A Perturbation Method for the 3D Finite Element Modeling of Electrostatically Driven MEMS

M. Boutaayamou, R. V. Sabariego, and P. Dular

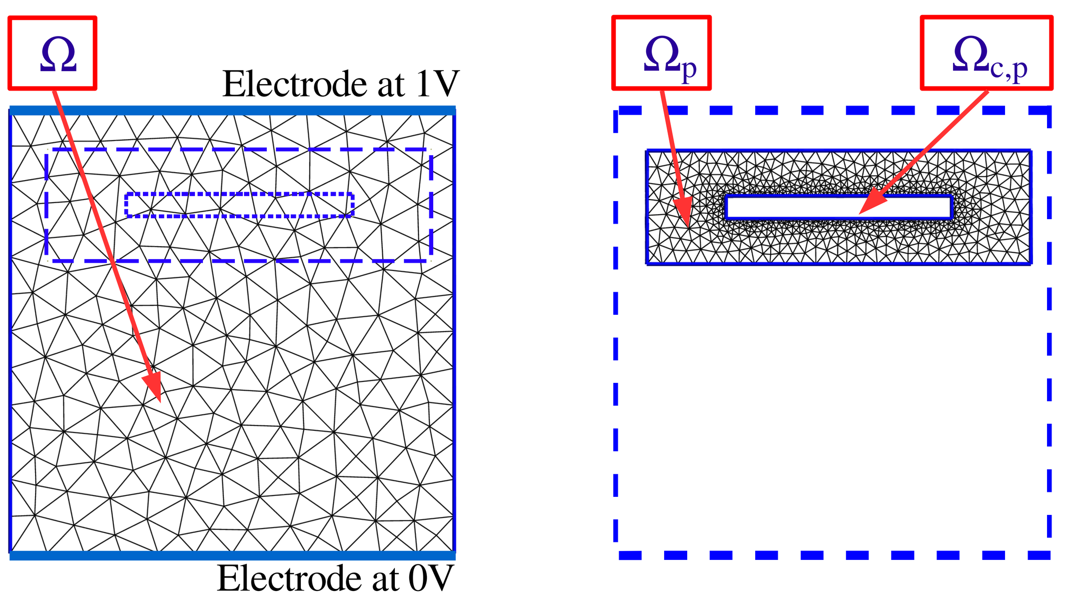
Department of Electrical Engineering and Computer Science (ELAP) **University of Liège, Belgium**

Modeling electrostatically driven MEMS

- Lumped spring-mass models
  - helpful for physical insight
  - neglect important effects such as bending of plates and the fringing field effects
- Finite element (FE) method
  - adapted for complex geometries
  - compute accurately the fringing field effects at the expense of dense discretization near corners
  - movement modeling usually requires successive meshing and computations for each new position
  - classical approach computationally expensive
- Perturbation method
  - unperturbed field computed in a global domain without the presence of some conductive regions
    - ✔ no mesh of perturbing domains
    - ✔ possible symmetries or analytical solution
    - ✔ applied source to the perturbation problems
  - perturbed field determined in a local domain
    - ✔ subproblem-adapted meshes
- Iterative sequence of perturbation problems
  - when coupling between sub-regions is significant
    - ✔ successive perturbations in each region are computed from one region to the other
    - ✔ each sub-problem gives a correction as a perturbation

Perturbation method

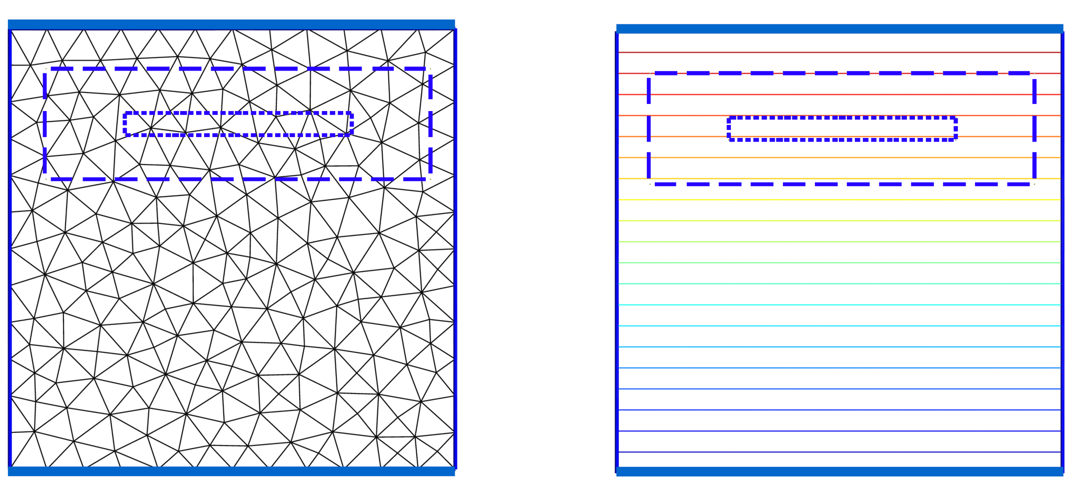
- Unperturbed domain: parallel-plate capacitor
- Perturbing domain: micro-beam (100μm × 10μm)



Unperturbed problem

- Unperturbed electric scalar potential FE formulation

$$(-\varepsilon\, \mathbf{grad}\, v_u, \mathbf{grad}\, v')_{\Omega} - \langle \mathbf{n}\cdot\mathbf{d}_u, v'\rangle_{\Gamma_u} = 0$$



Mesh of the unperturbed domain — Distribution of v_u

$\mathbf{grad}\, v_u$ is projected on $\partial\Omega_{c,p}$ ...

Perturbation equations

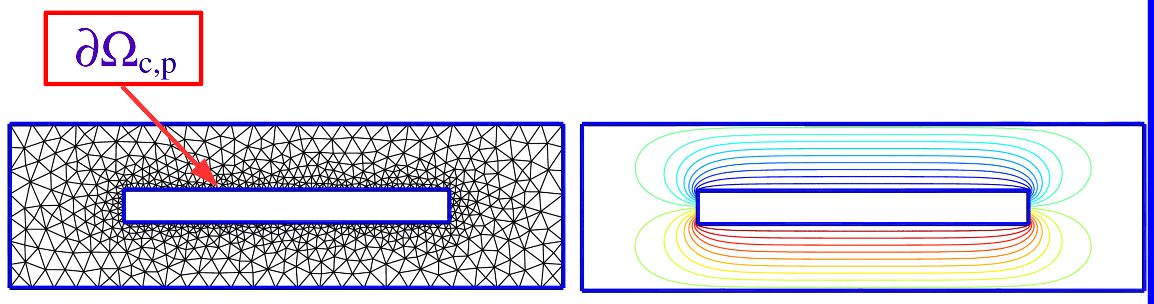
- Perturbed electric scalar potential FE formulation

$$\langle \mathbf{grad}\, v_s, \mathbf{grad}\, v'\rangle_{\partial\Omega_{c,p}} - \langle \mathbf{grad}\, v_u, \mathbf{grad}\, v'\rangle_{\partial\Omega_{c,p}} = 0$$

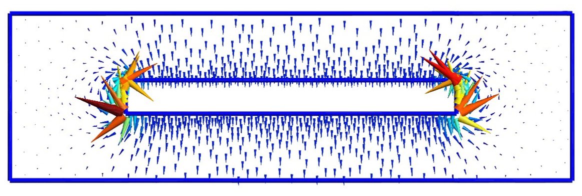
$$(-\varepsilon\, \mathbf{grad}\, v, \mathbf{grad}\, v')_{\Omega_p} - \langle \mathbf{n}\cdot\mathbf{d}_p, v'\rangle_{\Gamma_p} = 0$$

$$v = -v_s\big|_{\partial\Omega_{c,p}}$$

...with $v_u + v = v_p$ & $\mathbf{e}_u + \mathbf{e} = \mathbf{e}_p$

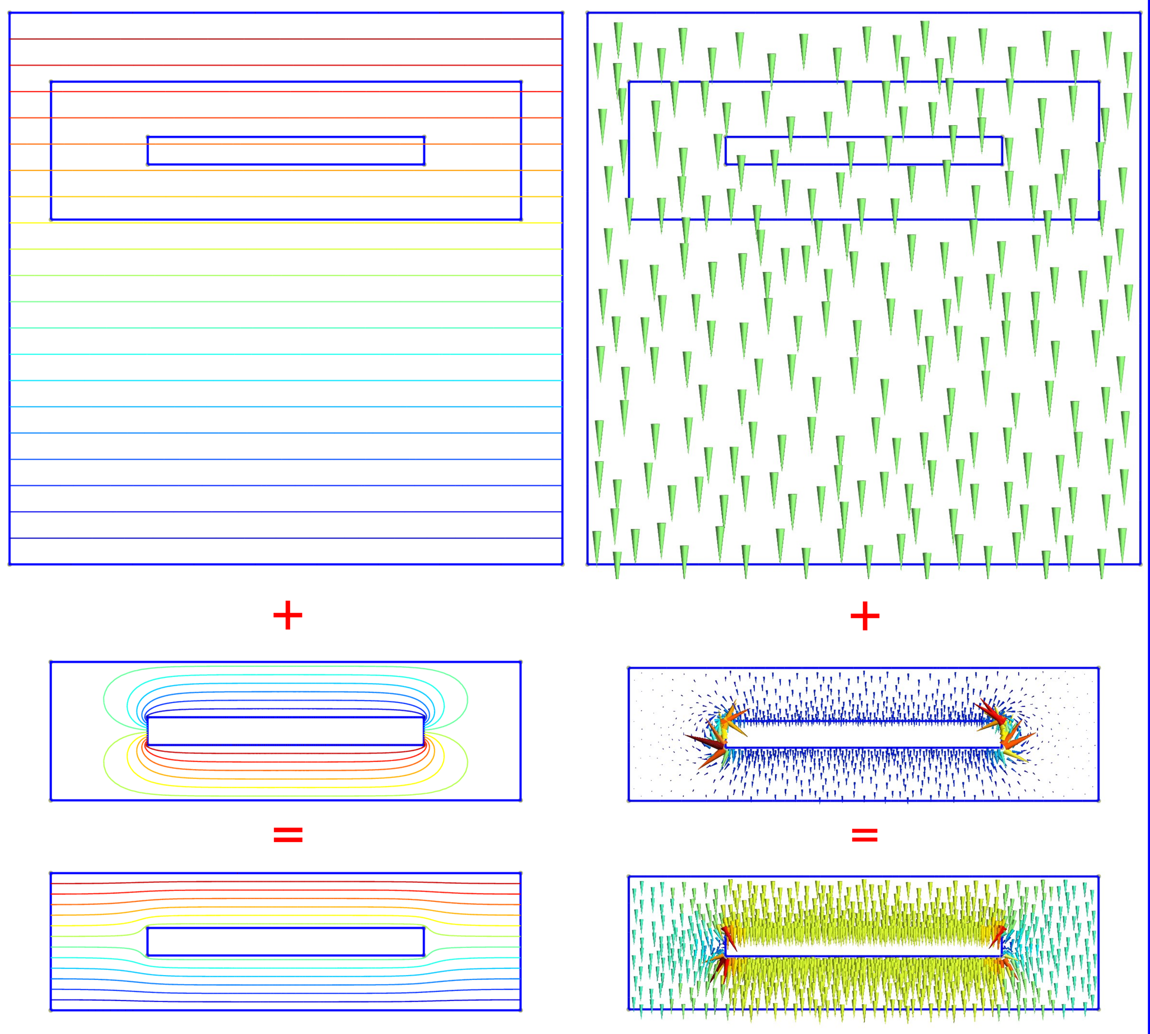


Adapted mesh of Ω_p — Distribution of v in Ω_p

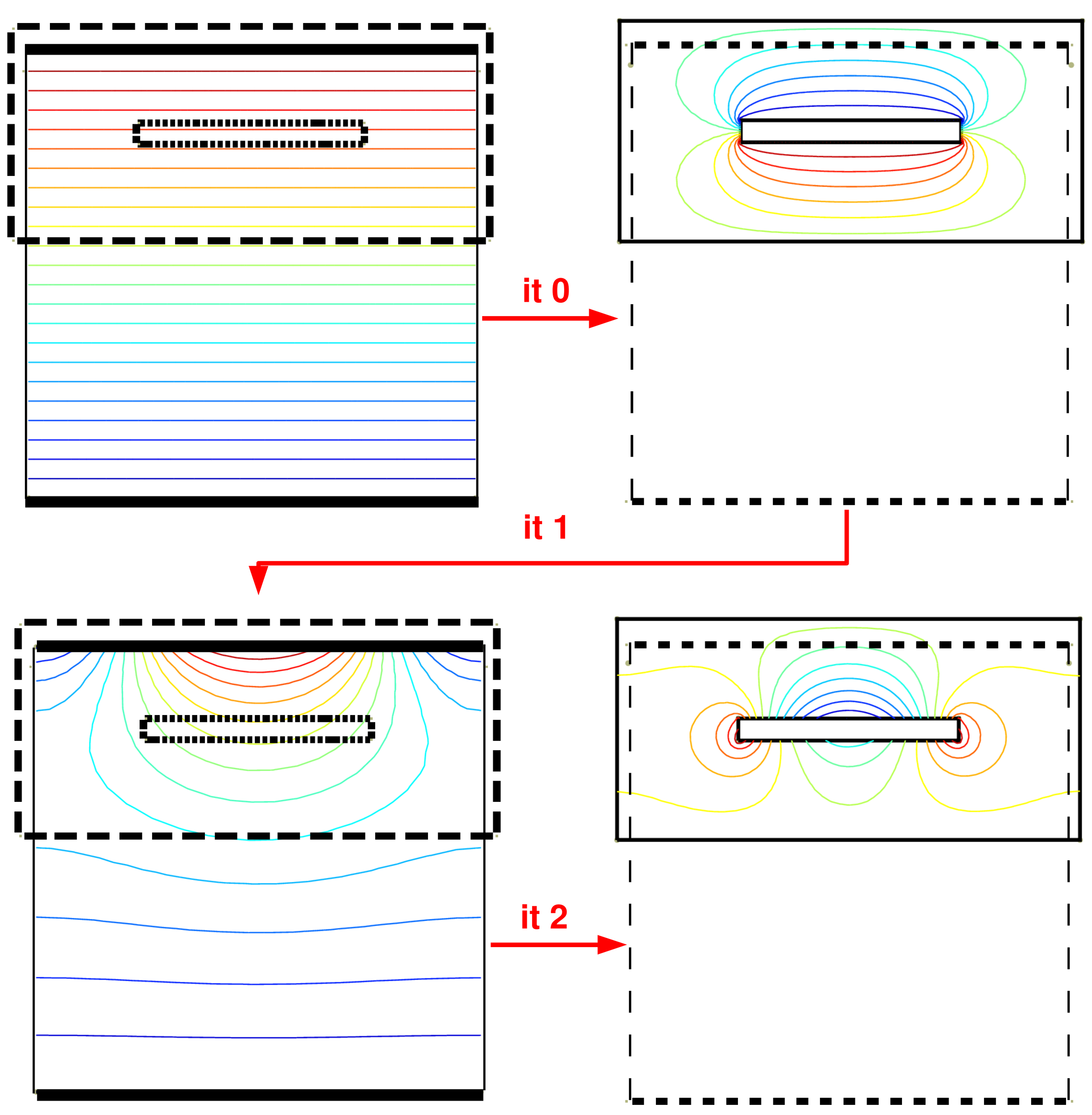


Distribution of $\mathbf{e}$ ($\mathbf{grad}\, v$) in Ω_p

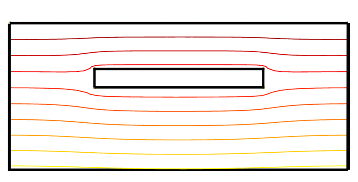
Post-processing



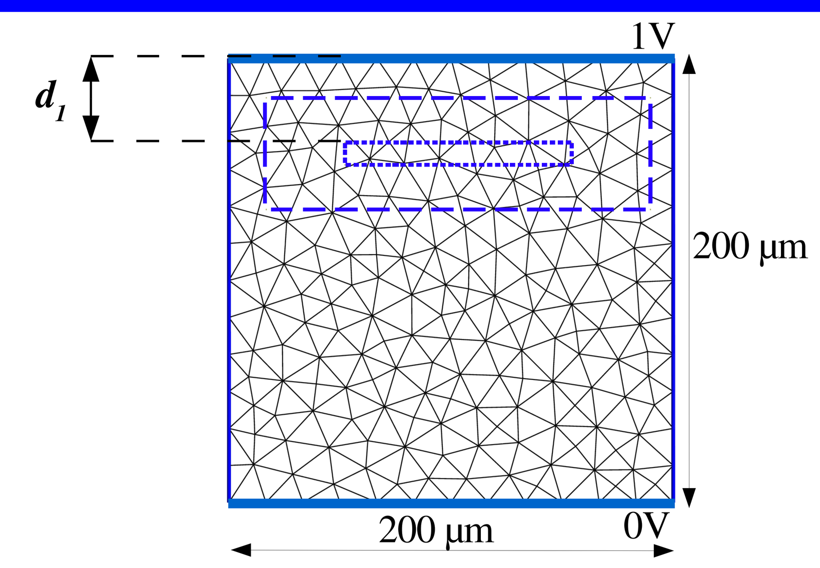
Iterative process of perturbation problems

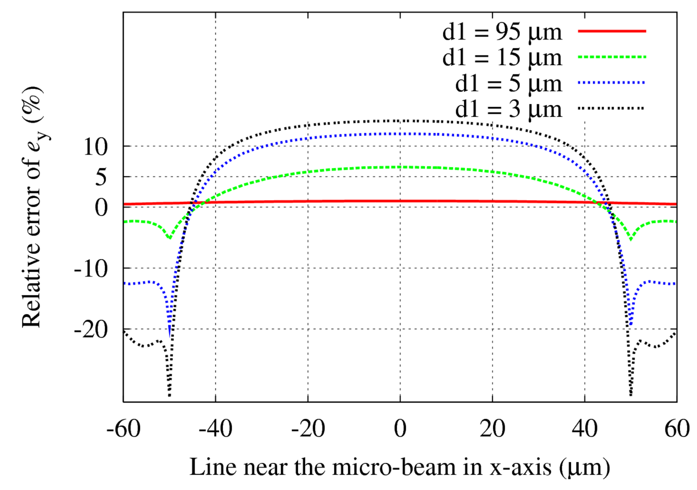


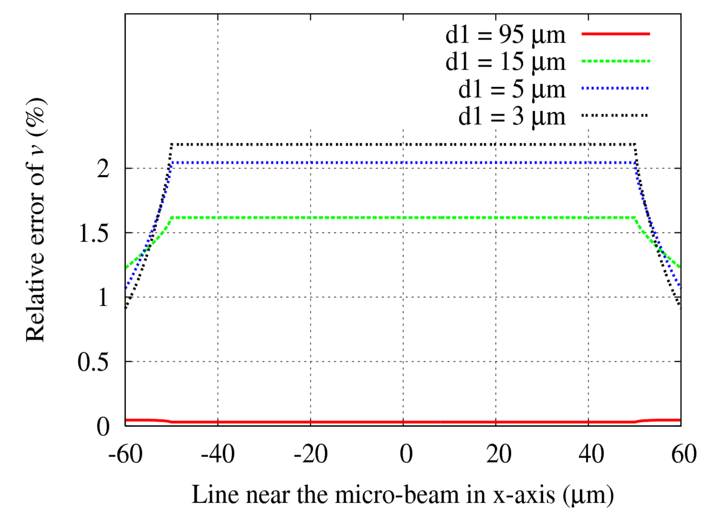
Unperturbed Solution + Perturbations = Perturbed Solution ⇒



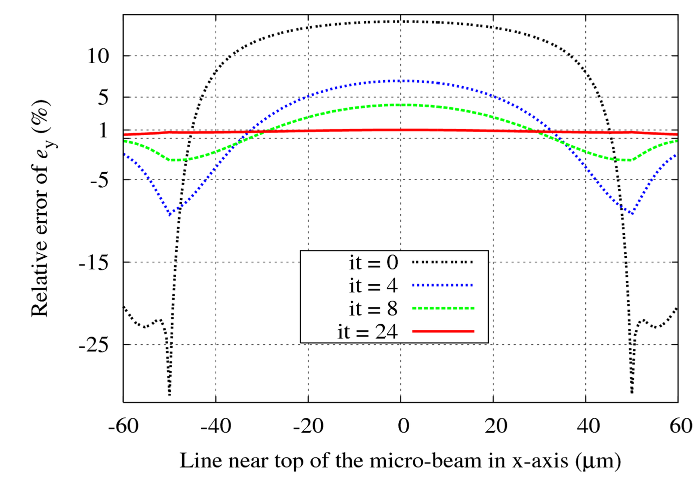
Application



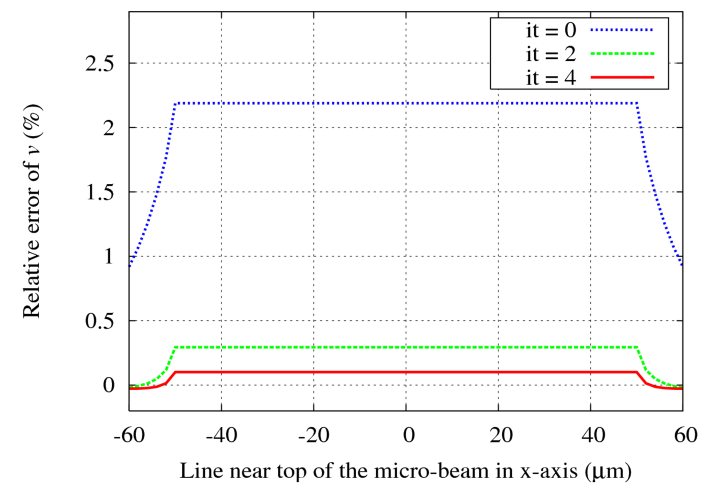


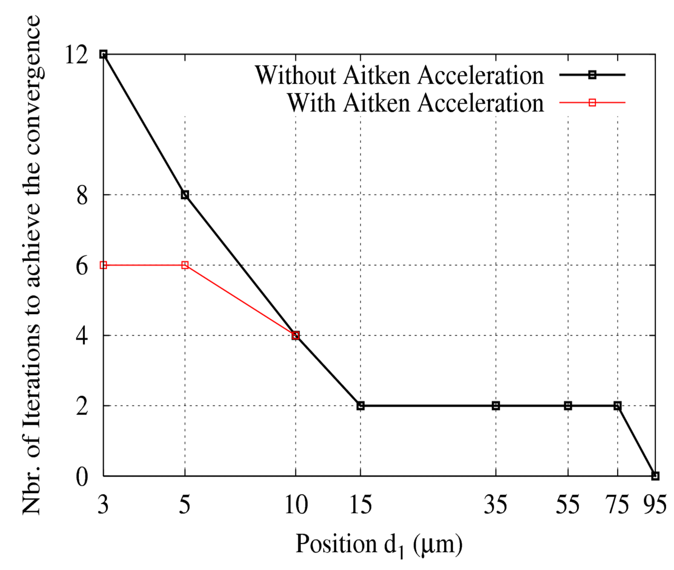


$d_l = 3$ μm



$d_l = 3$ μm





Tolerance = 1% (with respect to the regular FEM)

Conclusions

- A perturbation approach based on electric scalar potential FE formulation has been presented
  - adapted for complex geometries
  - unperturbed field computed in a global domain without the presence of conductive regions
  - unperturbed field applied as source in $\Omega_{c,p}$
  - perturbed field determined in reduced domain Ω_p
  - iterative procedure used when the coupling between the sub-regions is significant
- Convergence acceleration of iterative sequence in progress

Acknowledgements: This work is supported by the Belgian French Community (ARC 03/08-298) and the Belgian Science Policy (IAP P6/21). P. Dular is a Research Associate with the Belgian National Fund for Scientific Research (F.N.R.S.).