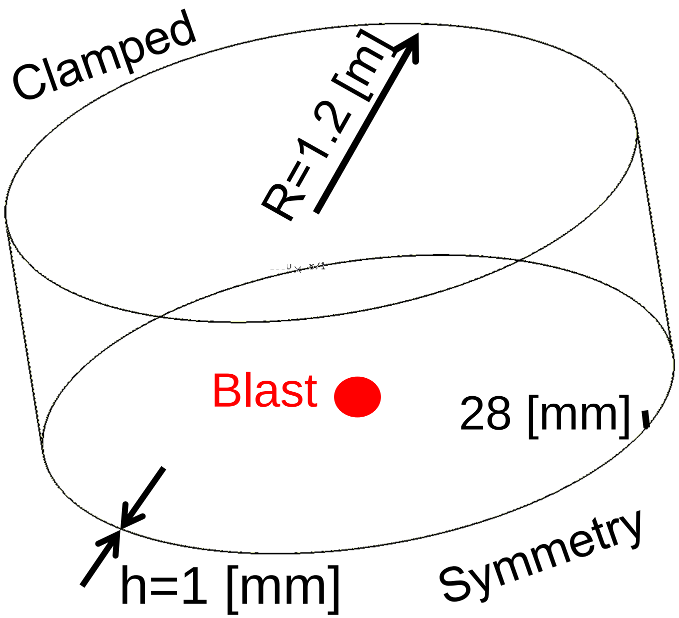


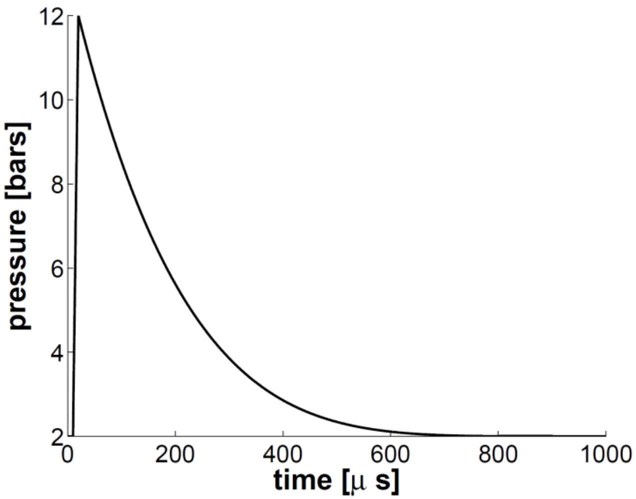




Applications of the DG/ECL framework

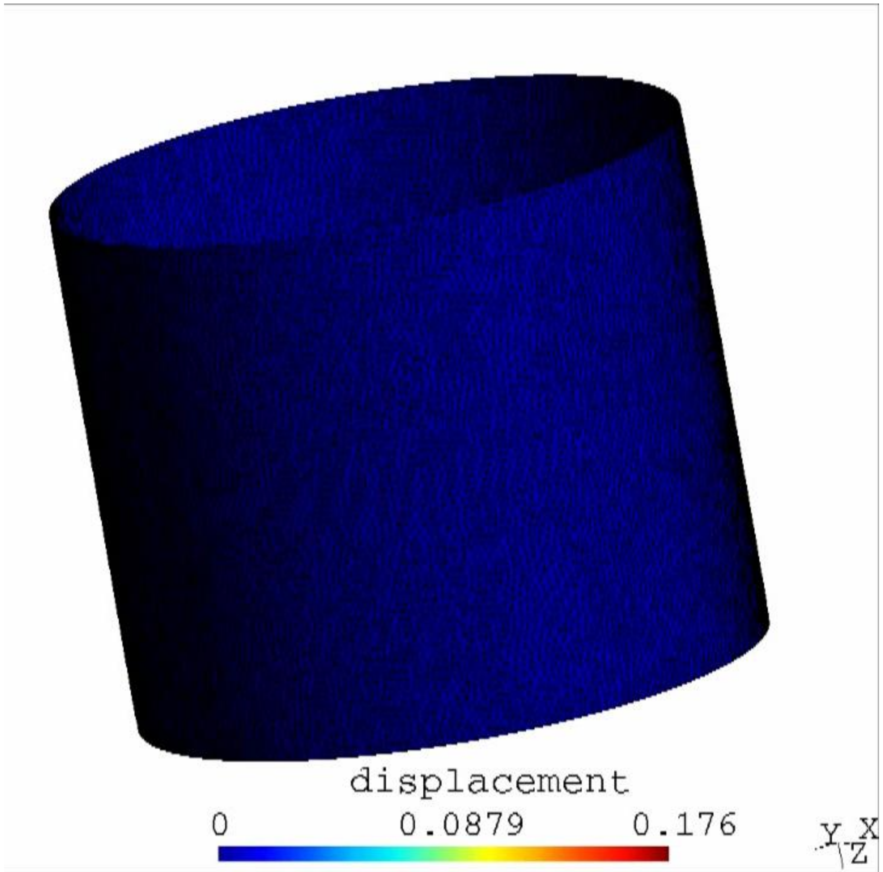
- Blast of an axially notched elasto-plastic cylinder (large deformations)



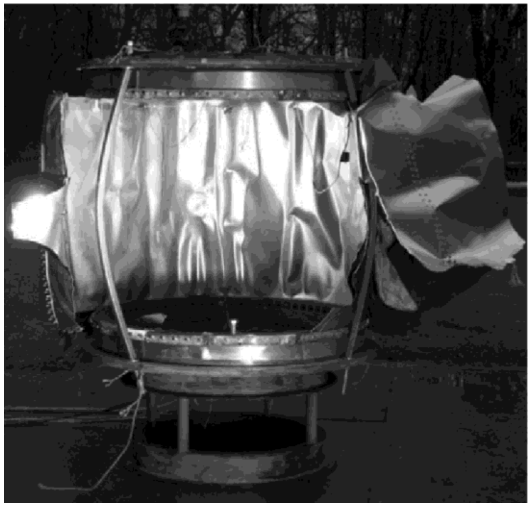


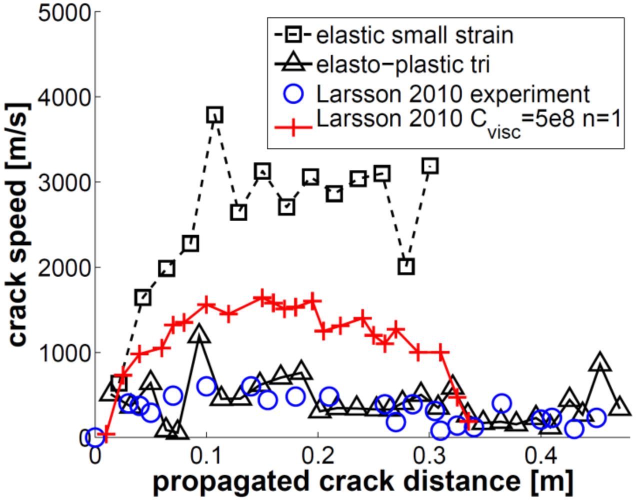
Applications of the DG/ECL framework

- Accounting for plasticity allows capturing the crack speed
  - Compare with the literature [*Larson et al ijnme2011*]

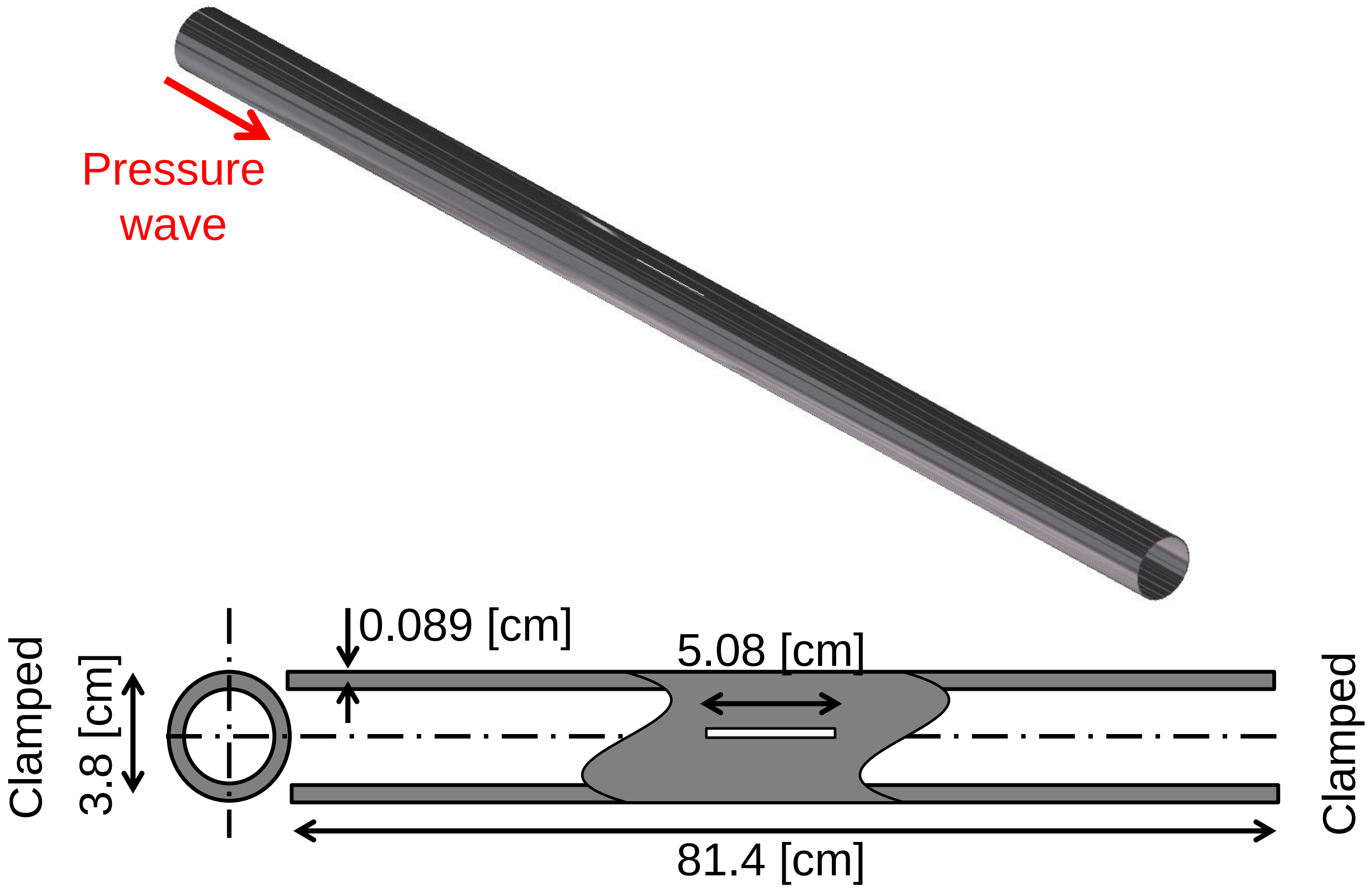


556 080 Dofs
±72h on 16 cpus



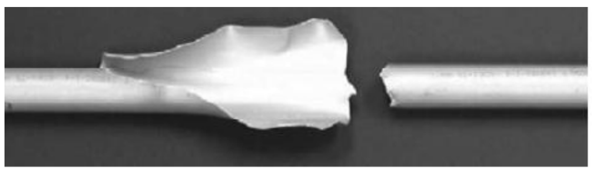


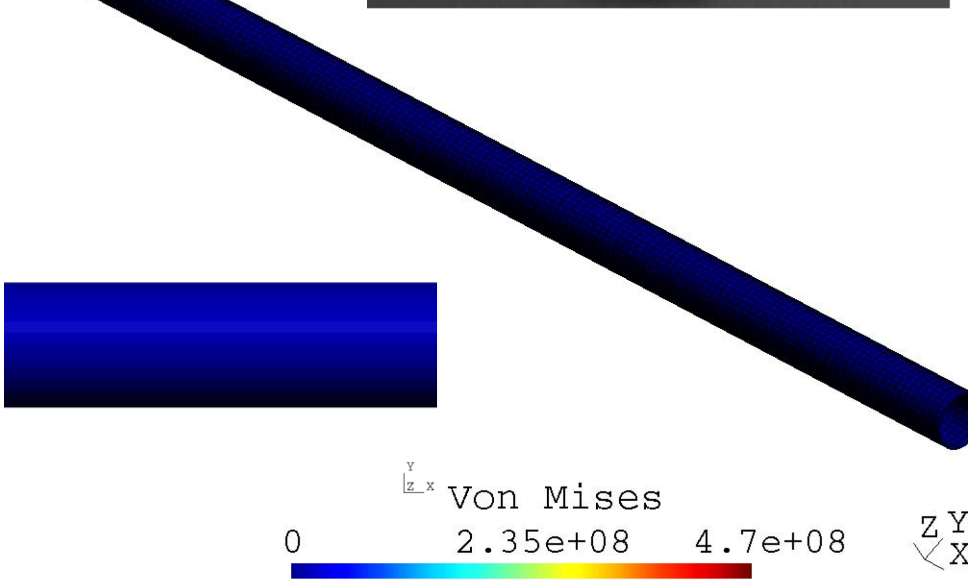
- Pressure wave passes through an axially notched elasto-plastic pipe (large deformations)



Applications of the DG/ECL framework

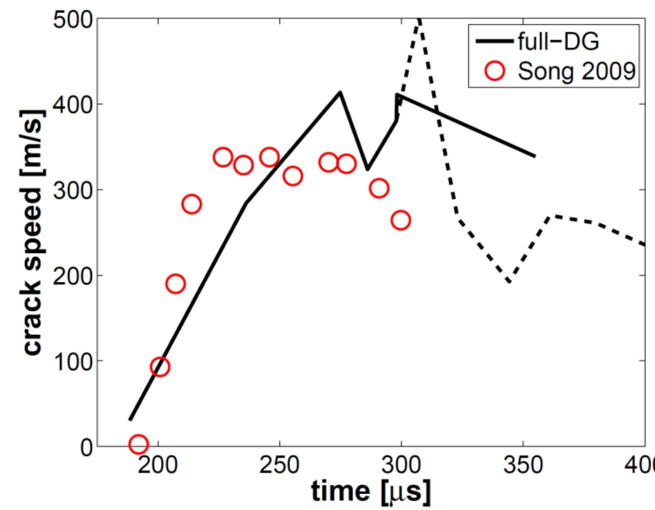
- Crack path and speed are well captured by the framework
  - Compare with the literature
    [*Song et al jam2009*]

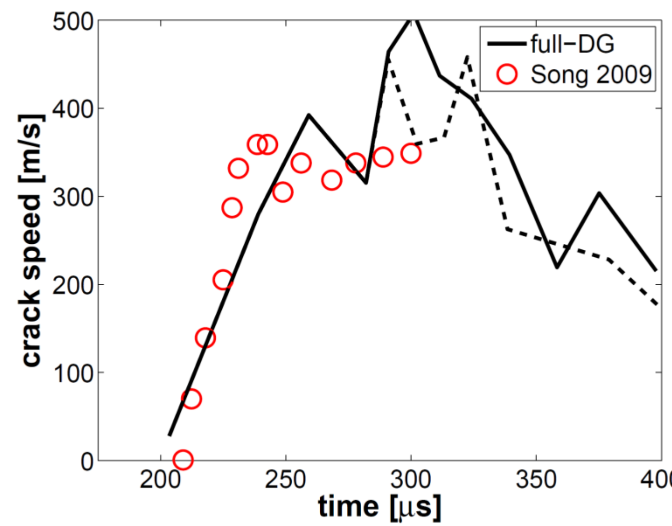




224 256 Dofs

±21 h on 12 cpus





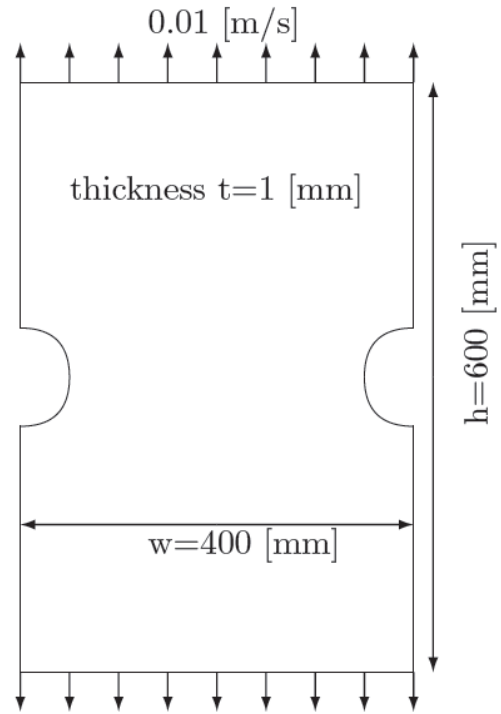
Conclusions

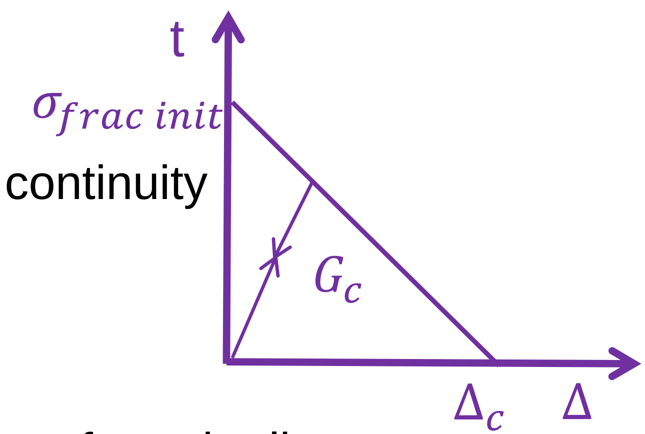
- Full-DG / ECL framework allows accounting for fracture in dynamic simulations of thin bodies
  - Crack propagation as well as fragmentation
  - Recourse to an elasto-plastic model is mandatory to capture crack speed
  - Affordable computational time for large models
- Main contributions
  - Full-DG model of linear Euler-Bernoulli beams and (non)-linear Kirchhoff-Love shell
  - Energetically consistent extrinsic cohesive law based on reduced stresses
  - Explicit Hulbert-Chung algorithm based on ghost elements (reduce MPI communication, independent of material law)

Future work

- Model the damage to crack transition by coupling a damage law with the full-DG/ECL framework
  - Replace the criterion based on an effective stress by a criterion based on the damage
  - Define the shape of the cohesive law

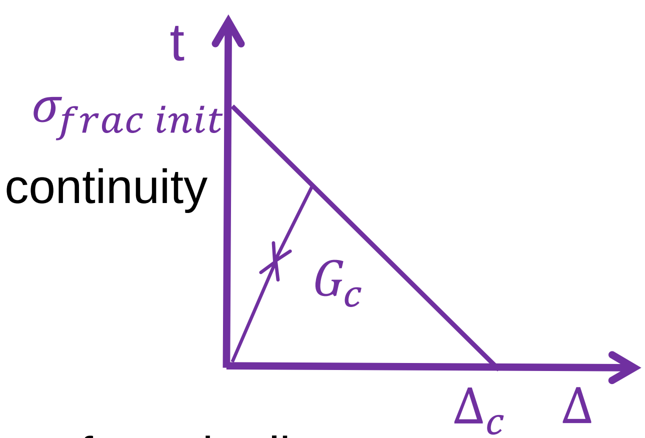
- An exploratory benchmark
  - Quasi-static (dynamic relaxation)
  - Linear damage theory
  - Fracture criterion $D > D_c$
  - Cohesive shape

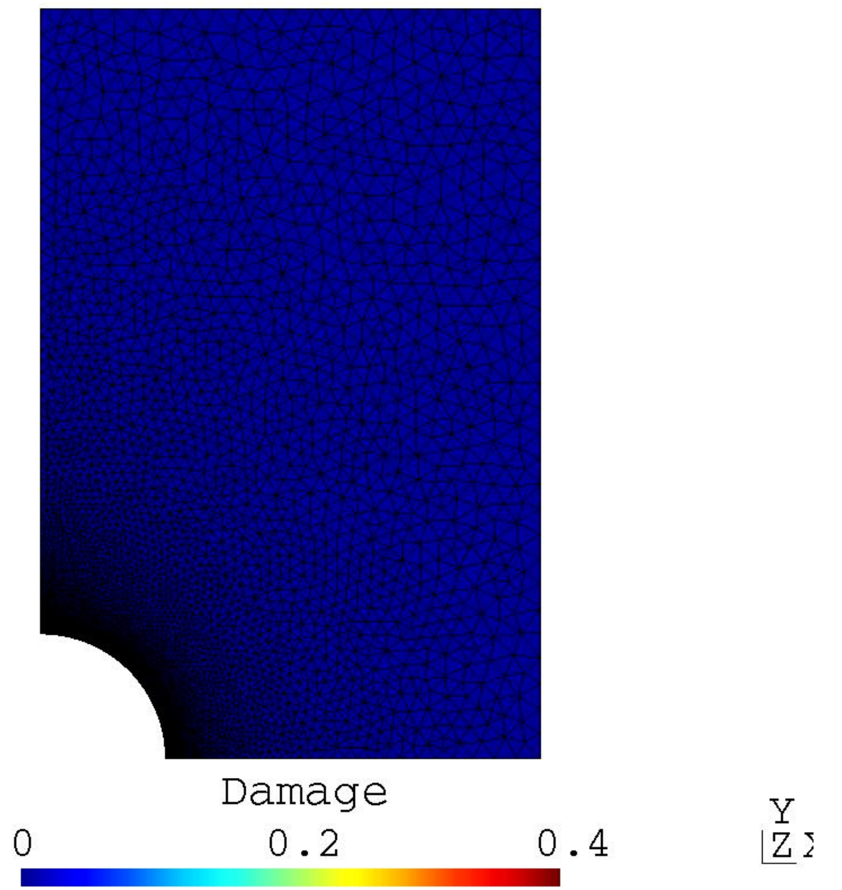




G_c from the literature [*Mazars et al ijss1996*]

- The benchmark shows encouraging perspectives
  - Linear damage theory
  - Fracture criterion $D > D_c$
  - Cohesive shape





G_c from the literature [*Mazars et al ijss1996*]

Future work

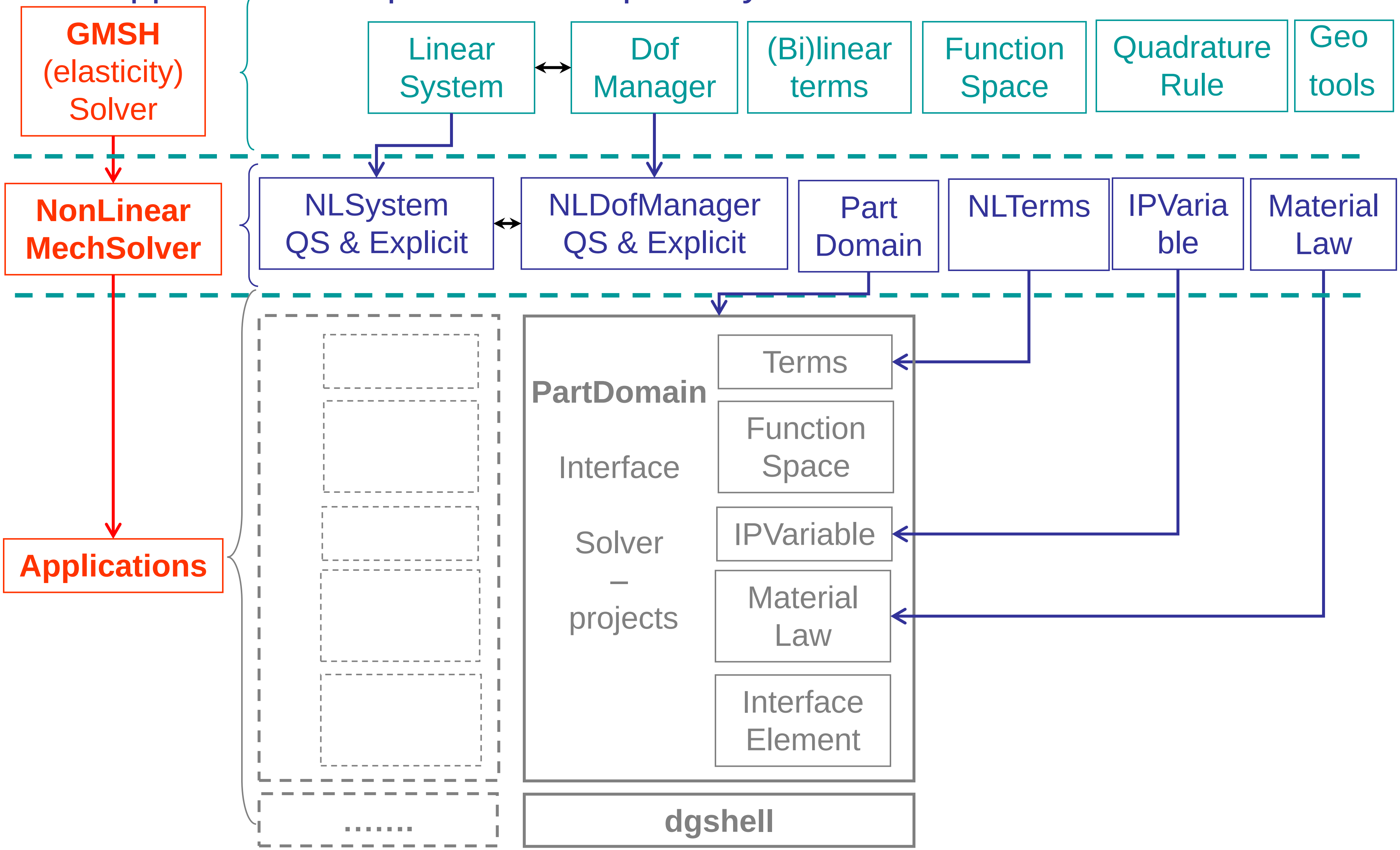
- The benchmark shows encouraging perspectives but many improvements are required
  - Non local damage model
  - Account for stress triaxiality (and out-of-plane shearing)
  - Shape of the cohesive law
  - …

Thank you for your attention

Appendix

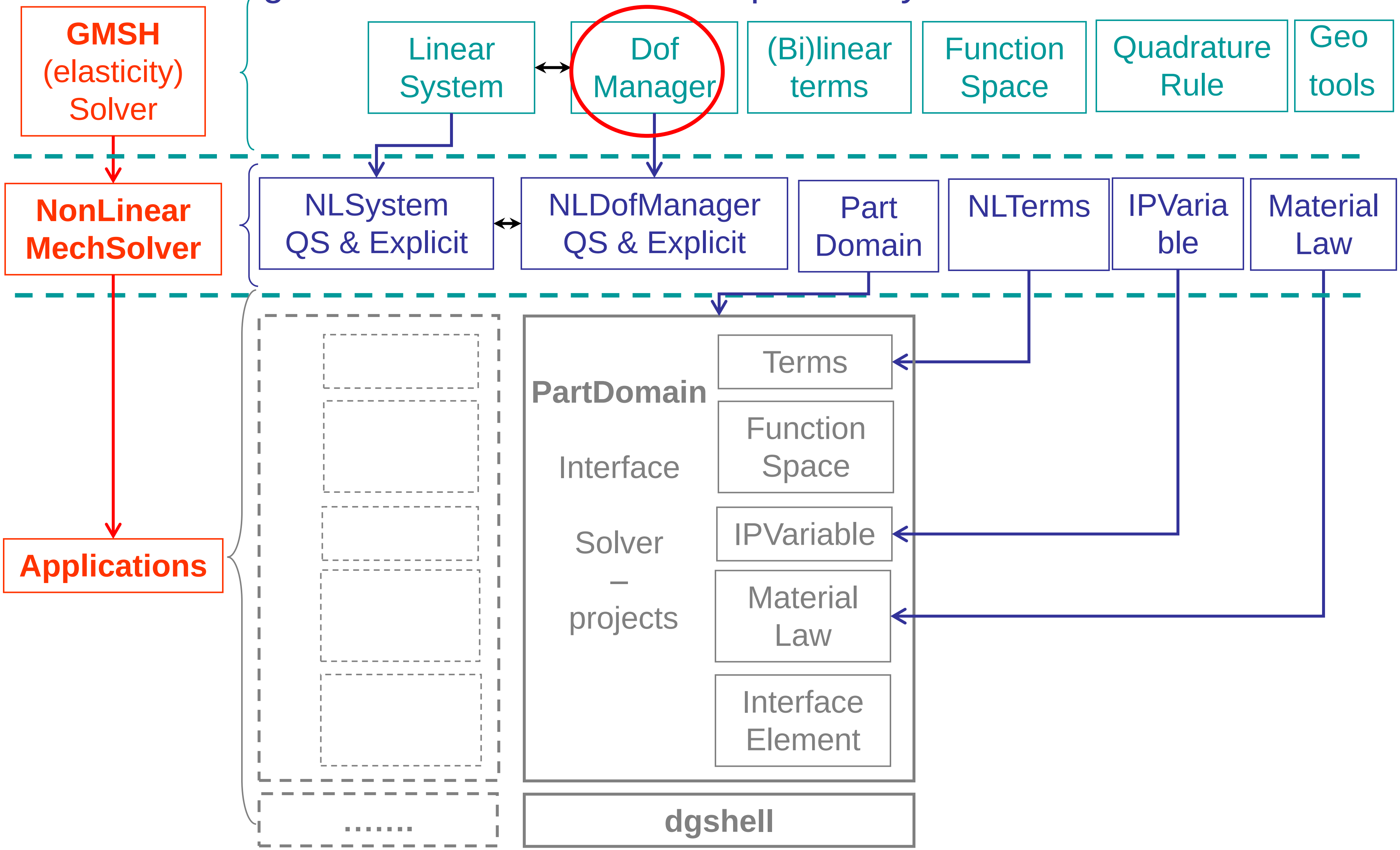
Implementation of the full-DG formulation of Kirchhoff-Love shells

- Application is implemented separately from the solver to be versatile



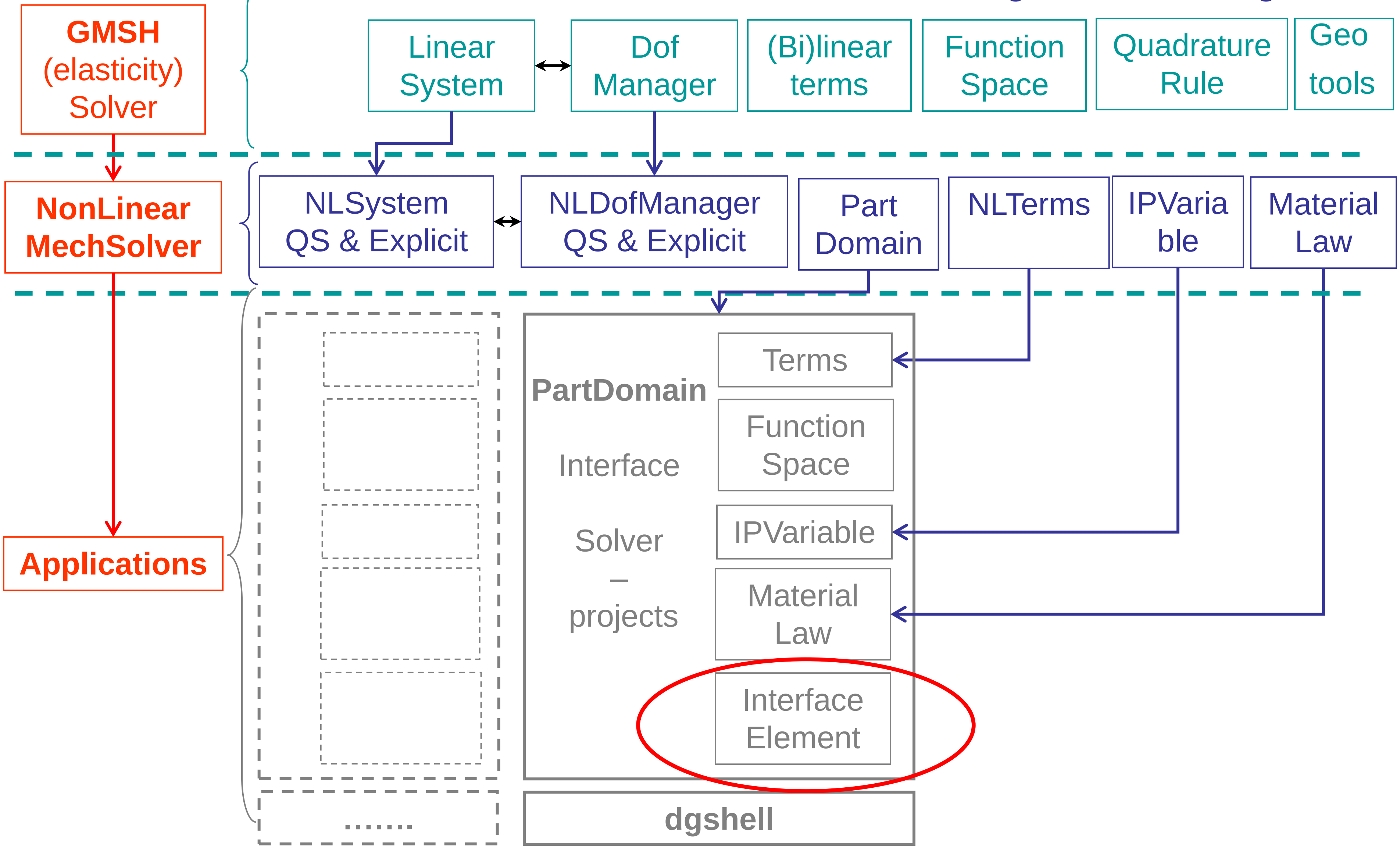
Implementation of the full-DG formulation of Kirchhoff-Love shells

- DofManager allows to define dof independently of the mesh



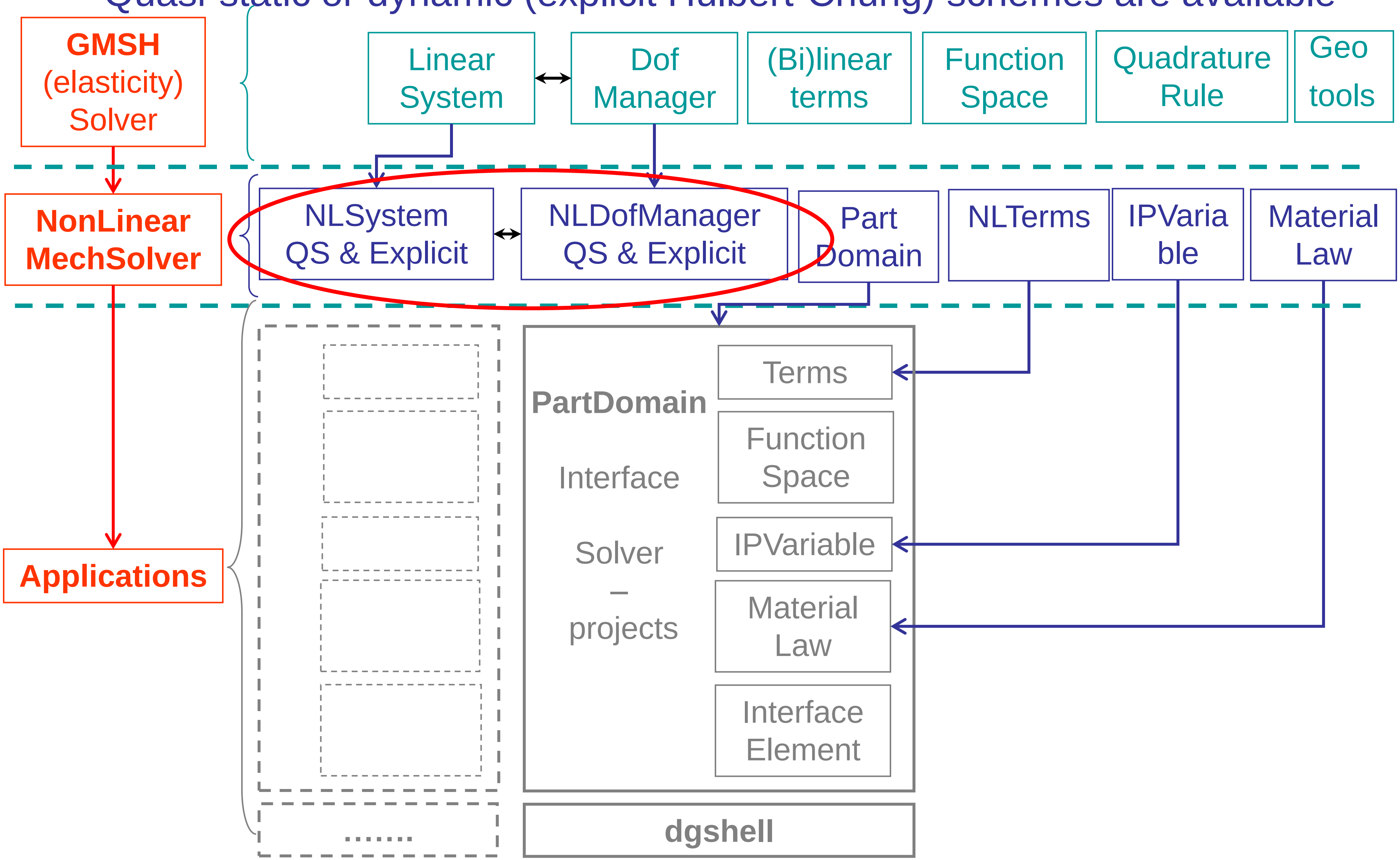
Implementation of the full-DG formulation of Kirchhoff-Love shells

- Continuous mesh is used as interface elements are generated in dgshell



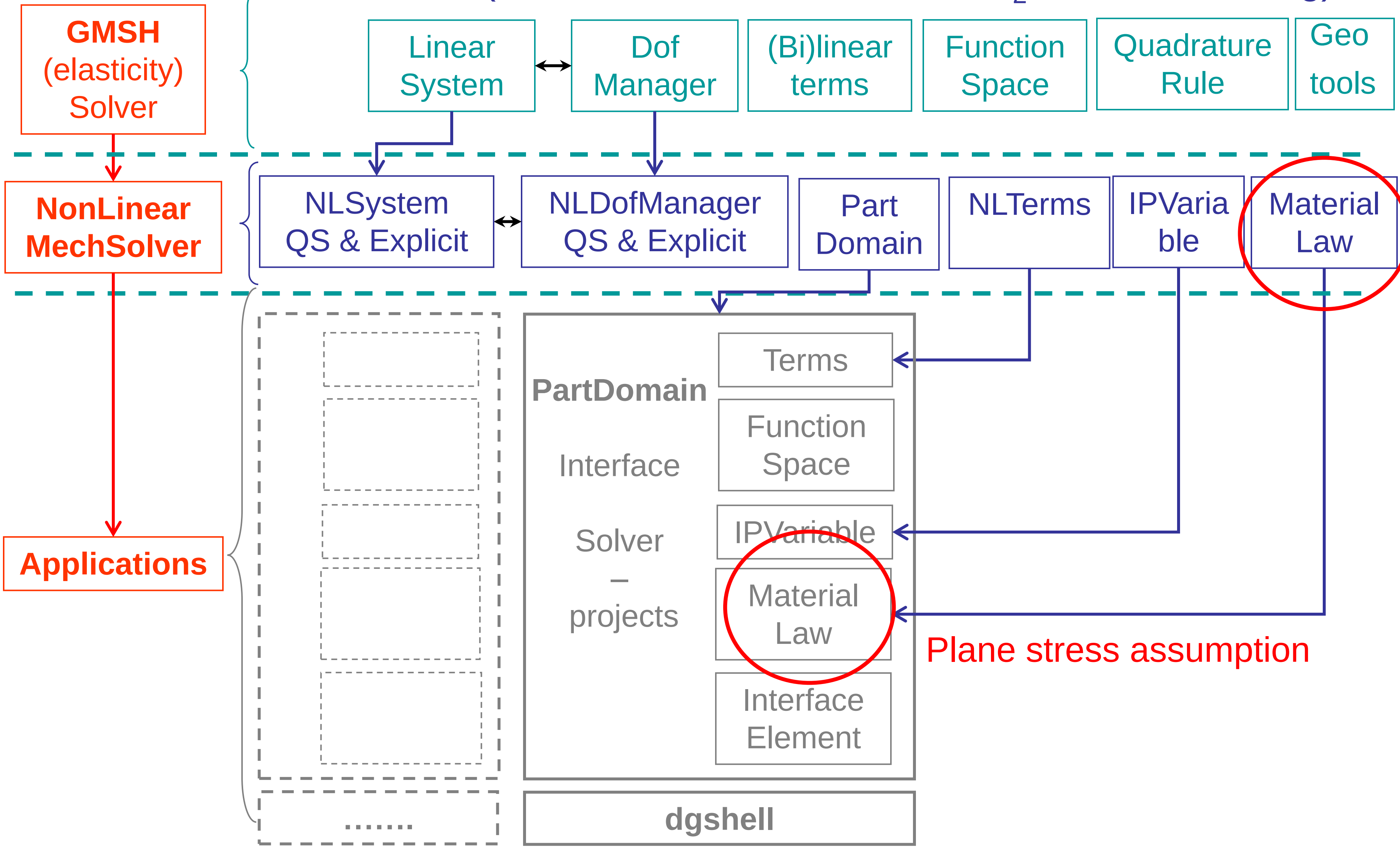
Implementation of the full-DG formulation of Kirchhoff-Love shells

- Quasi-static or dynamic (explicit Hulbert-Chung) schemes are available



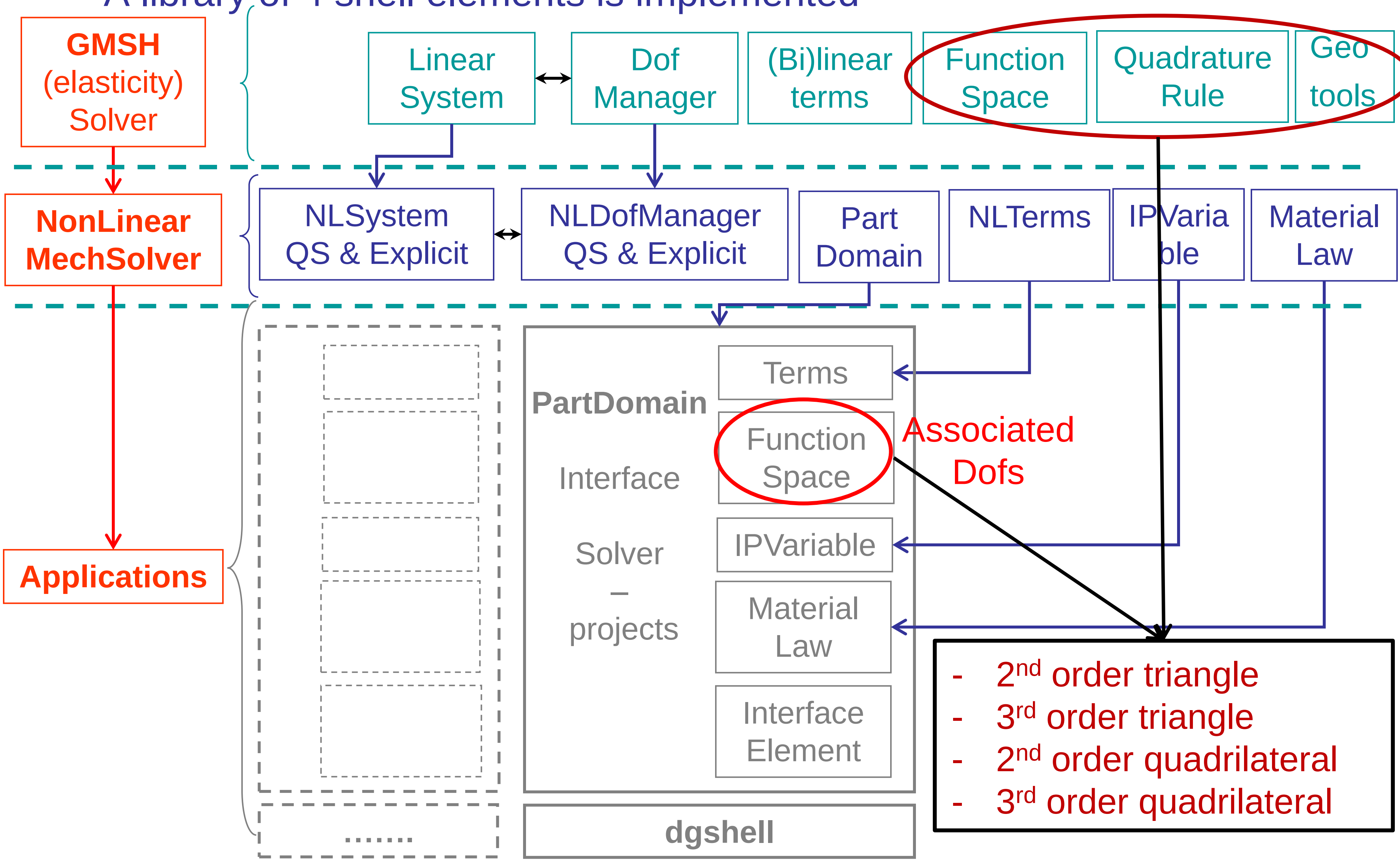
Implementation of the full-DG formulation of Kirchhoff-Love shells

- Different material law (elastic linear, neo-Hookean, J_2-linear hardening)



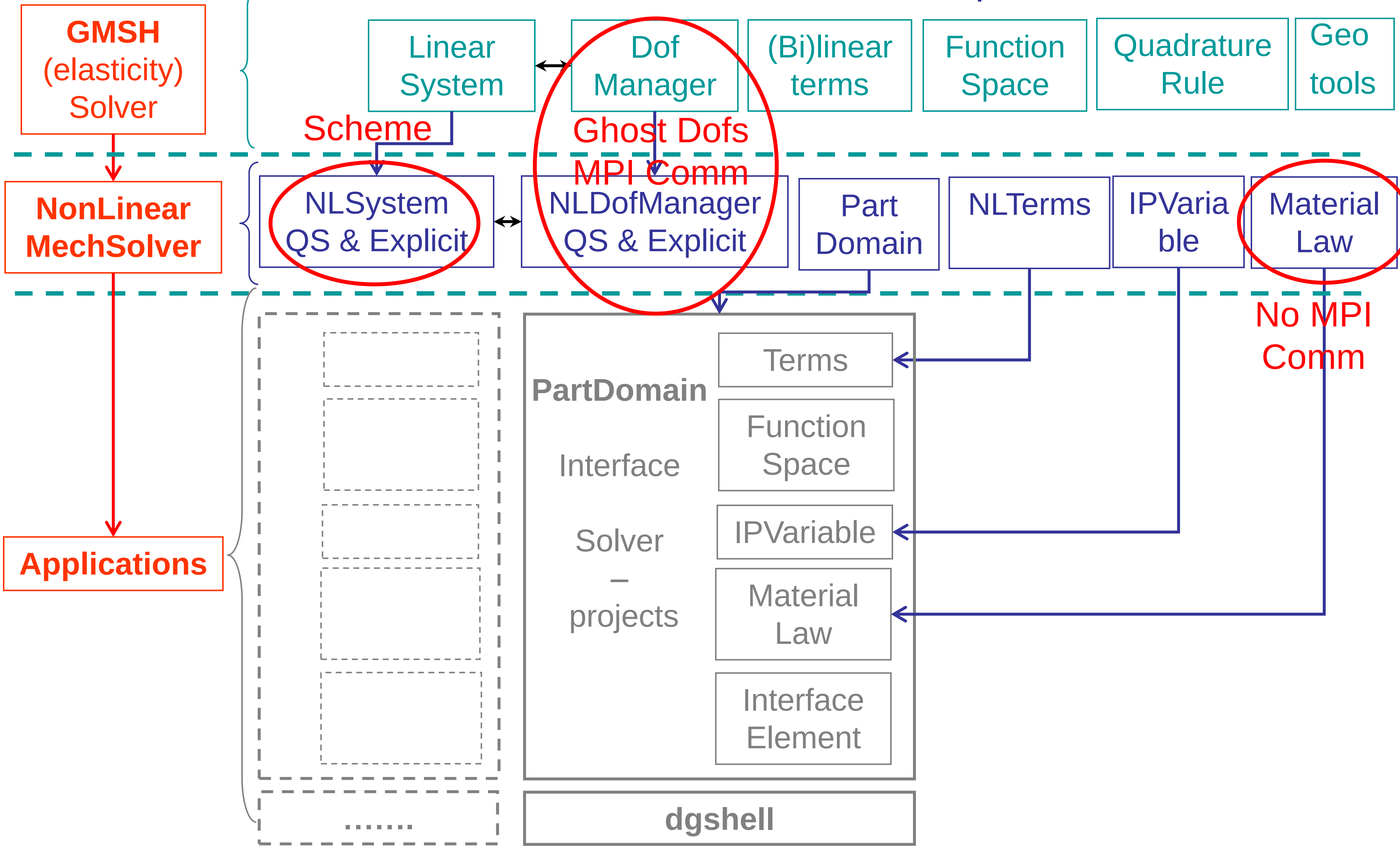
Implementation of the full-DG formulation of Kirchhoff-Love shells

- A library of 4 shell elements is implemented



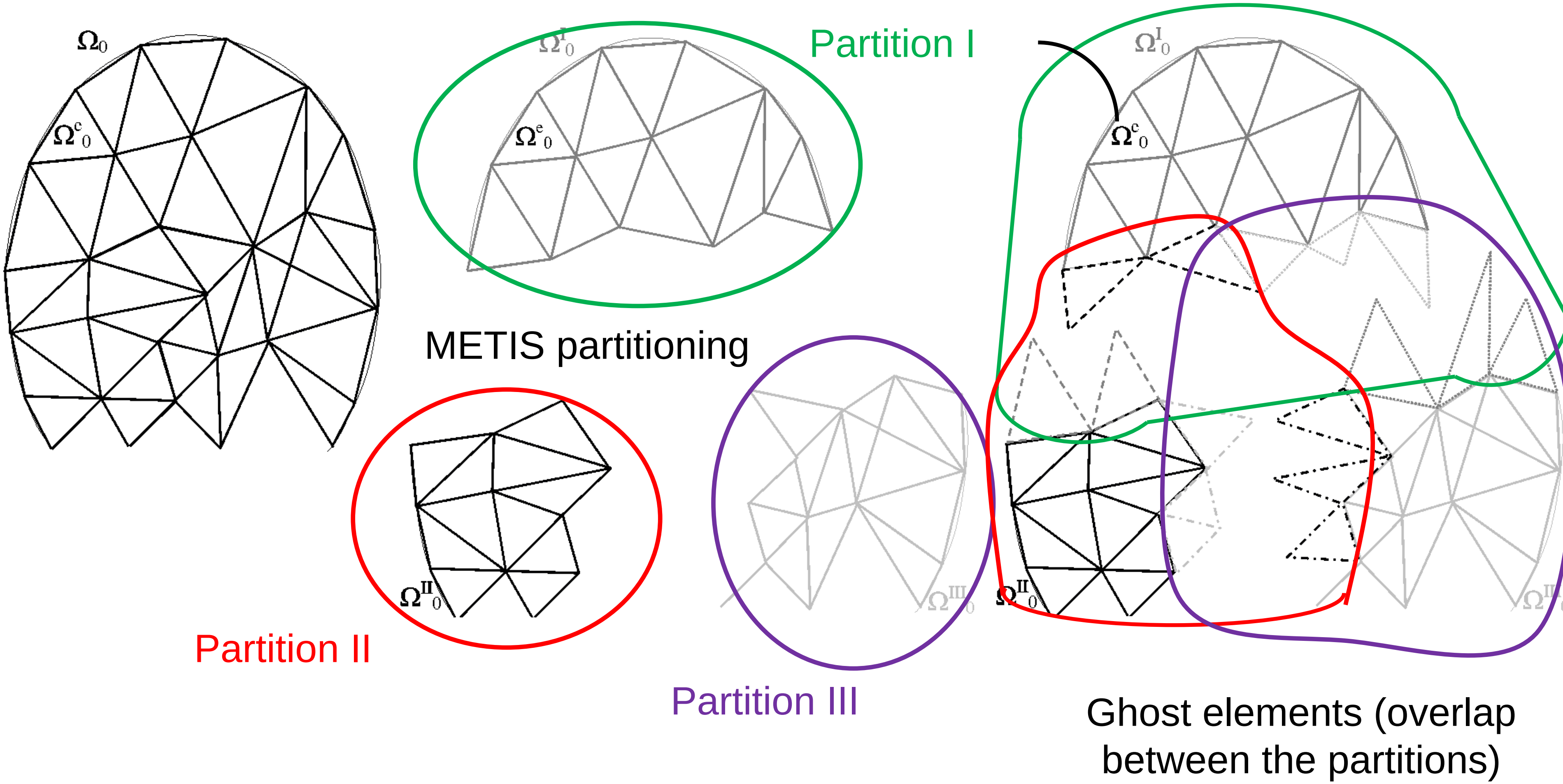
Implementation of the full-DG formulation of Kirchhoff-Love shells

- Parallel scheme is based on Ghost element concept



Parallel implentation of full-DG formulation of Kirchhoff-Love shells

- Ghost elements of a partition are the elements of other partitions which have a common interface with this partition



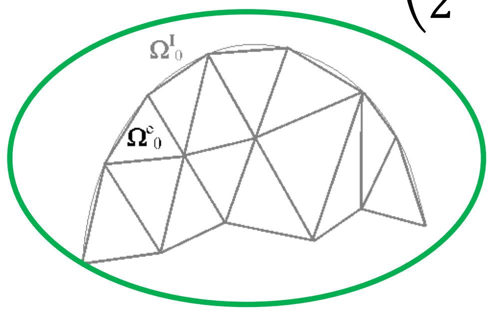
- Solve for the dofs of the elements of the partition linked to the processor

  – $\boldsymbol{M}$ is the diagonalized mass matrix

$$\ddot{\boldsymbol{x}}^{n+1} = \frac{1}{1-\alpha_M}\boldsymbol{M}^{-1}\cdot\left[\boldsymbol{F}^n_{ext} - \boldsymbol{F}^n_{int}\right] - \frac{\alpha_M}{1-\alpha_M}\ddot{\boldsymbol{x}}^n$$

$$\dot{\boldsymbol{x}}^{n+1} = \dot{\boldsymbol{x}}^n + \Delta t(1-\gamma)\ddot{\boldsymbol{x}}^n + \Delta t\gamma\ddot{\boldsymbol{x}}^{n+1}$$

$$\boldsymbol{x}^{n+1} = \boldsymbol{x}^n + \Delta t\,\dot{\boldsymbol{x}}^n + \Delta t^2\left(\frac{1}{2}-\beta\right)\ddot{\boldsymbol{x}}^n + \Delta t^2\beta\ddot{\boldsymbol{x}}^{n+1}$$

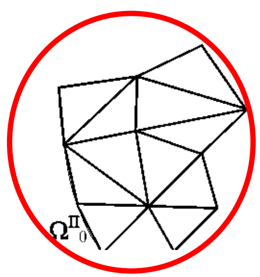


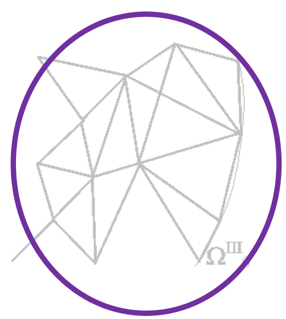
Processor I

$\boldsymbol{x}_{partition\ I}$, $\dot{\boldsymbol{x}}_{partition\ I}$, $\ddot{\boldsymbol{x}}_{partition\ I}$

Processor II

$\boldsymbol{x}_{partition\ II}$, $\dot{\boldsymbol{x}}_{partition\ II}$, $\ddot{\boldsymbol{x}}_{partition\ II}$



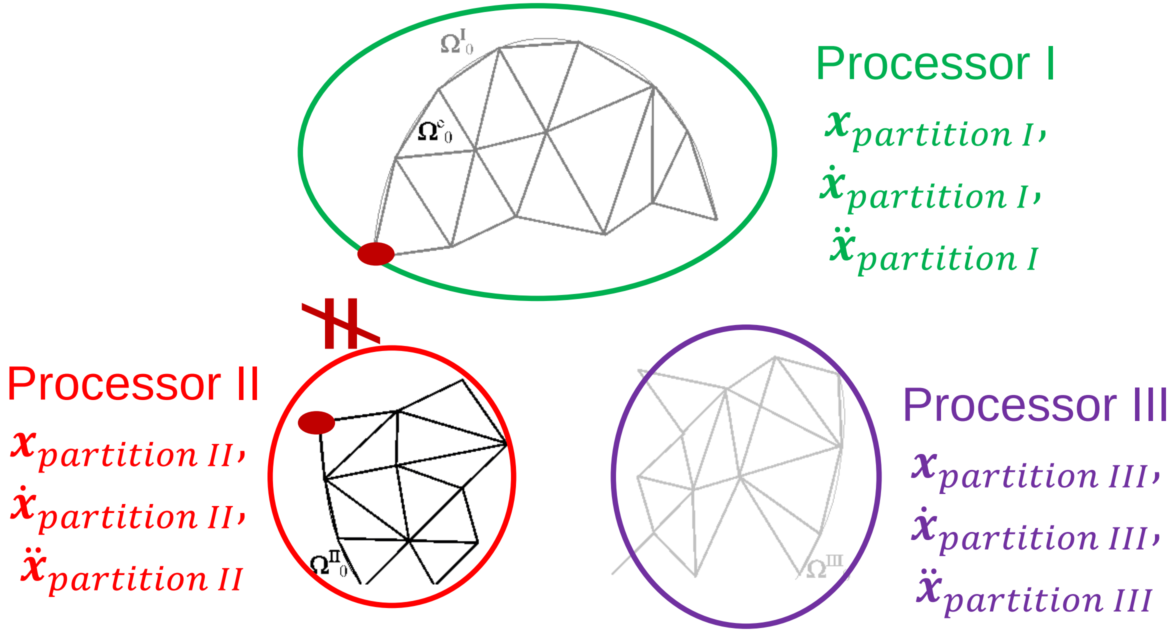


Processor III

$\boldsymbol{x}_{partition\ III}$, $\dot{\boldsymbol{x}}_{partition\ III}$, $\ddot{\boldsymbol{x}}_{partition\ III}$

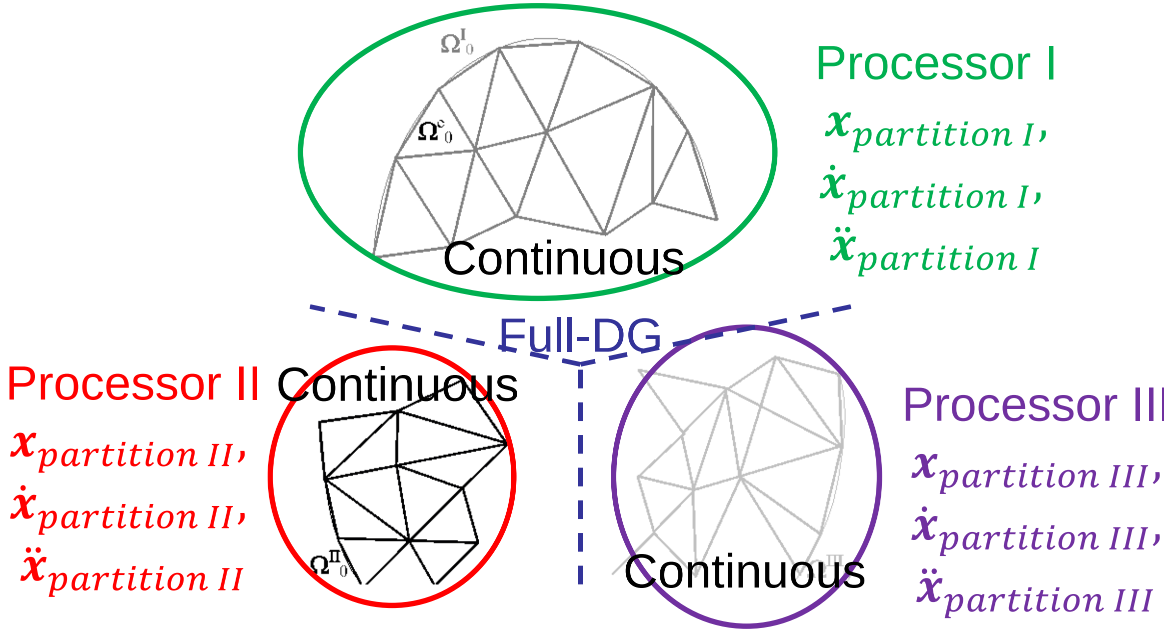
Parallel implentation of full-DG formulation of Kirchhoff-Love shells

- The elements have to be discontinuous between partition



- Recourse to the full-DG formulation between partitions to ensure continuity between them

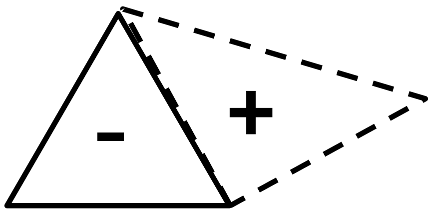
  – Extra dofs are only inserted between partitions

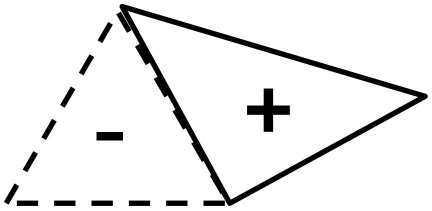


  – Interface elements have to be computed

Parallel implentation of full-DG formulation of Kirchhoff-Love shells

- Interface elements are computed on each partition (using Ghost elements)
  - Processor I
  - Processor II





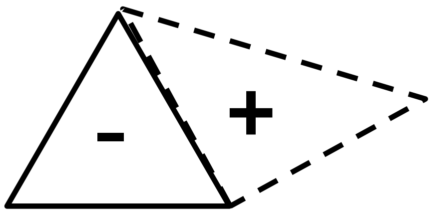
$$\boldsymbol{F}_{int} = \begin{bmatrix} \boldsymbol{F}_{int}^{-} \\ \boldsymbol{F}_{int}^{+} \end{bmatrix}$$

$$\boldsymbol{F}_{int} = \begin{bmatrix} \boldsymbol{F}_{int}^{-} \\ \boldsymbol{F}_{int}^{+} \end{bmatrix}$$

Parallel implentation of full-DG formulation of Kirchhoff-Love shells

- Only the part of $\boldsymbol{F}_{int}$ associated to the dofs of the partition is assembled

  – Processor I

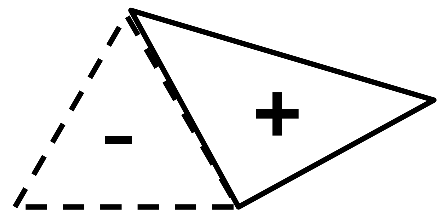


$$\boldsymbol{F}_{int} = \begin{bmatrix} \boldsymbol{F}_{int}^{-} \\ \boldsymbol{F}_{int}^{+} \end{bmatrix}$$

Assembled

Explicit system
(Partition I)

  – Processor II



$$\boldsymbol{F}_{int} = \begin{bmatrix} \boldsymbol{F}_{int}^{-} \\ \boldsymbol{F}_{int}^{+} \end{bmatrix}$$

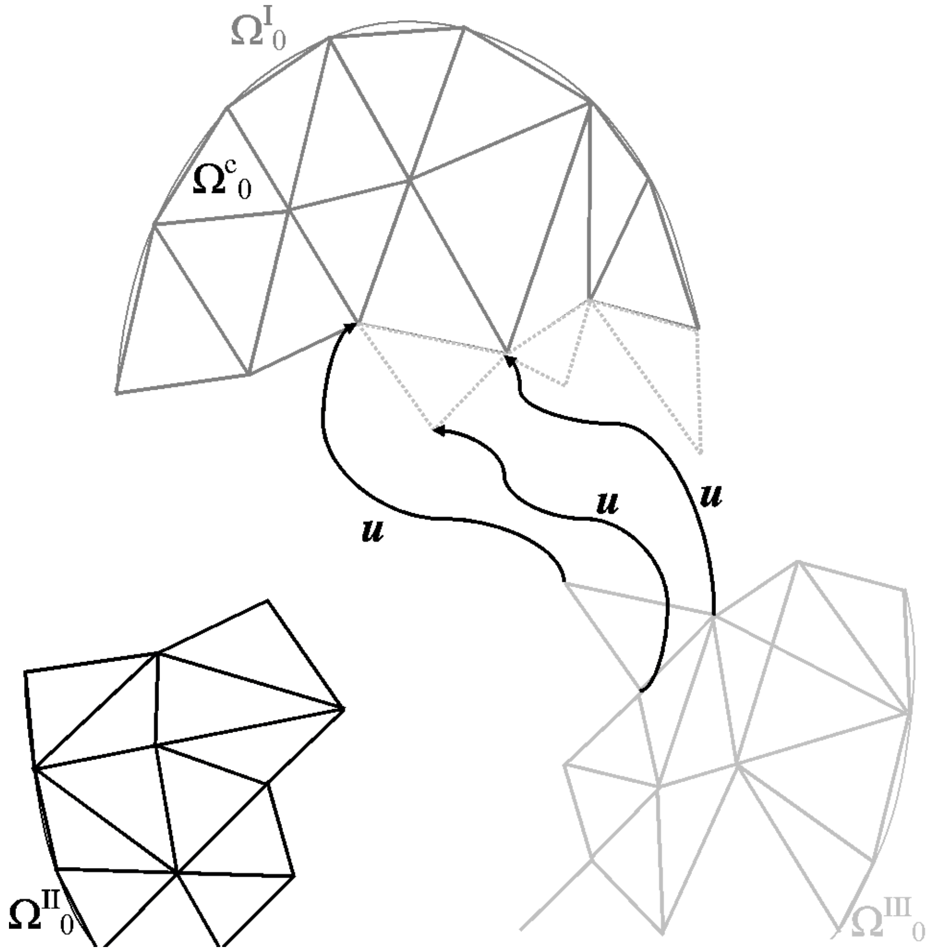
Assembled

Explicit system
(Partition II)

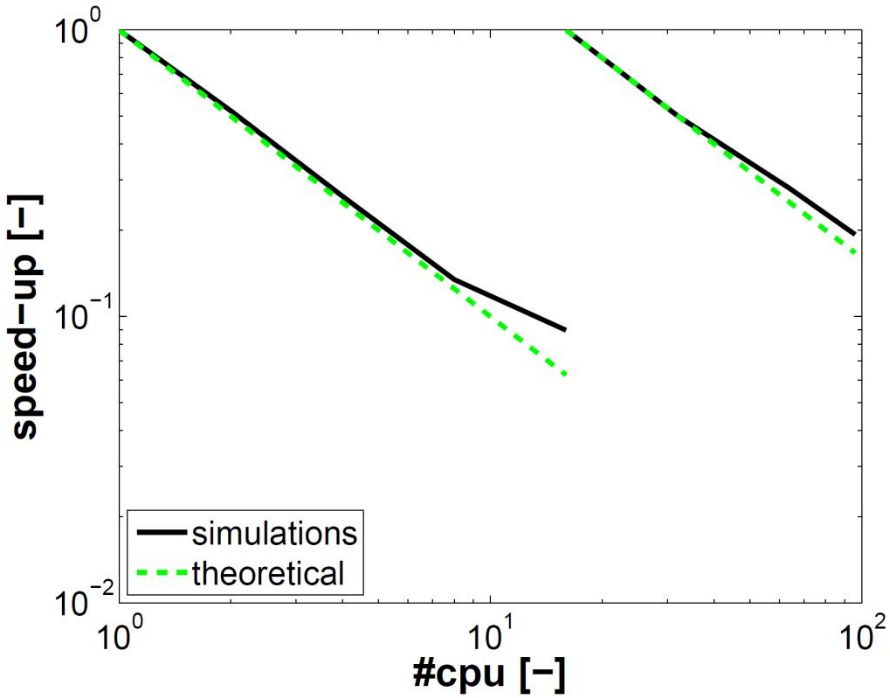
Parallel implentation of full-DG formulation of Kirchhoff-Love shells

- Only one MPI communication is required by time step

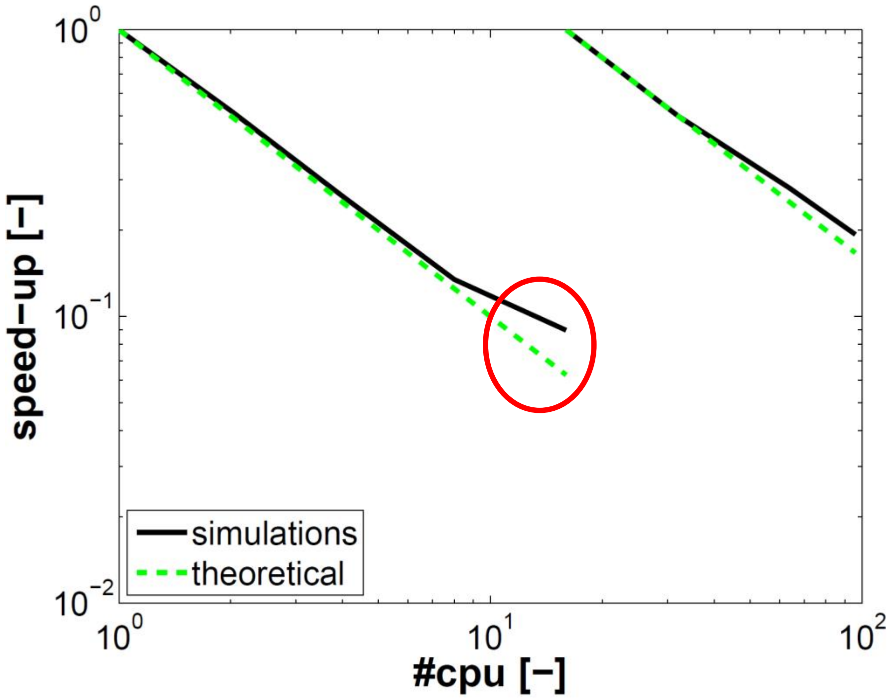
  – Unknowns are exchanged before the computation of F_{int}



- The parallel scheme is almost optimal
  - Theoretical speed-up = time n processor / time 1 processor = 1/n
  - Practically the speed-up is lower than expected (MPI communication)
  - Cylindrical panel benchmark on Nic3 (cluster with 8 cores 2.5Ghz per node)

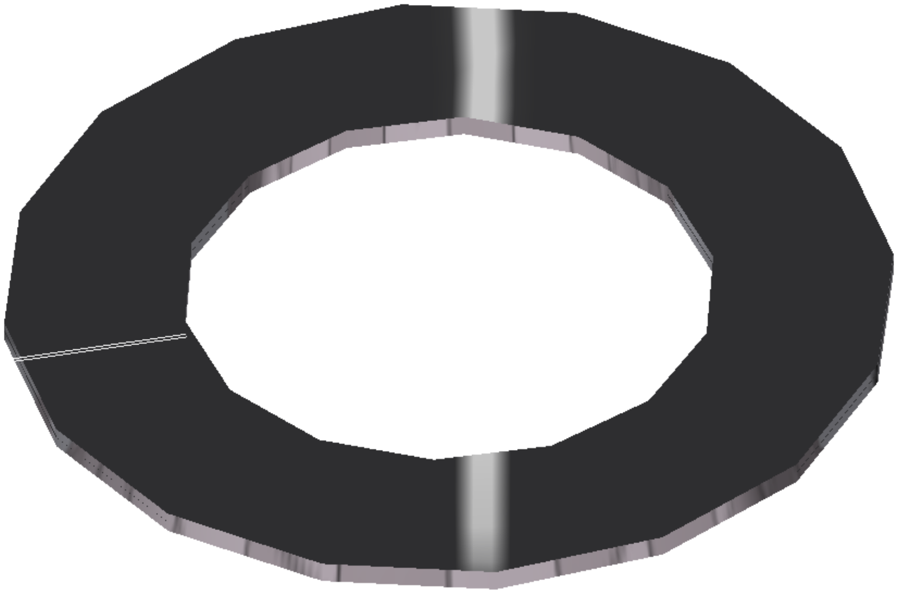


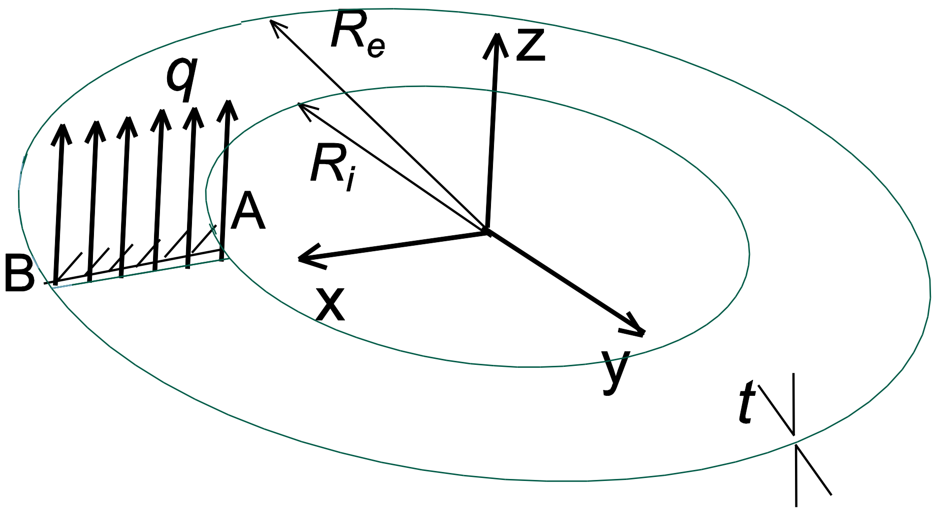
- The parallel scheme is almost optimal when the number of elements remains large compared to the number of interfaces (OK in practice)
  - Theoretical speed-up = time n processor / time 1 processor = 1/n
  - Practically the speed-up is lower than expected (MPI communication)
  - Cylindrical panel benchmark on Nic3 (cluster with 8 cores 2.5Ghz per node)



The number of interfaces is not negligible compared to the number of elements on each partition (±1 interface for 3 elements)

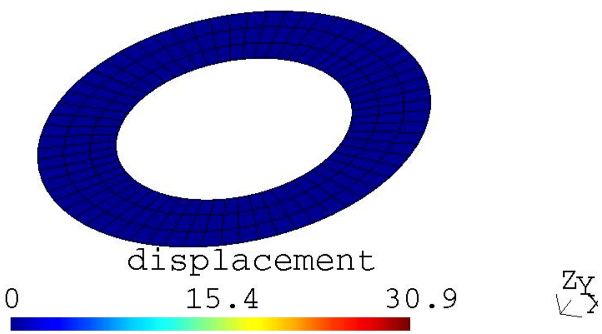
- Neo-Hookean (elastic large deformations) plate ring loaded in a quasi-static way

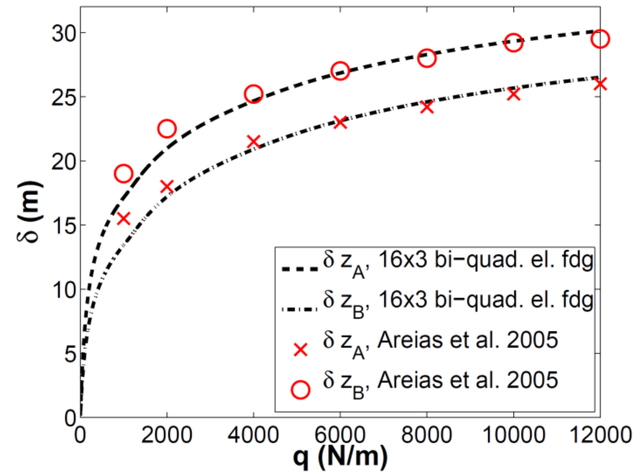




Full-DG formulation extra benchmarks

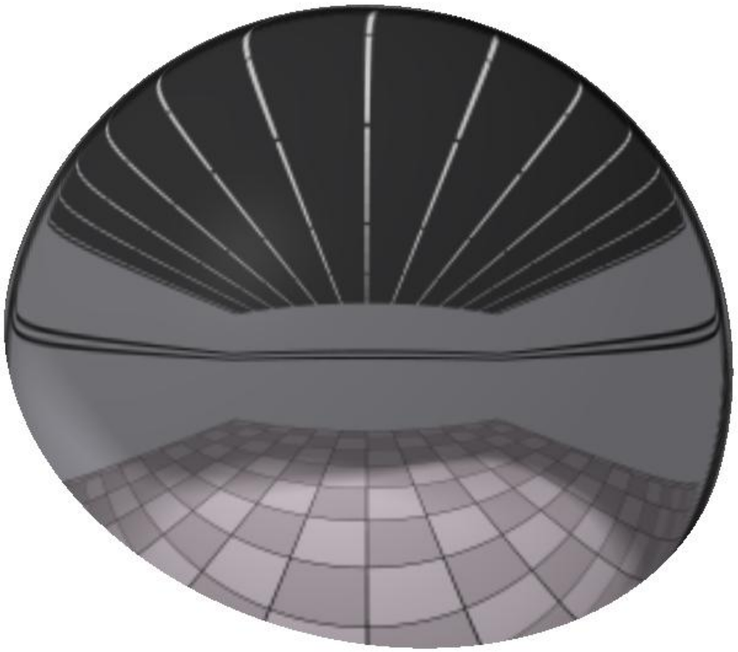
- Neo-Hookean (elastic large deformations) plate ring loaded in a quasi-static way
  - The method gives accurate results even in the case of large distortions

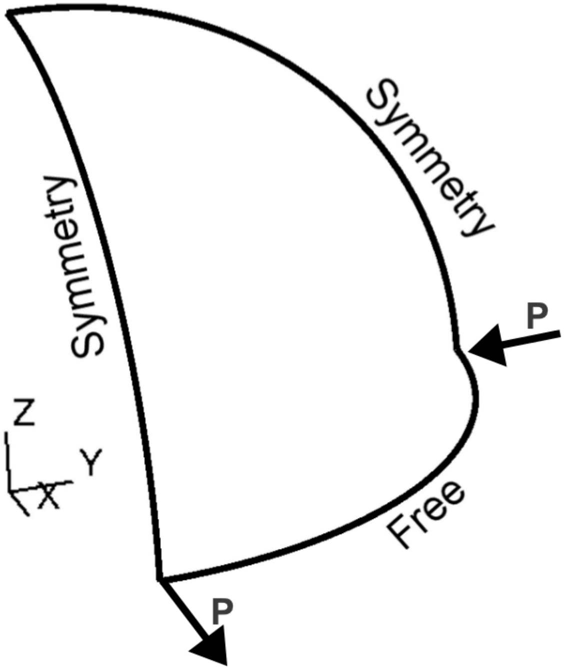




Full-DG formulation extra benchmarks

- J_2-linear hardening (elasto-plastic large deformations) hemisphere loaded in a quasi-static way





- J_2-linear hardening (elasto-plastic large deformations) hemisphere loaded in a quasi-static way
  - Same results for 2nd and 3rd order triangles