

State Reconstruction from Synchronized Phasor Measurements

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Abstract—The traditional state estimator relies on measurements affected by time skew and, while the whole system can be made observable, it cannot track the post-disturbance dynamics. Synchronized phasor measurements, on the other hand, can track those dynamics but, in a foreseeable future, they will be available in scarce configurations that do not allow estimating the whole system state. State reconstruction precisely aims at using the available phasor measurements to track the evolution of the power system - at least in a region of interest - in between two runs of a traditional state estimator. It minimizes under constraints a weighted least square objective involving both the phasor measurements and bus power pseudo-measurements relative to a reference state. The placement of phasor measurement units at the generator buses is recommended for accuracy. Tests are reported with phasor measurements obtained from time simulation of post-disturbance system evolution. The results deal with the effect on accuracy of the relative weights assigned to phasor and pseudo-measurements, the benefit of taking the reference bus powers from the last reconstructed instead of the pre-disturbance state, and the rate of the state reconstructions.

Index Terms—situational awareness, synchronized phasor measurements, unobservability, state estimation, weighted least squares, constrained optimization

I. INTRODUCTION

Traditional state estimators run every few minutes and rely on measurements provided by the Supervisory Control And Data Acquisition (SCADA) system. Apart from areas which are unobservable due to temporary unavailability or permanent lack of data, the SCADA measurements allow estimating the whole state vector of complex bus voltages. However, these measurements are affected by time skew, i.e. during transients they do not all refer to the same state, which adds to the noise stemming from sensors. Therefore, when the power system is subject to a disturbance, the standard state estimator gives an “average picture” of the system state but cannot track its dynamics.

Phasor Measurement Units (PMUs) are accurately time synchronized power system instruments able to gather voltage and current phasors at high rate (from 30 to 120 samples per second) [1], [2]. Supported by advances in computational facilities, networking infrastructure and communications, this technology has opened new perspectives of real-time applications. These advanced sensors provide coherent, time-synchronized

and generally more accurate measurements. With their high sampling rates, they can track the system dynamics following an event [1]. However, they are available in present-day power systems in scarce configurations since the upgrade of existing power system infrastructures requires investments in these technologies and only incremental upgrades are realistic. As a consequence, unlike standard SCADA measurements, synchronized phasor measurements alone do not allow estimating the whole state vector of a power system.

So far, most of the efforts have concentrated on using PMUs to improve the estimate provided by standard state estimators [1], [3], [4]. Synchronized measurement snapshots taken in almost the same time window as the SCADA measurements can be used to reinforce the redundancy of the latter. Existing formulations, such as the traditional weighted least square estimation method have been adapted to incorporate the available (desirably optimally placed) PMU measurements [1]. Another approach consists of post-processing the PMU measurements separately [3], [4], which offers the advantage of leaving unchanged the available state estimation software. Nevertheless, the so enhanced estimators are still run at the rate of the traditional ones.

The issue dealt with in this paper can be summarized as follows: can the available synchronized phasor measurements be used to track the evolution of the power system - at least in a region of interest - in between two runs of the classical state estimator ?

This challenge has been recognized by power system practitioners and research community and efforts have been undertaken to use PMU data to increase situational awareness [5], [6], [7], [8]. Several concepts have been studied to this purpose: incomplete observability through formation of PMU observable islands [5], hybrid [6] and PMU morphed [7], [8] power flow formulation.

The approach advocated in [9] and further elaborated in this paper consists of “reconstructing” the states (of at least a region of interest) at successive time instants between two runs of the classical state estimator. Each state reconstruction uses, on one hand, the synchronized phasor measurements relative to a given time and, on the other hand, pseudo-measurements relative to a “reference” state of the system, i.e. the state at a recent past time. Those pseudo-measurements allow resolving the already mentioned unobservability issue. In [9] the reference state was provided by the last run of the classical state estimator. In this paper, the approach is extended by taking the last reconstructed state itself as reference.

State reconstruction is formulated as an optimization problem with linear equality constraints. The constraints relate to

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network equations and zero bus injections. In our previous work [9], all synchronized phasor measurements were considered critical (i.e. removing any of them would decrease the number of bus voltages that can be computed from the PMU and zero injection data). In this paper, the assumption about criticality of PMU measurements is removed and these measurements are treated in the least-square sense, together with the pseudo-measurements.

Finally, the important issue of placing PMUs in order to reconstruct states with better accuracy has been considered. The placement of PMUs near generators appears to be advantageous in this respect.

The targeted application of state reconstruction is the tracking of the changing system state after a disturbance. This could contribute to a better monitoring of the system evolution, in particular anticipating its near-future evolution and early detecting possible cascading effects. This application is illustrated with the results obtained from a dynamic simulation of the Nordic32 test system. The paper also considers the effect of: the chosen reference state, the relative weights assigned to synchronized and pseudo-measurements, and the rate at which states are reconstructed.

The rest of this paper is organized as follows. Section II presents the principle of state reconstruction and the corresponding mathematical formulation. The rationale for specific PMUs placement is given in Section III, while Section IV presents the simulation results. A summary and future research directions are given in Section V.

II. PRINCIPLE AND FORMULATION OF STATE RECONSTRUCTION

A. Leading ideas behind state reconstruction

The main purpose of state reconstruction is to track the changing system state and fill the gap between successive classical state estimations by exploiting the coherent, time-synchronized phasor measurements [9]. The underlying ideas can be summarized as follows:

- 1) the PMU configuration provides scarce measurements, which do not make the system observable. As a consequence, there is an infinite number of states satisfying the available set of synchronized measurements;
- 2) we solve this indeterminacy by computing the state which yields the bus power injections closest (in the Euclidian-norm sense) to reference values;
- 3) these reference values can be obtained either from the last estimate provided by a classical state estimator [9] or from the last reconstructed state itself;
- 4) the reference bus power injections are treated as pseudo-measurements, and complement the synchronized bus voltage and branch current measurements;
- 5) both types of measurements are processed with proper weights in the least-square sense, together with sparse linear equality constraints relative to the network equations and zero injection information. The rectangular components of bus voltages and currents are taken as state variables.

As an alternative, one could think of taking complex bus voltages as reference values. However, following a disturbance, the complex bus voltages may change quite significantly while the bus power injections (and more particularly the load powers) are expected to vary to a lower extent under the effect of the disturbance. The latter are thus preferred. This point is further elaborated in Section III.

Other distances than the Euclidian one could be considered, such as the L_0 or L_1 norm used in other engineering problems facing the problem of indeterminacy [10].

B. Mathematical formulation

The network is modelled by the voltage-current relationships stemming from the bus admittance matrix formulation:

$$\mathbf{G}\mathbf{v}_x - \mathbf{B}\mathbf{v}_y - \mathbf{i}_x = \mathbf{0} \quad (1a)$$

$$\mathbf{B}\mathbf{v}_x + \mathbf{G}\mathbf{v}_y - \mathbf{i}_y = \mathbf{0} \quad (1b)$$

where \mathbf{v}_x , \mathbf{v}_y , \mathbf{i}_x and \mathbf{i}_y are vectors of rectangular components of voltages and currents, while \mathbf{G} and \mathbf{B} are the real and imaginary parts of the bus admittance matrix, respectively.

In addition to the above equations, the zero current injections at “transit” buses without load or generation connected, can be written as:

$$\mathbf{C} \begin{bmatrix} \mathbf{i}_x \\ \mathbf{i}_y \end{bmatrix} = \mathbf{0} \quad (2)$$

where \mathbf{C} is a matrix with 0's and 1's.

The objective is to compute a coherent, time-synchronized, system state that best fits the m available PMU data. The latter involve components of \mathbf{v}_x , \mathbf{v}_y , \mathbf{i}_x , \mathbf{i}_y according to:

$$z_i = \mathbf{a}_i \begin{bmatrix} \mathbf{v}_x \\ \mathbf{v}_y \\ \mathbf{i}_x \\ \mathbf{i}_y \end{bmatrix} + \omega_i \quad i = 1, \dots, m \quad (3)$$

where z_i is the i -th measured value, ω_i the corresponding noise, and \mathbf{a}_i is a unit row vector with the nonzero entry corresponding to the measured voltage or current component. Note that each voltage or current phasor measurement yields two equations of the type (3) corresponding to its real and imaginary parts, respectively.

Assuming that the PMU configuration provides scarce measurements, there will be typically much less equations (1, 2, 3) than unknown voltages and currents. Consequently, there is an infinite number of states satisfying the available set of synchronized measurements. As already mentioned, this issue of indeterminacy is solved by adding bus active and reactive power injections pseudo-measurements. These data relate to the voltages and currents through:

$$P_j^{ref} = (v_{xj}i_{xj} + v_{yj}i_{yj}) + \nu_{Pj} \quad j = 1, \dots, p \quad (4)$$

$$Q_j^{ref} = (v_{yj}i_{xj} - v_{xj}i_{yj}) + \nu_{Qj} \quad j = 1, \dots, p \quad (5)$$

where p is the number of pseudo-measurements, the ^{ref} superscript denotes reference values (see previous sub-section) and ν_{Pj}, ν_{Qj} can be interpreted as noise terms.

Some phasor measurements can be redundant with other phasor measurements, with zero injections, or with the pseudo-measurements. Hence, the phasor measurements are not treated as critical data, i.e. they are not imposed as equality constraints [9], but processed together with the pseudo-measurements in the Weighted Least-Square (WLS) objective function:

$$\begin{aligned} \min_{\mathbf{v}_x, \mathbf{v}_y, \mathbf{i}_x, \mathbf{i}_y} & \sum_{i=1}^m w_i \left(z_i - \mathbf{a}_i \begin{bmatrix} \mathbf{v}_x \\ \mathbf{v}_y \\ \mathbf{i}_x \\ \mathbf{i}_y \end{bmatrix} \right)^2 \\ & + \sum_{j=1}^p w_{Pj} \left(P_j^{ref} - v_{xj} i_{xj} - v_{yj} i_{yj} \right)^2 \\ & + \sum_{j=1}^p w_{Qj} \left(Q_j^{ref} - v_{yj} i_{xj} + v_{xj} i_{yj} \right)^2 \quad (6) \end{aligned}$$

Despite the fact that they remain affected by sensor and communication channel errors [11], synchronized measurements are expected to be more accurate than classical SCADA measurements. They are also expected to be more accurate than the pseudo-measurements. Hence, in the objective (6), the weights w_i assigned to phasor measurements should be larger than the weights w_{Pj}, w_{Qj} assigned to pseudo-measurements.

Clearly, if any pseudo- or phasor measurement is critical, the reconstructed state will be identical, whatever the weight w_i assigned to that measurement.

The objective (6) is minimized subject to the equality constraints (1) and (2), with $\mathbf{v}_x, \mathbf{v}_y, \mathbf{i}_x$ and \mathbf{i}_y as independent variables.

III. ON THE PLACEMENT OF PMUS

Typically, when a PMU is placed at one bus, it measures the complex bus voltage at that bus and the complex currents in all branches incident to that bus¹. These data enter the first term of the objective (6). In this case, there is no need to consider power injection pseudo-measurements at that bus, which is not involved in the second and third terms of (6).

If the PMU configuration can be chosen or reinforced, a relevant issue is to identify the buses which should be provided with PMUs. In Ref. [9], we advocated their placement at the buses of generators involved in frequency and/or voltage control, so that the generated powers are measured.

The rationale behind this choice is the following. A situation of interest for exploiting synchronized phasor measurements is when a disturbance takes place after the execution of the classical state estimator. In such a case, load powers change owing to their sensitivity to voltage and frequency. However, this change is usually in the order of a few percents. Hence, their pre-disturbance values determined by the state estimator or the latest values obtained from state reconstruction (as discussed in Section IV.B) constitute appropriate P_j^{ref}, Q_j^{ref} pseudo-measurements. On the other hand, the powers produced by generators may vary significantly under the effect of the disturbance, owing to their participation in voltage and/or

¹assuming there is no restriction on the number of communication channels

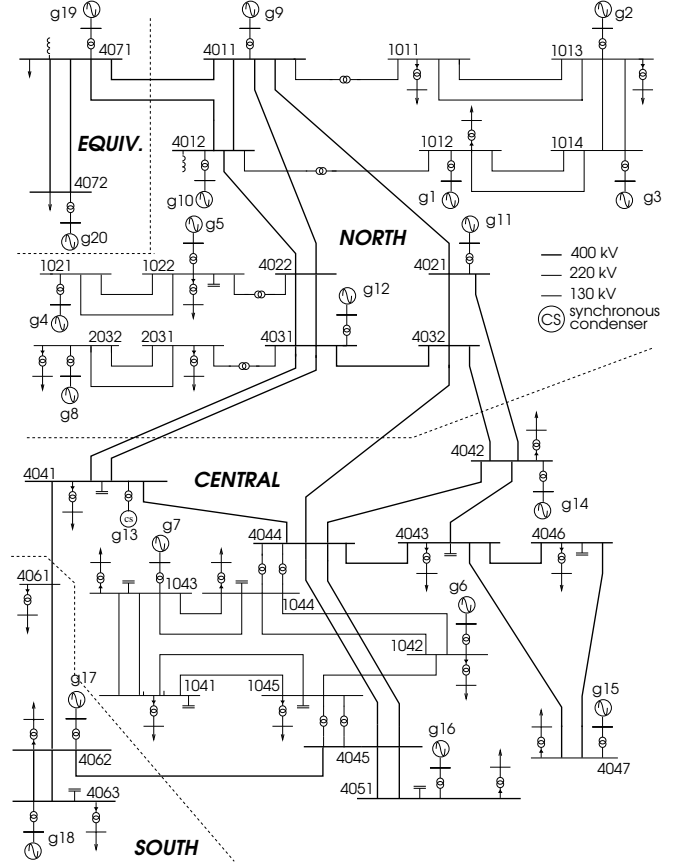


Fig. 1. Nordic32 test system

frequency control. For instance, the outage of transmission or generation equipments is reflected in the reactive powers of voltage controlled generators. Thus, it makes sense to collect real-time measurements of those quantities subject to larger deviations.

Of course, the same applies to any component participating in voltage control (such as static var compensators) as well as frequency control.

IV. SIMULATION RESULTS

A. Test system and simulation conditions

This section reports on simulation results obtained with the system used in [9], [12]. It is a variant of the Nordic32 test system, including 52 buses and 20 machines. Its one-line diagram is shown in Fig. 1.

The model includes a detailed representation of each synchronous machine, with models of speed governors, hydro and steam turbines, automatic voltage regulators and OverExcitation Limiters (OELs). Loads behave as constant current for the active power and constant impedance for the reactive power. Each load is fed through a transformer with automatic Load Tap Changer (LTC); the various LTCs act with various delays.

Time-domain simulations of the model were performed with the Simulink-based software described in [13].

The scenario considered in this paper involves the outage of the transmission line 4032-4044 (see Fig. 1) at $t = 6$ s. In the long-term the system evolves under the effect of LTCs and

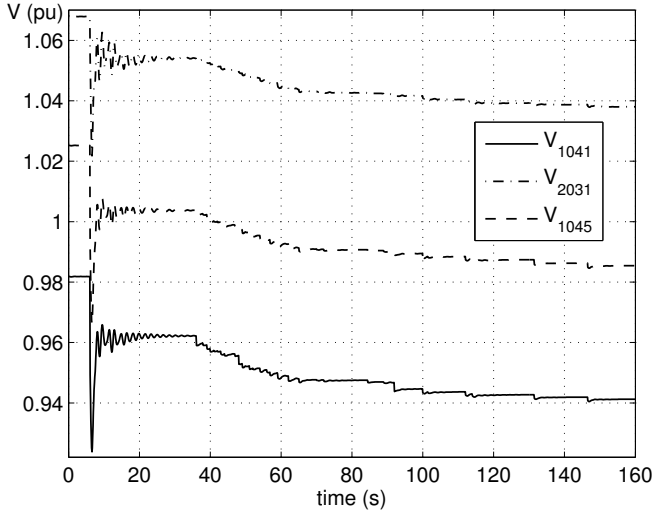


Fig. 2. Evolution of voltage magnitudes at three transmission buses

OELs. The system regains a new equilibrium with rather low voltage magnitude at bus 1041, as shown in Fig. 2.

As demonstrated in [9], the proposed method is able to focus on a region of interest. In the Nordic32 system, we assume that we are primarily interested in the Central region (see Fig. 1), whose voltages are much impacted by the outage of transmission lines in the corridor connecting this region to the North one. To reconstruct the state of that region, the closely located generators g6, g7, g14, g15, and g16 are assumed to be equipped with PMUs. Each PMU provides the complex voltage at the generator bus as well as the complex current injected by the generator. It would be more realistic to place PMUs on the high-voltage side of the step-up transformers. However, in this small test system, this PMU configuration would yield too good a coverage of the region and, hence, optimistic results.

The phasor data were obtained by sampling the rectangular components of the voltages provided by time simulation. Measurement noise was simulated by adding a random component with a Gaussian distribution to each voltage component v_x and v_y . The standard deviation of this noise was set to 0.003 pu, as suggested in [11], [14]². Noisy current measurements were simulated by computing the currents from the noisy voltages, using the network model.

At each state reconstruction point, the WLS objective (6) was minimized under the constraints (1) and (2) using the GAMS-IDE (General Algebraic Modeling System) environment, interfaced with MATLAB through the MATGAMS interface documented in [15]. The primal-dual interior-point nonlinear solver (with filter line-search method) IPOPT was used [16].

The change in topology (line outage) is assumed to be known by the state reconstruction procedure.

²thus, the noise lies in $[-0.01 + 0.01]$ pu with 99.9 % probability

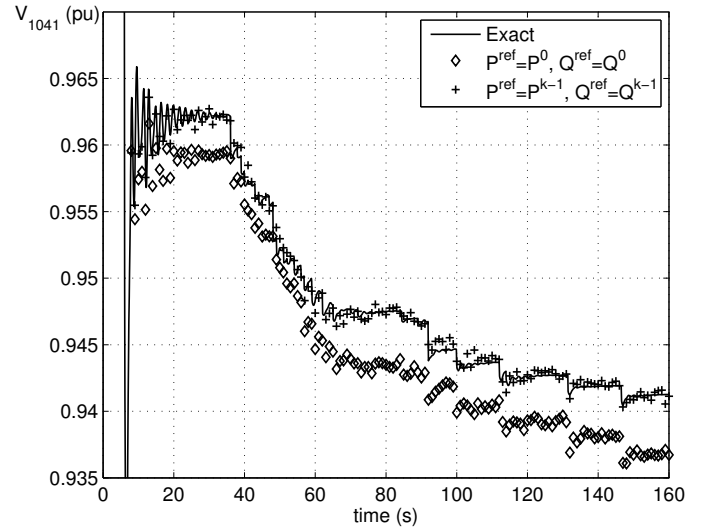


Fig. 3. Exact and reconstructed voltage at bus 1041; $w_{Pj} = w_{Qj} = 1$ ($j = 1, \dots, p$) and $w_i = 100$ ($i = 1, \dots, m$)

B. Consideration of different reference states and measurement weights

We consider hereafter the voltage magnitude at bus 1041 reconstructed every second. That bus was chosen as it experiences the largest drop under the effect of the line outage.

Figure 3 shows the exact and the reconstructed voltages obtained by assigning $w_{Pj} = w_{Qj} = 1$ ($j = 1, \dots, p$) to all pseudo-measurements and $w_i = 100$ ($i = 1, \dots, m$) to all phasor measurements, while using as pseudo-measurements the bus power injections:

- in the pre-disturbance state (i.e. at $t = 0$): the corresponding curves are labeled $P^{ref} = P^0, Q^{ref} = Q^0$ in all figures;
- provided by the previous state reconstruction: the corresponding curves are labeled $P^{ref} = P^{k-1}, Q^{ref} = Q^{k-1}$ in all figures.

Figures 4 and 5 show the corresponding results obtained with the phasor measurement weights set to respectively 500 and 10000 times the pseudo-measurement weights.

Similar results for the active power of the load at bus 1041 are given in Figs. 6, 7 and 8, again with phasor measurement weights set to respectively 100, 500 and 10,000 times the pseudo-measurement weights.

Although their quality varies with the chosen weights and reference bus power injections, the results show that state reconstruction can be remarkably accurate.

The following observations can be made from the results obtained when taking pre-disturbance bus powers as reference throughout the whole sequence of state reconstructions:

- high weights have to be assigned to phasor measurements to obtain accurate results;
- the reconstructed voltages are very accurate when setting the phasor measurements weights 10,000 times higher than the pseudo-measurements weights;
- with these high phasor measurement weights, the reconstructed powers are also better on the average. How-

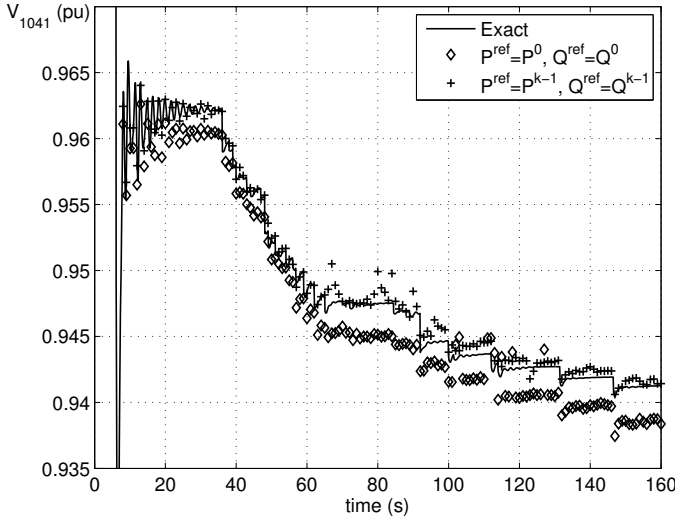


Fig. 4. Exact and reconstructed voltage at bus 1041; $w_{Pj} = w_{Qj} = 1$ ($j = 1, \dots, p$) and $w_i = 500$ ($i = 1, \dots, m$)

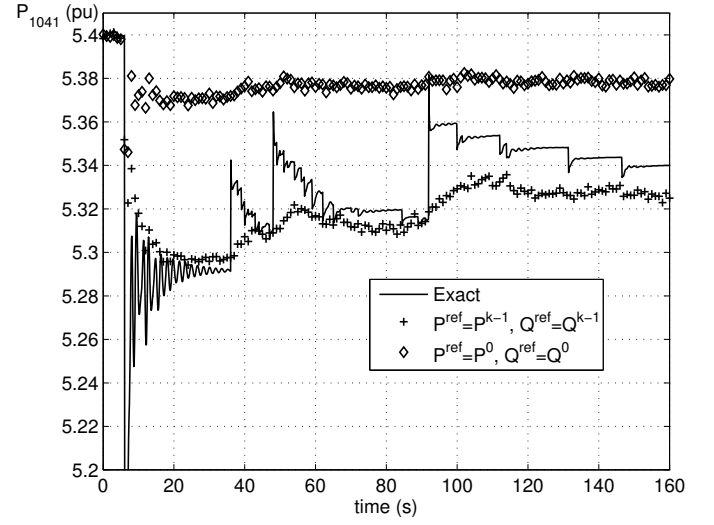


Fig. 6. Exact and reconstructed load active power at bus 1041; $w_{Pj} = w_{Qj} = 1$ ($j = 1, \dots, p$) and $w_i = 100$ ($i = 1, \dots, m$)

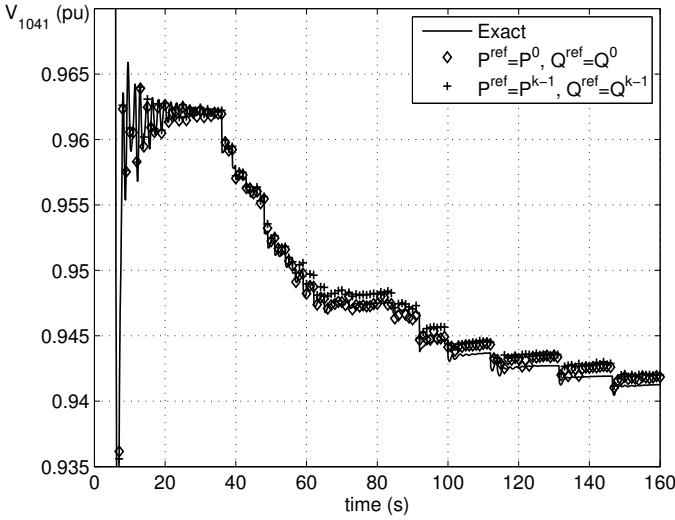


Fig. 5. Exact and reconstructed voltage at bus 1041; $w_{Pj} = w_{Qj} = 1$ ($j = 1, \dots, p$) and $w_i = 10,000$ ($i = 1, \dots, m$)

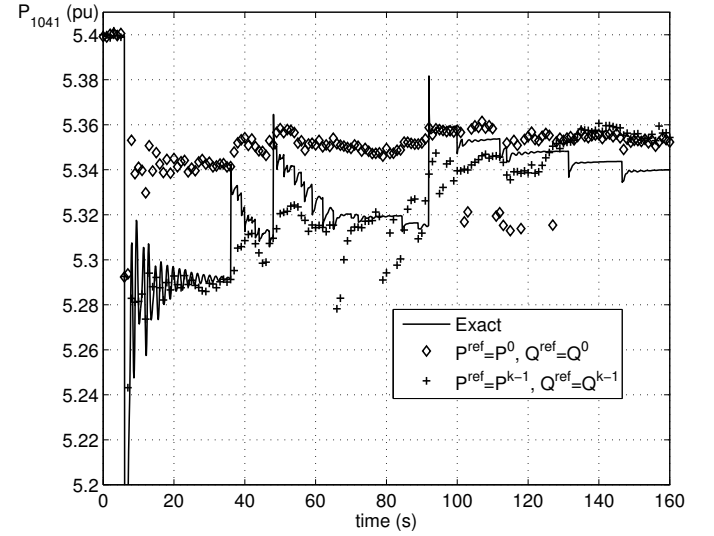


Fig. 7. Exact and reconstructed load active power at bus 1041; $w_{Pj} = w_{Qj} = 1$ ($j = 1, \dots, p$) and $w_i = 500$ ($i = 1, \dots, m$)

ever, they experience “jumps” that do not correspond to changes in system state but might indicate sensitivity to measurement noise;

- in addition, considering that the weights should be equal to the reciprocal of the measurement noise variances, a ratio of 10,000 would correspond to a ratio of 100 of the standard deviations, which is difficult to justify.

The following is observed when setting reference bus powers to the values provided by the previous state reconstruction:

- the reconstructed voltages are already accurate when setting the phasor measurements weights 100 times higher than the pseudo-measurements weights;
- they appear even more accurate when increasing this ratio to 10,000; however, the intermediate case with a ratio of 500 does not confirm this trend of the accuracy;
- the reconstructed powers follow the exact values, but in a

smoother manner when setting the phasor measurements weights 100 times higher than the pseudo-measurements weights;

- when this ratio is increased, the reconstructed powers experience “jumps” that are not correlated with changes in system state and might indicate sensitivity to measurement noise;
- a ratio of 100 in the weights is more reasonable from the already mentioned viewpoint of noise standard deviation.

The most accurate results appear to be obtained when taking the last reconstructed bus power injections as reference, and assigning reasonably (in the example, 100 times) larger weights to phasor measurements than to pseudo-measurements.

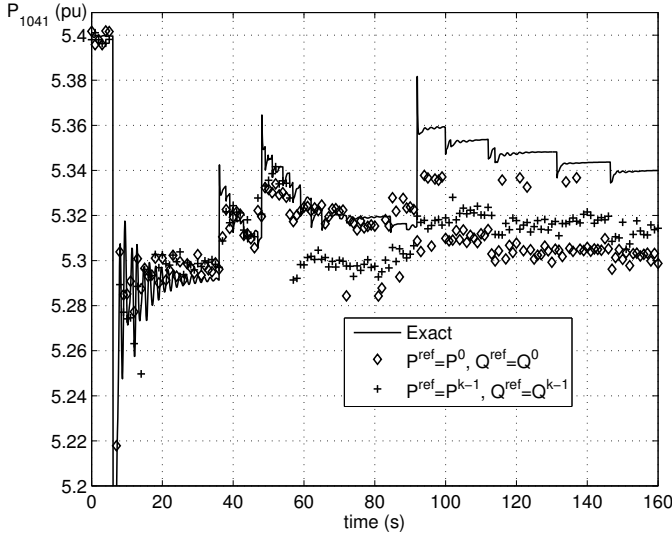


Fig. 8. Exact and reconstructed load active power at bus 1041; $w_{Pj} = w_{Qj} = 1$ ($j = 1, \dots, p$) and $w_i = 10,000$ ($i = 1, \dots, m$)

C. Consideration of different state reconstruction rates

In the simulations presented so far, phasor measurement snapshots were assumed to be taken every second. This is of course a large period considering the rate at which PMU data are available. Clearly, the rate at which states can be reconstructed is limited by the time it takes to solve the constrained optimization problem. This can be quite reasonable if only a region is monitored, and the unknowns are initialized from the previous reconstructed state.

In this section, the effect of processing the phasor measurement snapshots at different rates is illustrated (irrespective of the time taken by each reconstruction). Of course, this rate has no effect when the pseudo-measurements P_j^{ref}, Q_j^{ref} relate to a single point in the past, typically the last run of the state estimator³. Instead, the situation of interest is when they are taken from the last reconstructed state.

Figure 9 shows the exact evolution of the voltage at bus 1041 as well as the values reconstructed when processing a phasor measurement snapshot every 0.2 and 2.0 seconds, respectively. Similar plots are given in Fig. 10 for the active power of the load at the same bus. In all cases, the phasor measurements weights were set 100 times higher than the pseudo-measurements weights.

The following observations can be made:

- as expected, performing state reconstruction more frequently allows a better tracking of the system dynamic evolution;
- there is no significant degradation of state reconstruction accuracy when sampling every 2 seconds, except during sharp transients;
- those fast changes are nicely captured by the states reconstructed every 0.2 second, while both reconstructed trajectories converge to each other when the system is more quiet.

³coming back to computational effort, in this case, the reconstruction rate can be increased using several processors, each processing a different snapshot

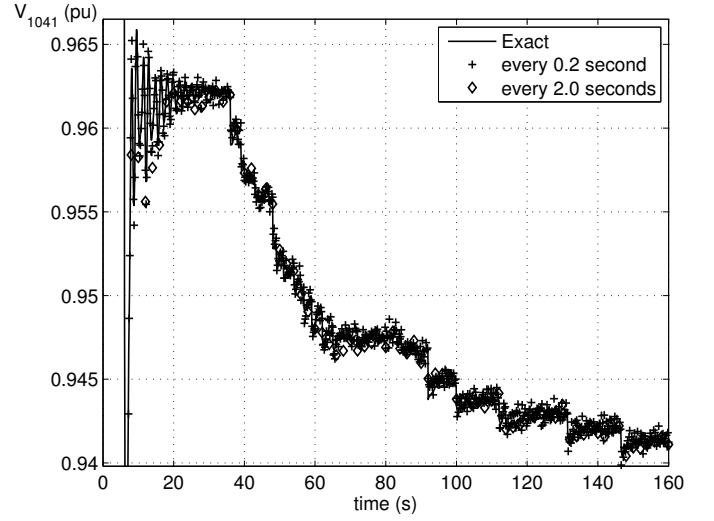


Fig. 9. Exact and reconstructed voltage at bus 1041 for different state reconstruction rates

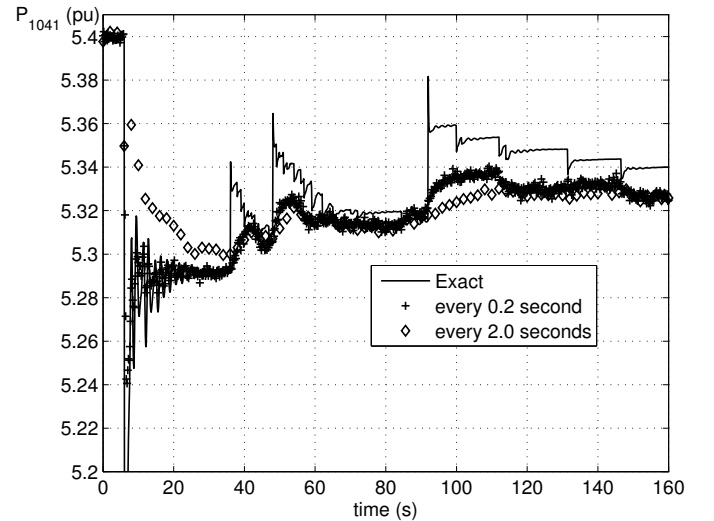


Fig. 10. Exact and reconstructed load active power at bus 1041 for different state reconstruction rates

The rate at which the changing system state should be reconstructed is a compromise between the required accuracy and the capabilities of the optimization solver. This last aspect is currently being investigated.

D. Example of quantity monitored from reconstructed states

An example of quantity that can be monitored from reconstructed states is the current in a transmission line impacted by the disturbance. This is illustrated in Fig. 11 relative to line 4032-4042, located in the same corridor as the outaged line. The figure shows respectively the exact current and the current computed from reconstructed bus voltages. The phasor measurement weights were set 100 times larger than the pseudo-measurement weights, and the bus powers determined at the previous state reconstruction were taken as reference.

As can be seen, the jump caused by the outage and the increase that accompanies the restoration of distribution volt-

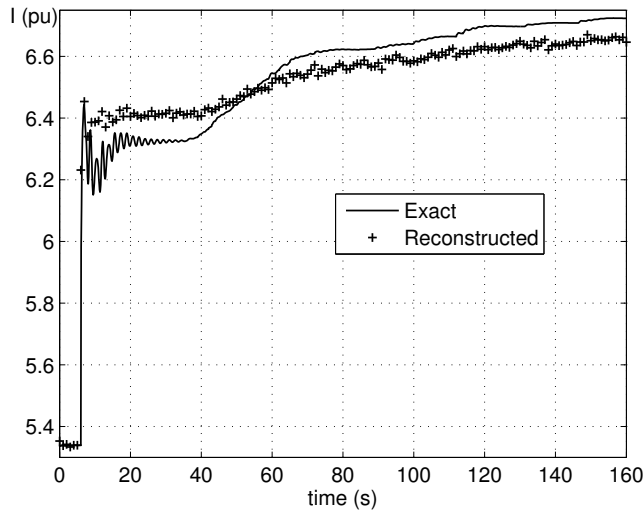


Fig. 11. Exact and reconstructed current in line 4032-4042

ages by LTCs are restituted with good accuracy, showing the capability of state reconstruction to track the system evolution and hence, anticipate possible cascading effects.

The accuracy is noteworthy considering that none of the ending bus of the line is provided with a PMU. Even more, the transmission line of concern connects the Central area to the North one, where no PMU has been considered (in the one-line diagram of Fig. 1, there is no PMU above the dotted line separating the North and Central regions). Hence, both terminal voltages of the line were reconstructed.

V. CONCLUSION

The purpose of state reconstruction is to fill the gap between successive classical state estimations by exploiting the time-synchronized phasor measurements to reconstruct coherent states of the system at various time instants during its post-disturbance evolution. This contributes to increasing situational awareness, for instance after an incident.

Among the features of the approach, let us quote that:

- it relies on constrained optimization in which computational efficiency can be achieved by exploiting the sparsity and linearity of the equality constraints;
- PMUs should be located at generator buses, which is also a proper location for assessing electromechanical oscillations;
- it is possible to accurately reconstruct the state of only a sub-network, using PMUs on the nearby generators. Outside the region of interest, the reconstructed state is less accurate, since it relies on pseudo-measurements only.

This paper has reported on extensions of our previous work in [9]. They deal with the processing of properly weighted phasor and pseudo-measurements together in the least-square sense, and the update of the reference bus powers used as pseudo-measurements. The corresponding results show the capability of filtering the noise on phasor measurements. They also suggest that the most accurate reconstruction is obtained with the reference bus powers taken from the last

state reconstruction and with reasonably larger weights assigned to phasor measurements. Expectedly, better tracking can be obtained by increasing the rate at which states are reconstructed, at the price of a higher computational effort.

Issues and improvements under investigation consists of:

- further testing the capability of local state reconstruction. To this purpose, PMUs located on the tie-lines linking the region of interest to the rest of the system could supplement those on the nearby generators;
- evaluating the benefits of state reconstruction to improve classical state estimation;
- investigating the best possible use of the PMU data gathered in between two state reconstructions;
- testing the quality of the reconstructed state in demanding applications, such as voltage instability detection [12]
- devising a dedicated algorithm to solve the optimization problem.

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