A Perturbation Finite Element Technique for Modeling Electrostatic MEMS

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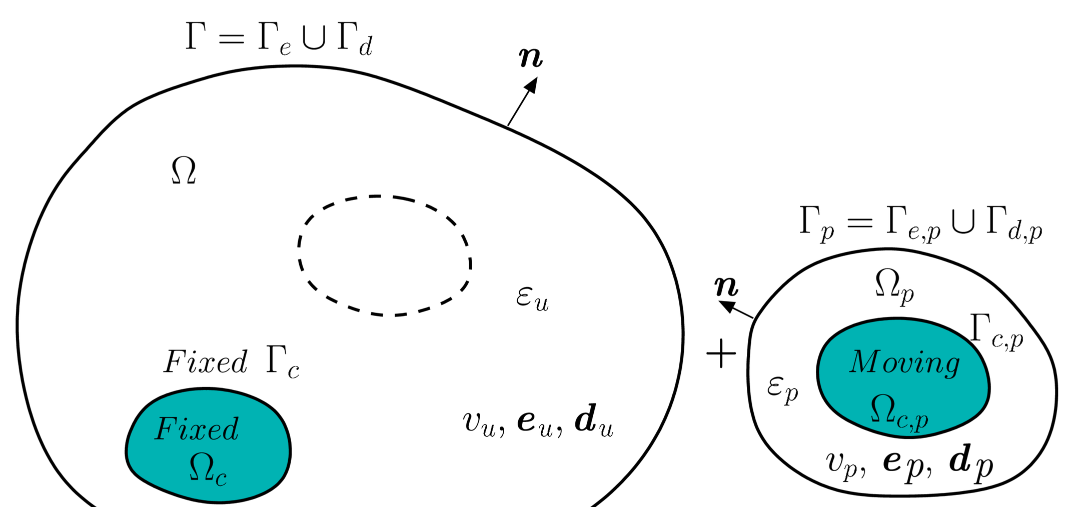
Modeling Electrostatic MEMS

- Lumped spring-mass models
  - helpful for physical insight
  - neglect important effects such as bending of plates and fringing field effects
- Finite element (FE) method
  - adapted for complex geometries
  - fringing field effects accurately computed at the expense of dense discretization near corners
  - movement modeling often requires a completely new computation and mesh for each position
  - computationally expensive
- Perturbation finite element method (PFEM)
  - unperturbed field computed in a global domain without the presence of some conductive regions
    - ✔ no mesh of perturbing domains
    - ✔ possible symmetries or analytical solution
    - ✔ source to the perturbation problems
  - perturbed field determined in a local domain
    - ✔ subproblem-adapted meshes
    - ✔ no degenerated elements when some moving regions undergo critical deformations
- Iterative sequence of perturbation problems
  - needed when significant coupling between sub-regions
    - ✔ successive perturbations in each region are computed from one region to the other
    - ✔ each sub-problem gives a correction
    - ✔ the Aitken acceleration diminishes the number of iterations

Perturbation Finite Element Method



Decomposition of the classical electric problem defined in domain Ω into **unpetrubed** (left) and **perturbation** problems (right)

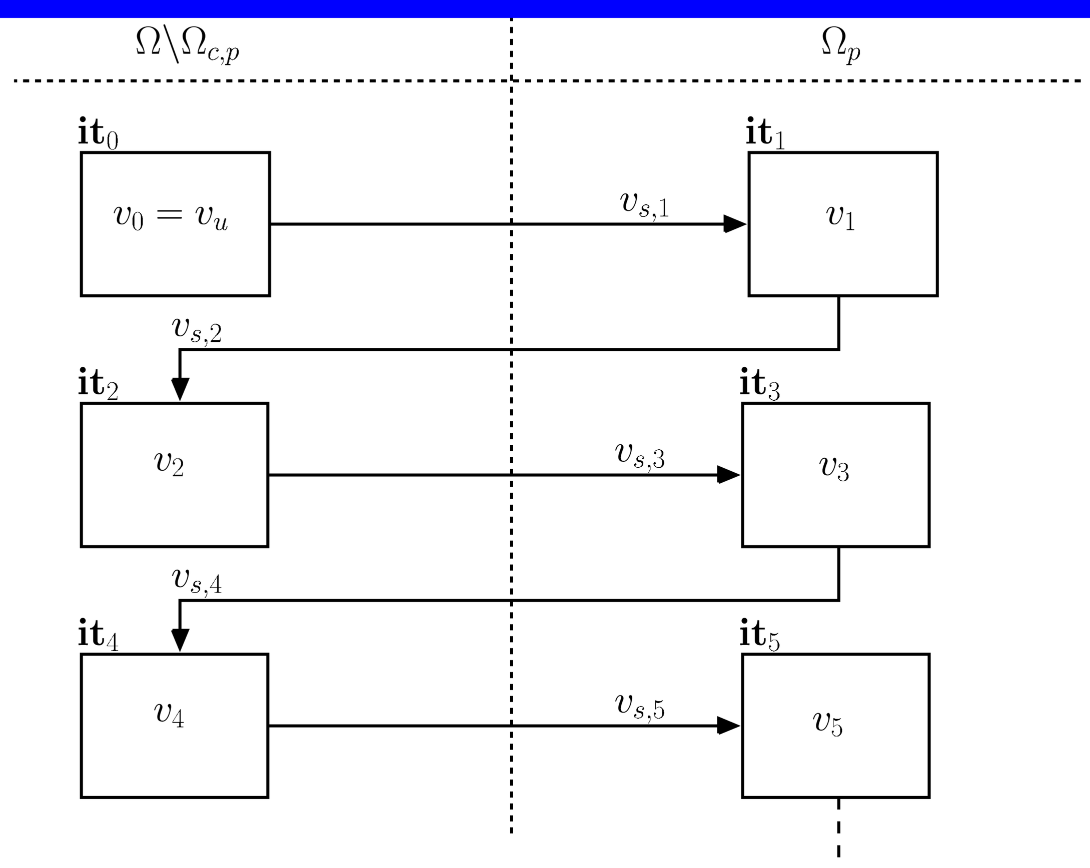
- Electric scalar potential formulation
  - **Unperturbed problem**
    - ✔ $(-\varepsilon \operatorname{grad} v_u, \operatorname{grad} v')_\Omega - \langle \mathbf{n}\cdot \boldsymbol{d}_u, v'\rangle_{\Gamma_d} = 0$

      $\boldsymbol{e}_u \times \mathbf{n}|_{\Gamma_e} = 0 \quad \& \quad \mathbf{n}\cdot \boldsymbol{d}_u|_{\Gamma_d} = 0$
  - **Perturbation problems**
    - ✔ $\langle \operatorname{grad} v_s, \operatorname{grad} v'\rangle_{\Gamma_{c,p}} - \langle \operatorname{grad} v_u, \operatorname{grad} v'\rangle_{\Gamma_{c,p}} = 0$
    - ✔ $(-\varepsilon \operatorname{grad} v, \operatorname{grad} v')_{\Omega_p} - \langle \mathbf{n}\cdot \boldsymbol{d}, v'\rangle_{\Gamma_{d,p}} = 0$

      $v|_{\Gamma_{c,p}} = -v_s \quad \& \quad \mathbf{n}\cdot \boldsymbol{d}|_{\Gamma_{d,p}} = -\mathbf{n}\cdot \boldsymbol{d}_u$

...with $v_p = v_u + v$ & $\boldsymbol{e}_p = \boldsymbol{e}_u + \boldsymbol{e}$

Iterative process of perturbation problems

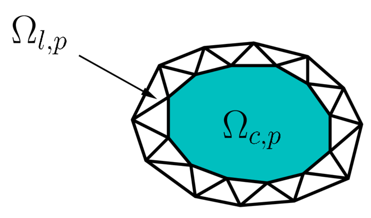


- In Ω_p: $v_p = v_{s,1} + v_1 + v_{s,3} + v_3 + v_{s,5} + v_5 + ...$
- Iterative process repeated till convergence

$$\left|\frac{v_{s,i} + v_i}{v_{p,i}}\right| \le err$$

Charges & Forces

- At discrete level, the domain of integration is limited to $\Omega_{l,p}$: layer of elements touching $\Gamma_{c,p}$ *in* $\Omega_p \setminus \Omega_{c,p}$



- Charges

$$Q_p = Q_s + Q$$

$$= -(-\varepsilon \operatorname{grad} v_s, \operatorname{grad} v_{c,p})_{\Omega_{l,p}} - (-\varepsilon \operatorname{grad} v, \operatorname{grad} v_{c,p})_{\Omega_{l,p}}$$

Case of iterative process:

$$Q_p = Q_{s,1} + Q_1 + Q_{s,3} + Q_3 + Q_{s,5} + Q_5 + ...$$

- Forces (by virtual work principle)

$$F_r = \int_\Delta [-\varepsilon \boldsymbol{e}_p J^{-1} \frac{\partial J}{\partial u} \boldsymbol{e}_p + \boldsymbol{e}_p \boldsymbol{e}_p \frac{\partial |J|}{\partial u}] \mathrm{d}\Delta$$

with: $\boldsymbol{e}_p = \boldsymbol{e}_{s,1} + \boldsymbol{e}_1 + \boldsymbol{e}_{s,3} + \boldsymbol{e}_3 + \boldsymbol{e}_{s,5} + \boldsymbol{e}_5 + ...$

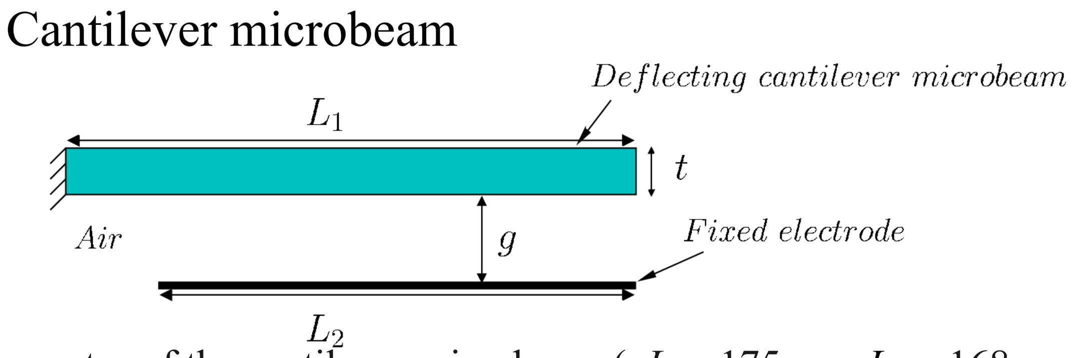
Elastic FE Model

- Linear mechanical problem: $\nabla[\mathrm{E}]\nabla\mathbf{u} + \mathbf{f_v} = 0$
- Smoothening of air domain Ω_p
- Updating nodal coordinates of the mesh of Ω_p
- Sequential electromechanical coupling repeated until:

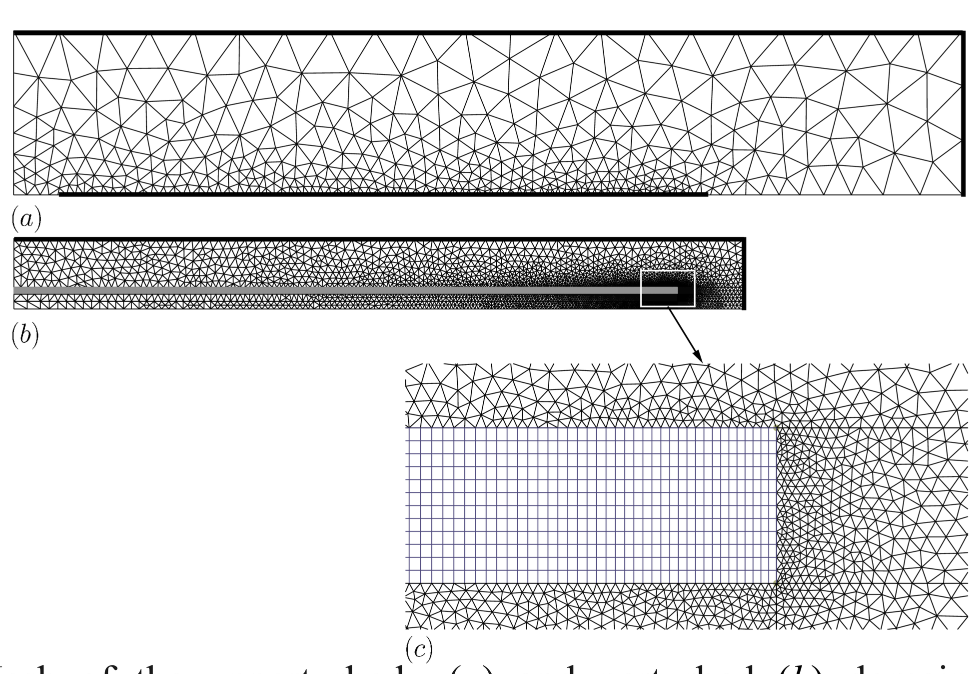
$$\frac{\|\mathbf{u}_{i+1} - \mathbf{u}_i\|_2}{\|\mathbf{u}_{i+1}\|_2} \le \alpha$$

Applications

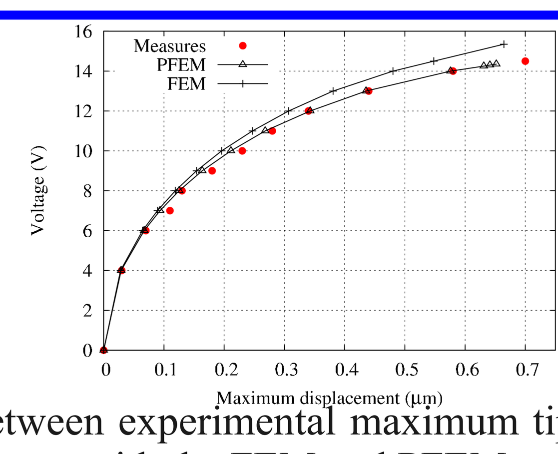
- Cantilever microbeam



Geometry of the cantilever microbeam (L_1= 175 µm, L_2 = 168 µm, t = 1.85 µm, g =2 µm, E = 135 GPa, ν = 0.22)

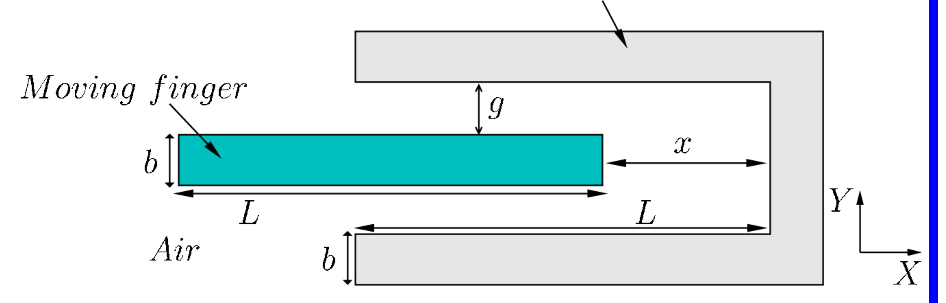


Mesh of the unperturbed (*a*) and perturbed (*b*) domains with infinite boundaries and a very fine mesh (*c*) in the vicinity of the corners of the perturbing cantilever microbeam to account for fringing field effects

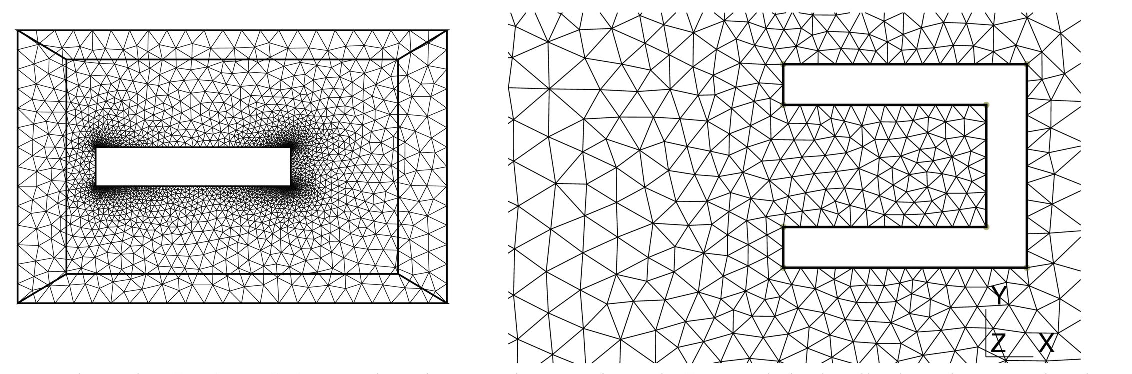


Comparison between experimental maximum tip displacement of the cantilever beam with the FEM and PFEM numerical results

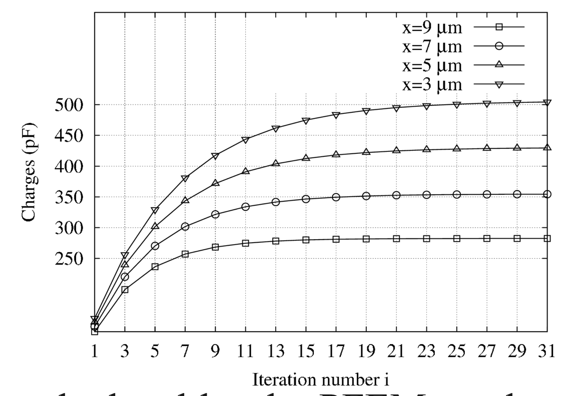
- Combdrive



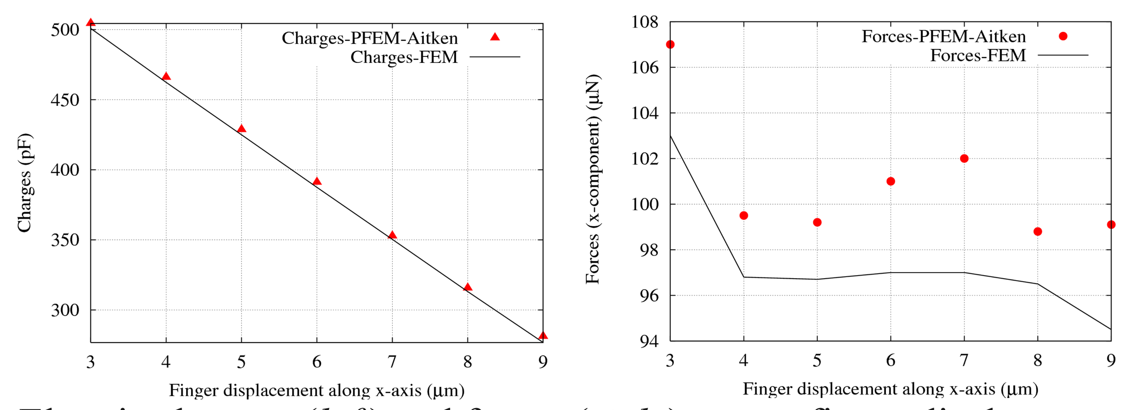
Geometry of the combdrive (L = 10 µm, b = 2 µm, g =2 µm)



Mesh of Ω (*right*) and adapted mesh of Ω_p with infinite boundaries around the moving finger (*left*)



Electric charges calculated by the PFEM on the moving finger for each position x as a function of iteration number i



Electric charges (*left*) and forces (*right*) versus finger displacement

Conclusions

- The analysis of the coupled electromechanical problem of electrostatic MEMS has been investigated by an iterative perturbation finite element technique
  - unperturbed field computed in a global domain without the presence of conductive regions
  - unperturbed field applied as source in $\Omega_{c,p}$
  - perturbed field determined in reduced domain Ω_p
  - iterative procedure used when the coupling between the sub-regions is significant
  - degenerated mesh elements avoided
- The accuracy of the PFEM is demonstrated by comparing the results with both measurements and the classical FE results
- Application of Aitken acceleration has been proven to be very efficient
- A significant speed-up in comparison to the conventional FEM is achieved

Selected Publications

- M. Boutaayamou, R.V. Sabariego and P. Dular, "A Perturbation Method for the 3D Finite Element Modeling of Electrostatically Driven MEMS," *Sensors*, Vol. 8, pp. 994-1003, 2008.
- M. Boutaayamou, R.V. Sabariego and P. Dular, " An Iterative Finite Element Perturbation Method for Computing Electrostatic Field Distortions," *IEEE Transactions on Magnetics,* 2008, in press.
- M. Boutaayamou, R.V. Sabariego and P. Dular, " A Perturbation Finite Element Method for Modeling Electrostatic MEMS without Remeshing," *Proc. 9th EuroSimE 2008, Thermal, Mechanical and Multiphysics Simulation and Experiments in Micro-Electronics and Micro-Systems,* Freiburg, 2008, in press.

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