Cuts from two rows of the simplex tableau

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Joint work with K. Andersen, R. Weismantel, L. Wolsey

Cutting plane generation in mixed-integer programming

The mixed-integer programming problem

$$\begin{aligned} \max\ & cx + dw \\ \text{s. t. } & Ax + Gw \leq b \\ & x \in \mathbb{Z}^{n_1}, w \in \mathbb{R}^{n_2} \end{aligned}$$

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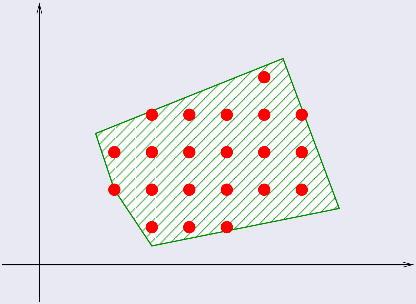

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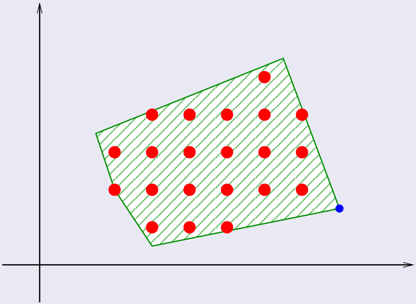

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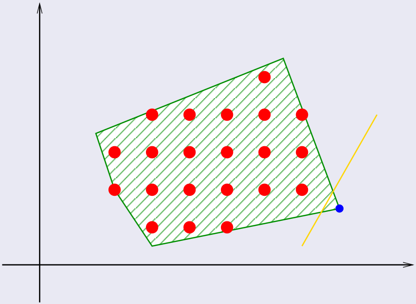

Central question : generation of cutting planes

1. Methods depending on the structure of the problem
   $\longrightarrow$ network problems, TSP, ...
2. General methods
   - Gomory mixed-integer cut (or MIR)

$$X = \{(x, y) \in \mathbb{Z} \times \mathbb{R}_+ | x + y \geq b\}$$

The only missing inequality is

$$x + \frac{1}{1 - \mathcal{F}(b)} y \geq \lceil b \rceil$$

   - Split cut
     Generalization of the Gomory cuts

All based on the information of one constraint or several constraints aggregated in one inequality.

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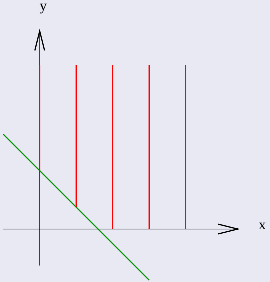

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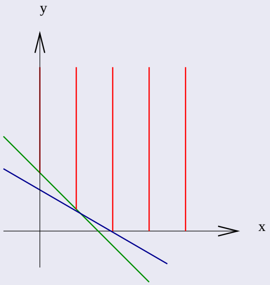

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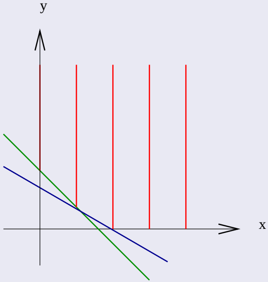

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Split cuts

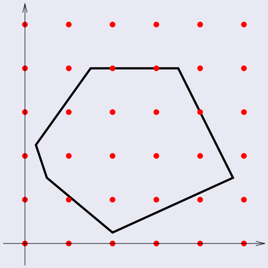
The geometry

Theoretical results

- Every cut that can be obtained from one row or from the aggregation in one row is a split cut.
  $\longrightarrow$ Very general class
- Includes MIR which is commonly used in commercial softwares.
- Using split cuts only does not guarantee to converge to the convex hull in finite time.

Split cuts

The geometry



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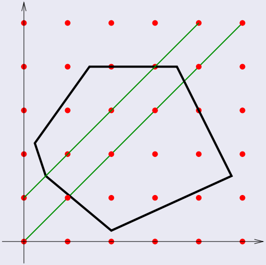

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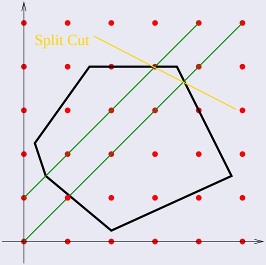

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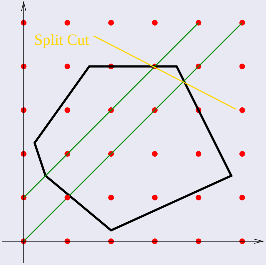

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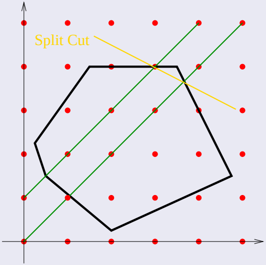

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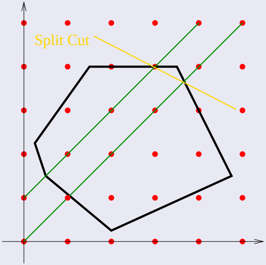

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Fundamental problem with two constraints

The simplex tableau

$$x_1 - \bar{a}_{11} s_1 - \cdots - \bar{a}_{1n} s_n = \bar{b}_1$$

$$\vdots$$

$$x_m - \bar{a}_{m1} s_1 - \cdots - \bar{a}_{mn} s_n = \bar{b}_m$$

- Select two rows
- Relax the integrality requirements of the non-basic variables
- Relax the nonnegativity requirements of the basic variables

The model

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} + \sum_{j=1}^{n} \begin{pmatrix} r_1^j \\ r_2^j \end{pmatrix} s_j$$

$$x_1, x_2 \in \mathbb{Z}, s_j \in \mathbb{R}_+$$

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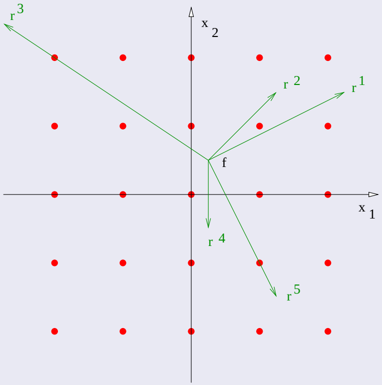
Fundamental problem with two constraints

The geometry

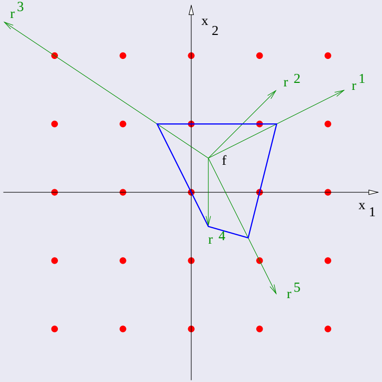
$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1/4 \\ 1/2 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} s_1 + \begin{pmatrix} 1 \\ 1 \end{pmatrix} s_2 + \begin{pmatrix} -3 \\ 2 \end{pmatrix} s_3 + \begin{pmatrix} 0 \\ -1 \end{pmatrix} s_4 + \begin{pmatrix} 1 \\ -2 \end{pmatrix} s_5$$



Fundamental problem with two constraints

Graphic representation of a facet

$$2s_1 + 2s_2 + 4s_3 + s_4 + \frac{12}{7}s_5 \geq 1$$



Fundamental problem with two constraints

Basic properties of the polyhedron

$$\text{conv}(P_I) = \text{conv}\{(x,s) \in \mathbb{Z}^2 \times \mathbb{R}^n_+ \mid \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} + \sum_{j=1}^{n} \begin{pmatrix} r^j_1 \\ r^j_2 \end{pmatrix} s_j \}$$

1. The dimension of the polyhedron is n (when nonempty)
2. The vectors (r^j, e_j) are extreme rays of the polyhedron
   $\rightarrow$ *If the rays are rational, one solution can always be extended*
3. Every facet can be written in the form
   $$\sum_{j=1}^{n} \alpha_j s_j \geq 1,$$
   with $\alpha_j \geq 0$.
4. Extreme point = One integer point and two non-zero rays

Fundamental problem with two constraints

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Fundamental problem with two constraints

Main results

1. Every facet is tangent to exactly 3 or 4 important integer points
2. Every facet is completely determined by exactly 3 or 4 rays
3. Classification of the facets in 3 categories :
   disection cuts, lifted 2 variable-cuts, split cuts

   The facets from the first 2 categories are never split cuts
4. Combinatorial algorithm to compute the extreme points in polynomial time

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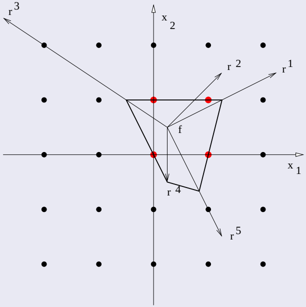

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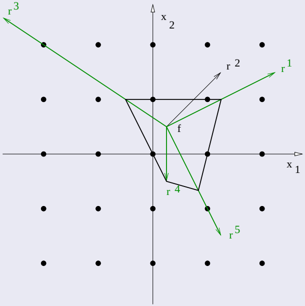

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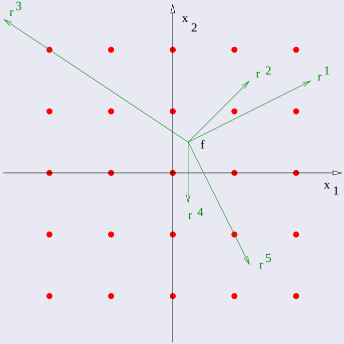
The projection picture

$$2s_1 + 2s_2 + 4s_3 + s_4 + \frac{12}{7}s_5 \geq 1$$

- We project the $n + 2$-dim space onto the x-space
- The facet is represented by a polygon L_α
- There is no integer point in the interior of L_α
- The coefficients are a ratio of distances on the figure



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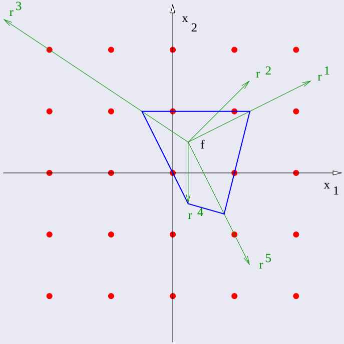

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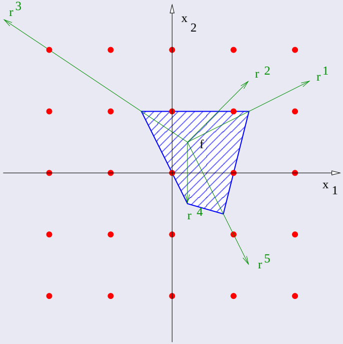

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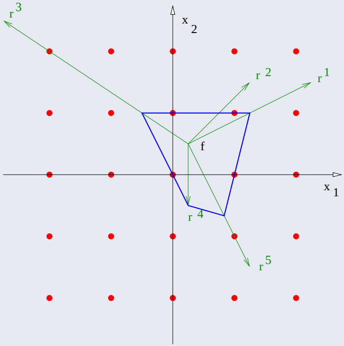

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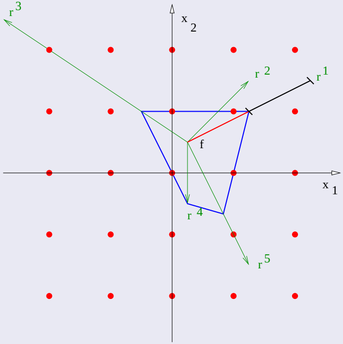

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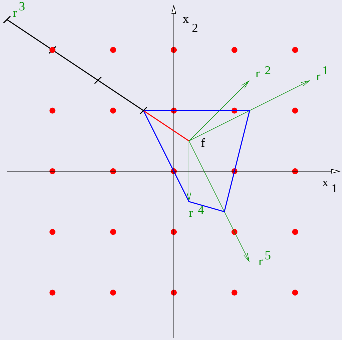
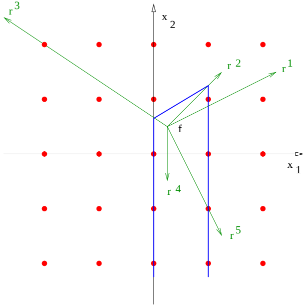


Facets with a 0 coefficient

Facets with a 0 coefficient are actually split cuts.

$$\frac{8}{3}s_1 + \frac{4}{3}s_2 + 12s_3 + 0s_4 + \frac{4}{3}s_5 \geq 1$$

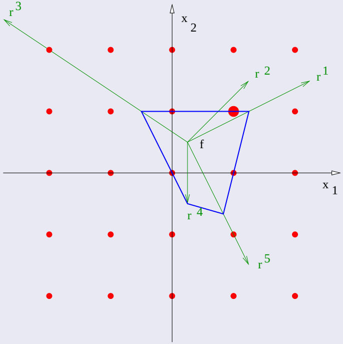


There cannot be a facet with two 0-coefficients unless two rays are parallel.

The projection picture

$$2s_1 + 2s_2 + 4s_3 + s_4 + \frac{12}{7}s_5 \geq 1$$

- What about tight integer points ?
- It can be represented as a minimal combination of two s_j in several cones cone$\{r^1, r^3\}$ cone$\{r^2, r^5\}$
- In cone$\{r^1, r^3\}$ it has a representation that is tight wrt. the facet
- In cone$\{r^2, r^5\}$ it has not
- In total there are four tight integer points (in some representation)
- By maximality the vertices of L_α are on the rays



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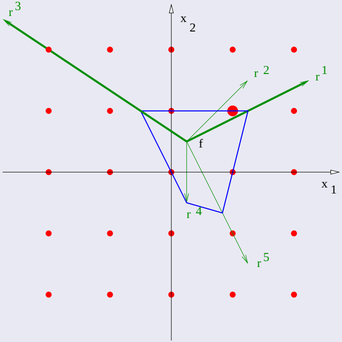

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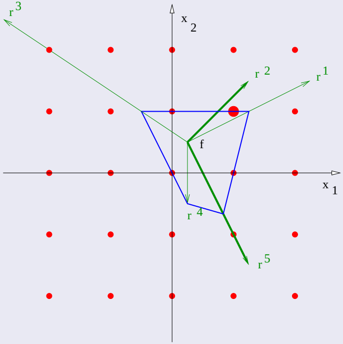

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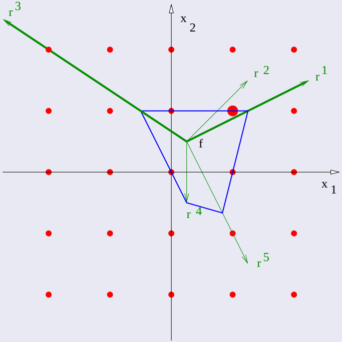

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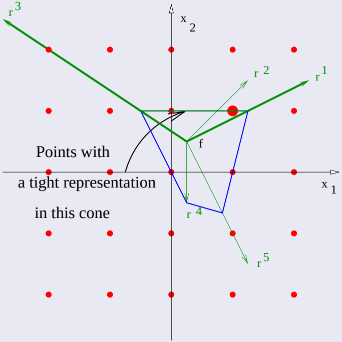

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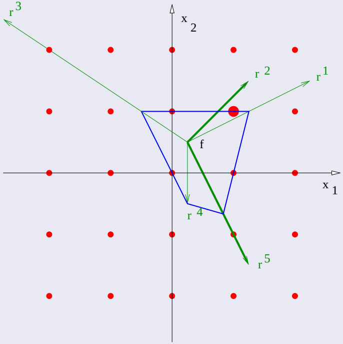

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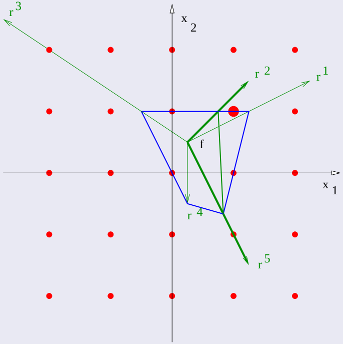

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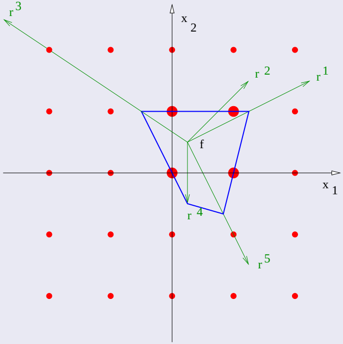

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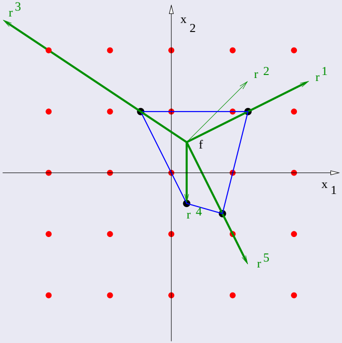

Polar

Why three or four integer points?

Theorem

Let P be a convex polygon (2D). If the vertices of P are integer and if P has no integer point in its interior, then P has at most four vertices.

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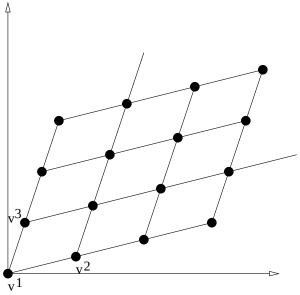

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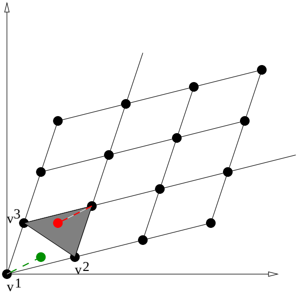

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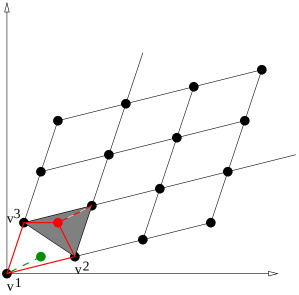

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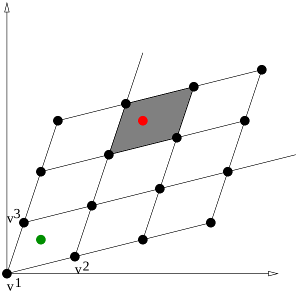

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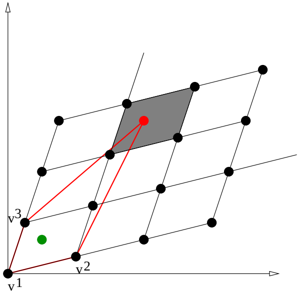

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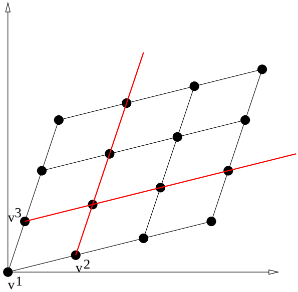

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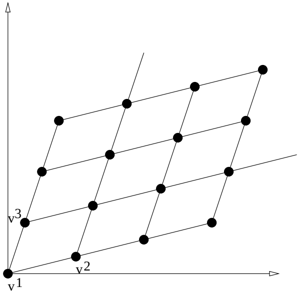

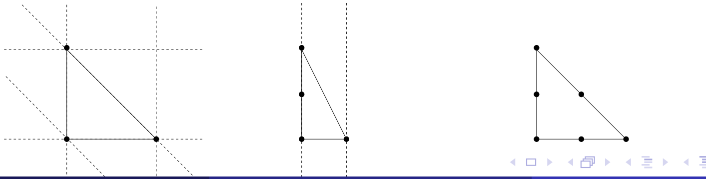

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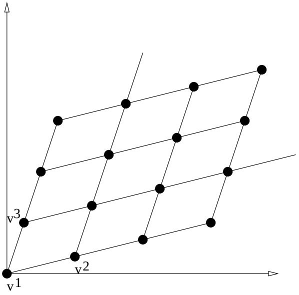

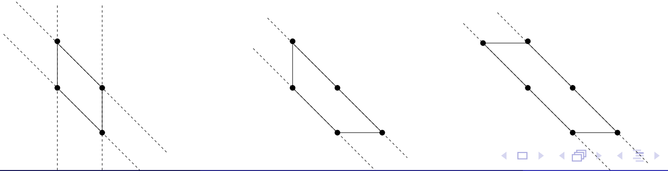

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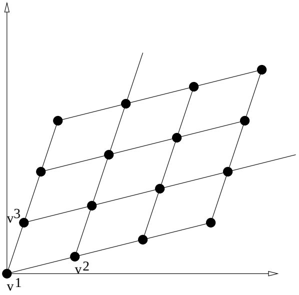

No split cuts

Why three or four rays ?

- There exists a minimal system that determines the facet
- Every tight integer point provides an equality

$$\alpha_i \gamma_i + \alpha_j \gamma_j = 1$$

in one tight representation, where (γ_i, γ_j) is the representation of the point in the cone (r^i, r^j).

- All other tight representations are automatically satisfied
- To have the minimally uniquely determined system
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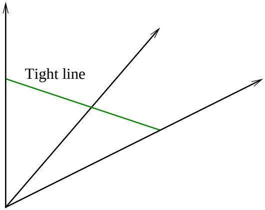

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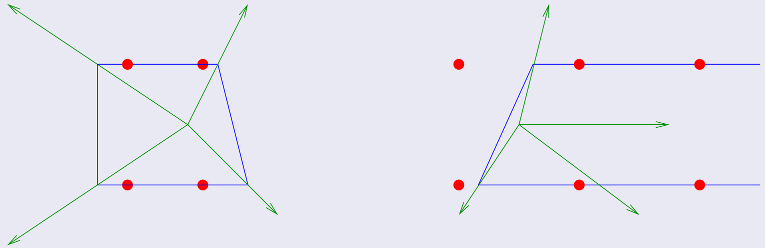
Classification of the facets

Split cuts



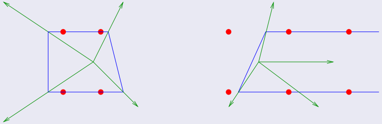
Disection cuts and Lifted 2-var cuts

These are never split cuts !

An important pathological case

Classification of the facets

Split cuts



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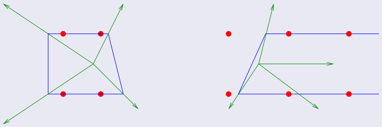

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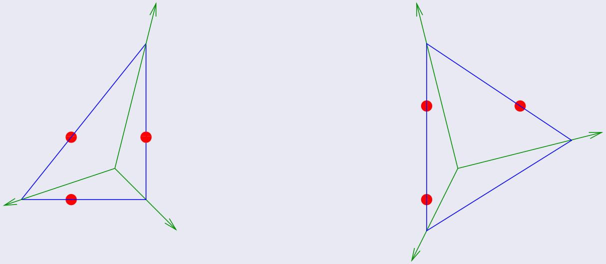

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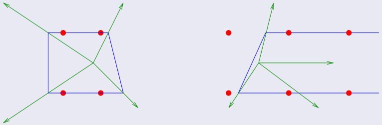

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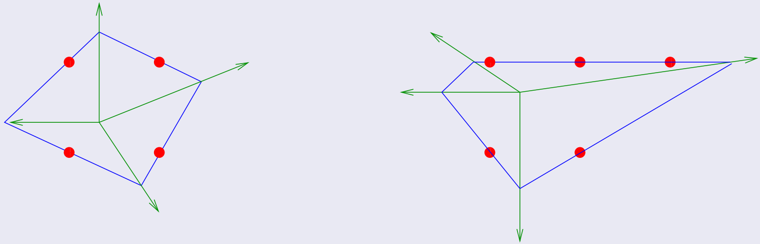

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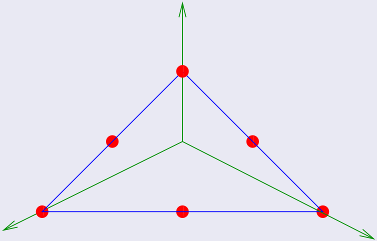

Simultaneous lifting by reading the coefficients on the figure

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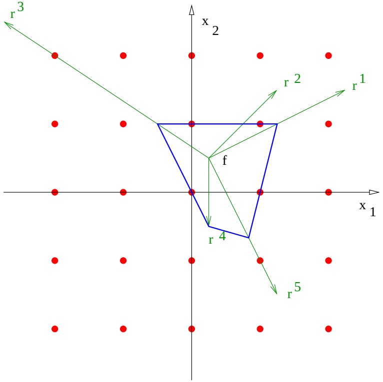

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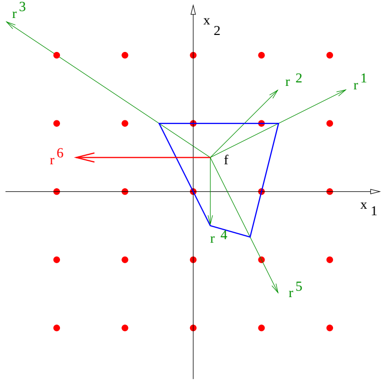

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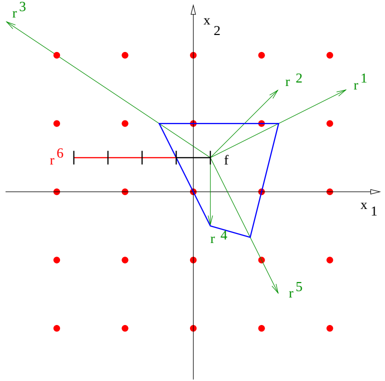

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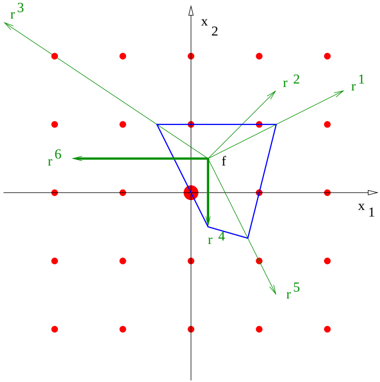
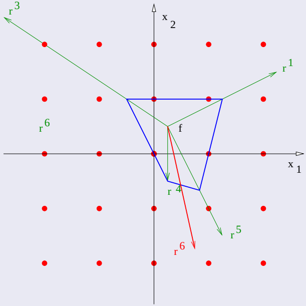


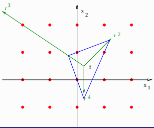
It is a facet !

Lifted 2 variable cuts



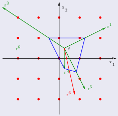
The lifted valid inequality is not a facet !

Disection cuts are universally liftable



Issues related to lifting

Lifted 2 variable cuts



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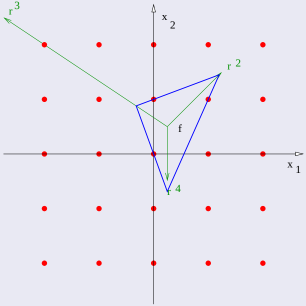

Remaining questions

Towards practice ?

1. General principle to generate cuts
   - Choose 3 or 4 integer points
     Which points ?
   - Choose 3 or 4 rays
   - Compute the cut
2. Important related questions
   - Strengthen an inequality by considering the integrality of a ray ?
   - Bounds on x and on y
   - Generalization to several constraints
     Includes a generalization of the split to a d-dimensional body with no integer point in its interior

Finite convergence for a cutting plane algorithm

1. Which black box is necessary to converge in finite time ?
2. Problem with m constraints $\longrightarrow$ black box with m constraints ?

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