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Liège, May 27, 2011

Dear Editor,

In this revised version of Manuscript GEOMOR-2447, we followed all recommendations of the reviewer. The content modifications appear in red in the text. As requested, the main change is the addition of a new section 5 dealing with the comparative analysis of the various types of adjustment based on a series of synthetic data sets. This brought also us to add a table and two figures to the previous material and to revise our conclusions.

We hope that the reviewer will accept this revised version. We are looking forward to reading his reaction.

With many thanks again for all the reviewing and editing work.

Sincerely yours,

Alain

Response to Reviewers

All your comments were welcome and very constructive. We have followed all recommendations and thank you sincerely for the appreciable improvement of the manuscript that they allowed. Not only is now its meaning much increased, but the conclusion is also somewhat changed.

***Highlights**

Adjustment of the stream power model to knickpoint data yields significantly different results depending on the misfit function, based either on distance or time residuals; the analysis of synthetic data sets shows that the latter approach is more accurate; the value of the m exponent is then slightly decreased.

On different types of adjustment usable to calculate the parameters of the stream power law

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Abstract

Model parameterization through adjustment to field data is a crucial step in the modeling and the understanding of the drainage network response to tectonic or climatic perturbations. Using as a test case a data set of 18 knickpoints that materialize the migration of a 0.7-Ma-old erosion wave in the Ourthe catchment of northern Ardennes (western Europe), we explore the impact of various data fitting on the calibration of the stream power model of river incision, from which a simple knickpoint celerity equation is derived. Our results show that statistical least squares adjustments (or misfit functions) based either on the stream-wise distances between observed and modeled knickpoint positions at time *t* or on differences between observed and modeled time at the actual knickpoint locations yield significantly different values for the m and K parameters of the model. As there is no physical reason to prefer one of these approaches, an intermediate least rectangles adjustment might at first glance appear as the best compromise. However, the statistics of the analysis of 200 sets of synthetic knickpoints generated in the Ourthe catchment indicate that the time-based adjustment is the most capable of getting close to the true parameter values. Moreover, this fitting method leads in all cases to a m value lower than that obtained from the classical distance adjustment (for example, 0.75 against 0.86 for the real case of the Ourthe catchment), corresponding to an increase in the non linear character of the dependence of knickpoint celerity on discharge. Accordingly, most published values of m might be somewhat overestimated.

Keywords. bedrock channel incision; stream power model; knickpoint propagation; drainage network.

1. Introduction

In studies of the river incision response to base level change, it is usual to analyze the upstream propagation of a wave of regressive erosion in the frame of a detachment-limited model expressing the "erosion" rate ε (more strictly, the rate of elevation change) in a bedrock channel as

$$\varepsilon = dz/dt = U - I = U - KA^{m}S^{n} \tag{1}$$

where U and I are respectively the uplift and incision rates, *A* is the drainage area (a surrogate for discharge) and *S* is the channel gradient. K is a coefficient of erosional efficiency and m and n are derived from exponents involved in various relations linking drainage area, discharge and channel width. Estimates of the m and n exponents depend on each specific physical model but the m/n ratio keeps constant at ~0.5, with m ≈ 0.5 and n ≈ 1 in the unit stream power model, and m ≈ 0.33 and n ≈ 0.67 in the shear stress model (Whipple & Tucker, 1999). In this respect, the stream power model of Seidl and Dietrich (1992) is unique in estimating m = n ≈ 1. Therefore, most applications of such models to case studies assumed that n ≈ 1, so that equation (1) is in fact a kinematic wave equation from which one may derive a simple expression of the celerity c at which a knickzone propagates within a drainage network

$$c = KA^{m} \tag{2}$$

Note that, if n ≠ 1,

$$c = KA^{m}S^{n-1} \tag{3}$$

which becomes, using the expression for S at steady state (Whipple & Tucker, 1999),

$$c = \mathcal{K}^{1/n}A^{m/n} \tag{4}$$

where $\mathcal{K} = KU^{n-1}$.

Based on these equations, many workers used field observations collected in drainage networks affected by knickpoint (or knickzone) migration to estimate the parameters m (or m/n) and K (or $\mathcal{K}^{1/n}$). As such studies rely on sets of knickpoints present at some distance upstream of the initial location of the base level change, and as the (unit) stream power models maintain best the shape of a retreating knickpoint, one generally expects m to fall in the range between 0.5 and 1. In the most frequent case, the data set comprises the current location of knickpoints associated with an erosion wave whose starting time (or age) is more or less well constrained and the parameterization of (2) is done by least square minimization on the stream-wise distances between the observed and modeled position of the knickpoints at a given time (e.g., Crosby & Whipple, 2006). Following this approach, Berlin and Anderson (2007) obtained for example m = 0.54 based on the analysis of a 7.8 ma-old erosion wave in western Colorado, whereas Crosby and Whipple (2006) calculated m = 1.125, not significantly different from 1, for knickpoints very close to the headwaters in New Zealand. In fewer studies, the analysis uses age data showing that a particular terrace level was abandoned diachronically, depending on the propagation of the erosion wave. In such a case, the data set gathers snapshots of the temporal evolution of one (or several) knickpoints rather than knickpoints spatially spread but belonging to a single generation (Rixhon et al., 2010). The same type of adjustment based on distances may be applied in such instances.

Pointing that Crosby and Whipple (2006) already suggested that misfit criteria other than distance between modeled and observed knickpoints might be used, and although they found no significant difference between the model parameters obtained from the various fits they tested (based, e.g., on drainage area and elevation), we question here the choice of distance as the most efficient variable to calculate realistic parameter values.

2. Two ways of fitting knickpoint data

There are two possible expressions of the physical reality of knickpoint propagation, integrating either the knickpoint's travel distance over a given time span or the travel time corresponding to a given distance of propagation. However, because they both involve the variable drainage area, these expressions cannot be solved analytically and the problem is therefore always treated numerically. Working with a fixed-size square grid, one calculates the time needed by a knickpoint *i* to move in the drainage network from a grid point *j* to the neighbor point *j-1*, to which a fixed A_{j-1} (the area drained at the "entrance" in the grid point *j*) is associated

$$\Delta t_{i,j} = \int_{i,j}^{i,j-1} \frac{-1}{\mathrm{KA}^{\mathrm{m}}} dx \approx \frac{1}{\mathrm{KA}_{i,j-1}^{\mathrm{m}}} (x_{i,j} - x_{i,j-1}) \qquad (5)$$

The time needed for the erosion wave to reach any point *n* in the drainage network is then given by

$$t_{i,n} = \sum_{j=j_{outlet}}^{n} \Delta t_{i,j} \qquad (6)$$

which is the universal way to model knickpoint migration before fitting the modeled information to the field data. The misfit function is usually given by the sum of the squared differences between observed and modeled distances and implicitly assumes that time is the independent variable determining the location of the knickpoints. However, there is fundamentally no causal link between time and spatial location, which are two dimensions of a phenomenon depending on the process of bedrock erosion, i.e. ultimately on variables such as discharge (or drainage area) and channel gradient. In other words, it is not more justified to consider the location of knickpoints as time-dependent than to admit that the temporal existence of knickpoints depends on the fact that they evolve in a spatial dimension.

Consequently, there is no less reason to parameterize the knickpoint celerity relation by using a misfit function based on time (e.g., minimizing the sum of squared differences between observed and modeled time needed for the knickpoints to migrate to their actual location) rather than distance and, at first glance, a statistical least rectangles adjustment might theoretically be the best solution.

3. A test case in northern Ardennes (western Europe)

To test the consequences of choosing one or the other parameterization approach, we analyzed a set of 18 knickpoints mapped in the Ourthe catchment of the northern Ardennes massif (western Rhenish shield, western Europe) (Fig. 1). With a catchment size of ~3600 km^2, the Ourthe River is the main tributary of the Meuse River in the Ardennes massif. As witnessed by the incision history of the rivers (e.g., Van Balen et al., 2000), this area underwent an uplift pulse around 0.7 Ma, which caused an erosion wave to propagate by knickpoint migration in the drainage network and to leave behind it a prominent terrace level called the Younger Main Terrace (YMT) (Demoulin et al., 2009). Based on a digital elevation model of the Belgian National Geographic Institute with a 20 m resolution, we extracted 76 knickpoints from the slope/drainage area log/log plots of the streams pertaining to the Ourthe catchment. The knickpoints corresponding to the post-YMT incision phase were identified by their geometrical coincidence with the upstream end of the YMT profile (Fig. 2) and all knickpoints whose location was suspected of being influenced by the bedrock lithology were removed.

As the point at issue is to examine the differences in the parameter values obtained from adjustments either on distance, on time, or on both of them, we shall not focus anymore on the geomorphological meaning of the analyzed set of knickpoints. In each case, to constrain the parameters m (or m/n) and K (or $K^{1/n}$), we classically used the brute force two-parameter search first advocated by Stock and Montgomery (1999), with a 300x300 search matrix, m (or m/n) varying linearly between 0.2 and 1.3, and K (or $K^{1/n}$) logarithmically between 10^{-12} and 10^{-3}. For the numerical modeling, we used equations (5) and (6) to calculate the knickpoint travel time across each pixel of the drainage network and the cumulated travel time associated to each knickpoint. In the distance adjustment, we determined the best (m, K) couple by minimizing the sum of the squared stream-wise linear distances between the actual position of each knickpoint and its modeled position at time t = 700 ka, a negative residual indicating that the actual knickpoint is located downstream of its modeled position, i.e., its propagation was delayed. As for the time adjustment, it searched to minimize the sum of squared differences between 700 ka and the modeled times at which the knickpoints should have reached their actual location. In this case, a negative residual means that the knickpoint migrated faster than expected. Finally, a least rectangles adjustment was performed by minimizing the sum of the products (|time residual| x |distance residual|) of all knickpoints. The time and distance residuals were standardized in all adjustments, the reference time being 700 ka and the reference distance corresponding to the longest distance traveled by a knickpoint (145.843 km), so that both variables have the same weight and their combined use in the least rectangles approach makes sense. We also underline that, because of the non-linearity of the relation between c (or t) and A (or x), the least rectangles adjustment induces a slight bias, namely towards a greater m in this power law case. By the way, note that time and distance residuals may of course be calculated for each type of adjustment.

Table 1 shows the best fit results for the three adjustments, the parameter ranges for which the misfit values are comprised within 4% of the minimum misfit (or best fit), and the mean and standard deviation of time and distance residuals in each attempt. Although the standardized misfit values are not strictly comparable, they suggest that time is the most unstable variable and that time adjustment is thus most sensitive. Fig. 3 illustrates how several characteristics of the data set may potentially bias the least square estimation of m (or m/n) and K. Firstly, as the relation between time and position of the knickpoints is a power law (if we neglect the jumps in A at the junctions), the distribution of the data mainly either on the right side of the graph (in case of an older generation of knickpoints) or on the left side (for a more recent erosion wave) will determine the most sensitive adjustment (Fig. 3.A). Young knickpoints are still mostly located in the steeper part of the x = f(t) curve where the adjustment on distances is more sensitive than that on time, and the reverse is true for a set of older knickpoints appearing in the flattened part of the curve, as exemplified by the Ourthe data set. Consequently, if a data set contains knickpoints distributed over a large range of stream orders (and thus of drainage area), best fits based on distance or time will adjust preferentially on the knickpoints respectively located in the larger or smaller streams, thus skewing every comparison.

Secondly, while it is expected that a random distribution of the data above and below the fitting curve will not affect the latter's definition, actual data may display some systematic trend not captured in the modeling. Deviations from the model noted by Wobus et al. (2006) in the particular case of the hanging valleys of Taiwan correspond for example to a situation where the knickpoints tend, because of any control not included in *A* and *S*, to lag behind their modeled position in the smaller streams (the data points are situated below the fitting curve), and vice-versa in the larger rivers. Such a systematic distribution of the residuals, positive in the larger rivers and negative in the smaller streams as far as distance residuals are concerned, must result in a higher estimate of m through distance adjustment and a lower one through time adjustment (Fig. 3.B).

Although such a trend is less obvious in the Ardennian example, the location of the most delayed knickpoints (with respect to their modeled positions) in lowest-order streams of the downstream half of the catchment has a similar effect on the adjustment, so that one indeed obtains a lower m from the time adjustment (0.75) than from the distance adjustment (0.86). The (1.04*$misfit_{min}$) value taken as quality indicator of the adjustment was arbitrarily chosen for the sake of comparison with the figures given in Berlin and Anderson (2007). At this level, while the amplitude of the m range is very similar in the three adjustments, the best fit m value obtained from the distance adjustment differs significantly from those yielded by the two others.

4. Implications

As there is no physical reason to privilege distance over time, or conversely, in the adjustment, we might provisionally assume that the intermediate least rectangles approach is the most appropriate. Here, probably because the migrating erosion wave has already propagated far upstream in the

Ourthe catchment, the influence of the time variable predominates in the least rectangles adjustment, which yields parameter values closer to those of the time adjustment.

In order to examine the consequences of choosing one or another type of adjustment, one may first evaluate how they affect the knickpoint celerity calculated in the downstream and upstream regions of the catchment. At the Ourthe catchment's outlet, the distance, time, and least rectangles parameterizations of the celerity equation yield propagation speeds respectively of 0.87, 0.71 and 0.73 m/y, quite in the range of what has been measured or inferred elsewhere on a comparable time scale (Loget & Van Den Driessche, 2009). In upstream reaches, calculations for a drainage area of 15 km^2 give, in the same order, speeds of 0.008, 0.012 and 0.010 m/y. The latter figures, implying that the erosion wave needs about 100 ka to propagate 1 km farther in the low-order catchments, highlight the reason why the time variable is much more unstable than distance and, in general, why time residuals are significantly higher than distance residuals for knickpoints located close to the headwaters. But, though distance adjustment slightly favors faster propagation in the lower reaches and time adjustment in the upper reaches, the three models calculate very similar celerity values. The significant difference lies rather in the degree of non linearity of the relation linking *c* to *A*, and consequently in the relative importance of the controls included in the erosional efficiency coefficient K. With higher m values, the distance adjustment tends to exaggerate the impact of drainage area (i.e., discharge) on *c*, at the expense of other effects included in K, notably sediment supply and bedrock erodibility. In the test case, for a difference in m of 0.11, K differs by one order of magnitude between distance and time adjustments.

5. What is the most correct approach? A synthetic test case

It has been so far shown why the distance and time adjustments of knickpoint data yield significantly different results, leading us to hypothesize that a least rectangles approach might be preferable, but this still needs a demonstration that the results of such an adjustment are closest to the reality. In order to assess which of the three methods performs best, we therefore analyzed how close they respectively come to the true parameter values used to propagate a set of synthetic knickpoints.

The drainage network of the Ourthe basin was used to create two synthetic data sets by propagating an erosion wave in it at a celerity $c = 10^{-7} * A^{0.7}$ and noting the knickpoint location at times $t_1 = 400$ ka and $t_2 = 750$ ka in the 18 rivers used in the real case described above (at 400 ka, the erosion wave had not yet reached 6 of the sampled rivers, so that the corresponding data set contains only 12 knickpoints). These sets of perfectly adjusted knickpoints could obviously not serve directly to make comparative tests of various fitting methods before some noise was added to the data. A simple noise made of Gaussian perturbations affecting randomly the positions of the knickpoints in the drainage system led to unrealistic situations where some of the highest advances with respect to the model were ascribed to knickpoints in very small streams, whereas we know that knickpoints entering smaller tributaries are almost systematically delayed. The earlier a knickpoint migrates to a low-order stream in the downstream part of the catchment, the shorter its travel distance within a given time span will be, and this corresponds in general to a more negative distance residual or a higher time residual in the adjustment, probably because of an additional control not captured in *A*. This is illustrated by the good negative correlation between travel distances and time residuals for the real knickpoints of the Ourthe catchment (Fig. 4), corresponding to a positive correlation between travel distances and distance residuals, so that we chose to apply to the modeled knickpoint positions more realistic perturbations obtained by ranking 18 random values of a Gaussian noise and assigning them in order to the knickpoints with increasing travel distances. We created so 100 synthetic data sets for t_1 and t_2, with mean and standard deviation of the perturbations randomly taken respectively in the [-100, 100] and [1500, 2500] ranges (in meters), in agreement with the residual statistics observed at the scale of the Ourthe catchment.

The statistics of the m and K parameters obtained from the three types of adjustment applied to the synthetic data sets are presented in Table 2 and Figure 5. In both cases (400 and 750 ka), the adjustment based on the minimization of the differences between observed and modeled times yielded parameter values closest to the true values (m = 0.7 and K = 10^{-7}). The difference between these values and those obtained from the two other fitting methods is statistically significant at the 99% confidence level. Contrary to all theoretical expectations, the time adjustment worked statistically better not only for knickpoints located high in the drainage network (750 ka) but still more in the case of knickpoints situated more downstream (400 ka), suggesting that the relative higher sensitivity of the distance-based adjustment diminishes very rapidly when the knickpoints move away from the catchment's outlet and migrate to tributaries with much smaller drainage areas. It is also surprising that, despite the lesser efficiency of the distance-based fitting to reproduce the true values of the parameters, the distance variable seems on average to exert the main influence on the least rectangles adjustment, which produces very similar results and cannot thus compete with the time-based approach. However, confirming what was already noticed in the Ourthe real case, the results of the synthetic data sets (Fig. 5) emphasize that, if the unstable character of the time variable makes time adjustment more sensitive and, in general, more effective, it is also responsible for this approach's main drawback, namely a greater scatter of the results. Therefore, when a single small set of knickpoints is available for a regional calibration of the stream power model, one might wonder whether the chances of getting close to the true parameter values are higher with the accurate but not very precise time-based fit or the more precise but inaccurate distance-based fit. The answer is given by the mean deviation from the true parameter values of the values calculated for the individual synthetic data sets (in absolute value), which shows that the time-based approach hit on average closer to the true values than the other adjustments (Table 2) and that, all in all, though more sensitive to outliers, it should consequently be preferred for the model parameterization.

6. Conclusion

We have shown that misfit functions based respectively on distance and time residuals yield significantly different parameter values for the stream power law of river incision and knickpoint migration. As calibration of the models and understanding of the relationships between the factors that drive river incision rely heavily on field data, it is of utmost importance that the latter be exploited in the most effective way, which seems to generally imply that an adjustment based on the minimization of the differences between observed and modeled propagation times be employed. Indeed, all statistics based on 100 sets of synthetic knickpoints analyzed at two stages of their upstream migration show that this fitting method offers the highest probability of a right parameterization. In all cases, the time adjustment yielded m values lower than those obtained from the classical distance adjustment, which suggests that the published values of m, derived from the latter approach, might have generally be somewhat overestimated. The needed slight reduction of the m (or m/n) exponent implies a more non linear character of the relation between knickpoint celerity and discharge. Accordingly, the corresponding increase in the K coefficient enhances the role of bedload (via tool versus cover effects, and grain size effect) and bedrock erodibility in the stream power law.

Acknowledgments

We warmly thank the anonymous reviewer whose constructive comments contributed to greatly improve a first version of this note.

References

Berlin M., Anderson R., 2007. Modeling of knickpoint retreat on the Roan Plateau, western Colorado. *Journal of Geophysical Research* 112, F03S06, doi: 10.1029/2006JF000553.

Crosby B., Whipple K., 2006. Knickpoint initiation and distribution within fluvial networks: 236 waterfalls in the Waipaoa River, North Island, New Zealand. *Geomorphology* 82, 16-38.

Demoulin A., Hallot E., Rixhon G., 2009. Amount and controls of the Quaternary denudation in the Ardennes massif (western Europe). *Earth Surface Processes and Landforms* 34, 1487-1496.

Hack J., 1957. Studies of longitudinal stream profiles in Virginia and Maryland. *US Geological Survey Professional Paper* 294-B, 97 pp.

Loget N., Van Den Driessche J., 2009. Wave train model for knickpoint migration. *Geomorphology* 106, 376-382.

Rixhon G., Braucher R., Bourlès D., Siame L., Bovy B., Demoulin A., 2011. Quaternary river incision in NE Ardennes (Belgium) - Insights from $^{10}Be/^{26}Al$ dating of river terraces. *Quaternary Geochronology*, 6, 273-284.

Seidl M., Dietrich W., 1992. The problem of channel erosion into bedrock. In: Schmidt K., de Ploey J. (Eds.), *Functional Geomorphology*. Catena Supplement 23, 101–124.

Stock J., Montgomery D., 1999. Geologic constraints on bedrock river incision using the stream power law. *Journal of Geophysical Research* 104B, 4983-4993.

Van Balen R., Houtgast R., Van der Wateren F., Vandenberghe J., Bogaart P., 2000. Sediment budget and tectonic evolution of the Meuse catchment in the Ardenness and the Roer Valley Rift System. *Global and Planet. Change* 27, 113-129.

Whipple K., Tucker G., 1999. Dynamics of the stream-power river incision model: implications for height limits of mountain ranges, landscape response timescales, and research needs. *Journal of Geophysical Research* 104B, 17661–17674.

Wobus C., Crosby B., Whipple K., 2006. Hanging valleys in fluvial systems: Controls on occurrence and implications for landscape evolution. *Journal of Geophysical Research* 111, F02017, doi: 10.1029/2005JF000406.

Figure captions.

Fig. 1. Digital elevation model of the Ourthe catchment (delimited by the dashed line) locating the knickpoints identified (dots) and those selected in the data set as marking the current position of the post-YMT incision phase (circled dots). Many knickpoints not included in the data set might

nevertheless pertain to this same erosion wave and were removed only because of suspicion about possible lithological biases. The situation of the study area in western Europe is shown in the inset.

Fig. 2. A. Current river long profile and profile of the YMT in the Salm Valley. The knickpoint observed at the intersection of the two profiles marks the present position of the post-YMT erosion wave. B. Localization of the knickpoints in the slope/drainage area plot of the Salm River. The values of the concavity θ (= m/n) are given for the two profile segments upstream and downstream of the knickpoint. Compare the local value downstream of the Salm knickpoint (0.620) with the regional values yielded by the tested adjustments (0.752 for the time adjustment and 0.862 for the distance adjustment)

Fig. 3. Schematic diagram of the x = f(t) relationship of knickpoint propagation showing the influence of the positions of the knickpoints (A) either on the left (young) or right (old) side of the graph, and (B) either above or below the graph. For easier representation, we show the case of a diachronic data set made of successive positions of one knickpoint along a single stream, but the argumentation is identical for a synchronic data set comprised of current knickpoints distributed over streams of varying sizes. Note also that a real x = f(t) curve would be made of successive power law segments between discontinuities corresponding to the tributary junctions. A ν_t denotes the time residual and a ν_x the distance residual between the observation (black dot) and the fitting curve. See text for explanation.

Fig. 4. Negative correlation between the residuals of the time-based adjustment of the stream power model to the knickpoint data of the Ourthe catchment and the distance traveled by the knickpoints. The knickpoints that migrated less far show in general an additional delay not captured in the celerity equation. The more classical distance-based adjustment reveals an as good, but positive correlation.

Fig. 5. Box-whisker plots illustrating the distribution of the m and K parameter values obtained from distance-based, time-based, and least rectangles adjustments applied to 100 random sets of synthetic knickpoints in the Ourthe catchments for two epochs, respectively 400 and 750 ka after the erosion wave has started to propagate in the catchment. Though more scattered, the results of the time-based adjustment are on average closer to the true values (bold lines) than those of the other fitting methods.

Table 1

Table 1. Results of the three types of adjustment performed on the knickpoint data set of the Ourthe catchment. The standardized misfit values are obtained by dividing the misfit values respectively by the squared longest observed knickpoint travel distance (145.843 km), the squared travel time (700 ka) and the product of longest observed knickpoint travel distance by travel time.

	misfit	m		K (10^{-9} $m^{1-2m}y^{-1}$)		time residuals (ky)		distance residuals (km)	
	(standard.)	best fit	1,04 $misfit_{min}$	best fit	1,04 $misfit_{min}$	mean	st. deviation	mean	st. deviation
distance adjustment	0,0039	0.862	0.833 - 0.947	5.04	0,955 - 8.77	-41,305	225,248	0,035	2,202
time adjustment	1,2677	0.752	0.686 - 0.814	46.3	14.3 - 161.2	68,272	178,007	-0,836	2,300
least rectangles	0.0583	0.785	0.730 - 0.833	23.1	9.40 - 70.2	20,608	192,460	-0,258	2,282

Table 2

Table 2. Results of the three types of adjustment performed on two batches of 100 sets of synthetic knickpoints obtained by adding a noise (see text for detail) to the knickpoints positions modeled in the Ourthe catchment at two different times with m = 0.7 and K = 10^{-7}.

			distance-based fit	time-based fit	least rectangles
t_1 = 400 ka	m	mean	0.80	0.71	0.79
		st. deviation	0.03	0.09	0.05
		mean |m-0.7|	0.10	0.08	0.09
	log K	mean	-7.86	-7.11	-7.78
		st. deviation	0.27	0.83	0.43
		mean |log K+7|	0.86	0.70	0.81
t_2 = 750 ka	m	mean	0.82	0.76	0.82
		st. deviation	0.04	0.08	0.06
		mean |m-0.7|	0.12	0.09	0.12
	log K	mean	-8.01	-7.54	-7.98
		st. deviation	0.36	0.69	0.52
		mean |log K+7|	1.01	0.74	0.98

Figure 1
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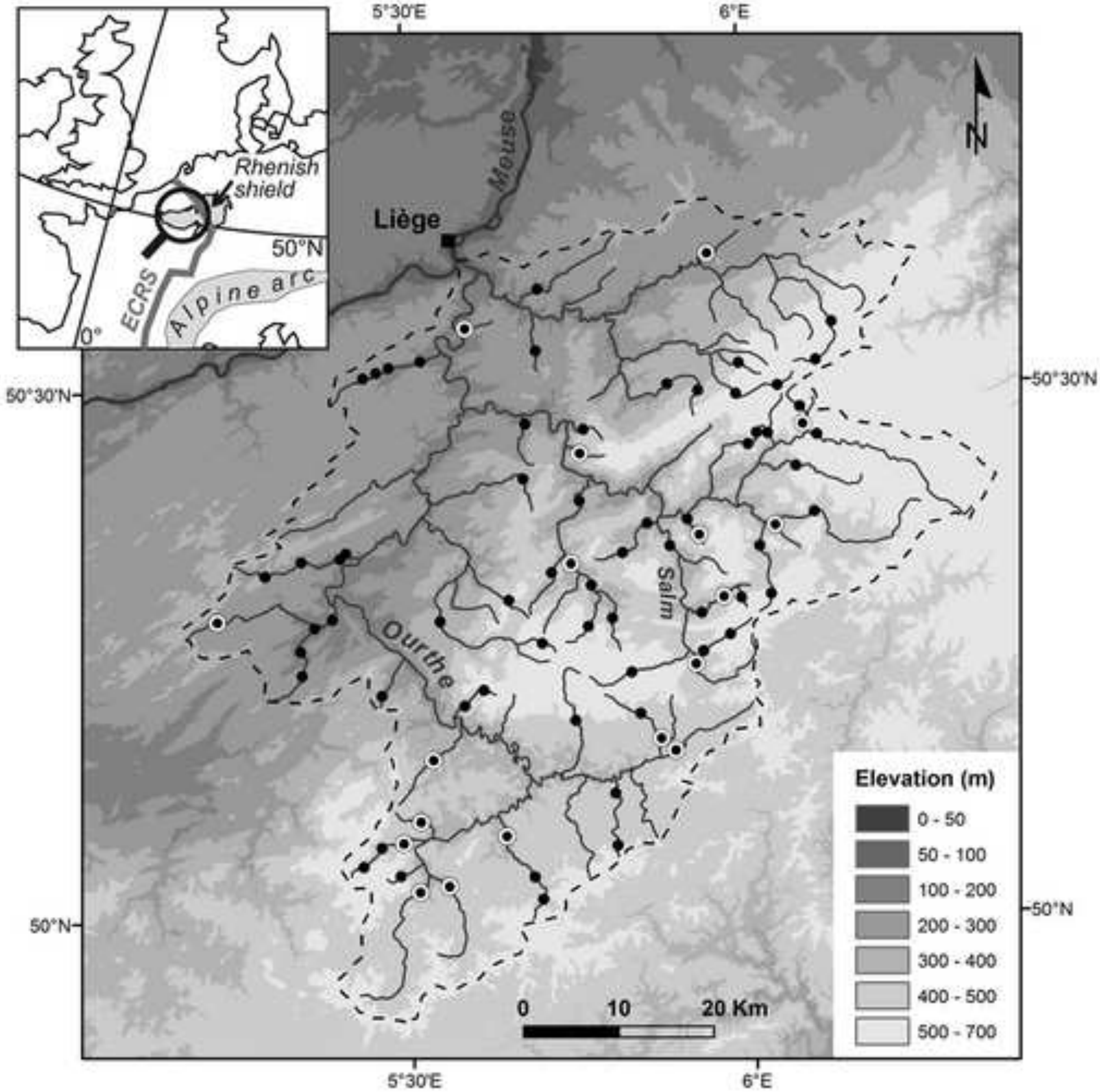

Figure 2

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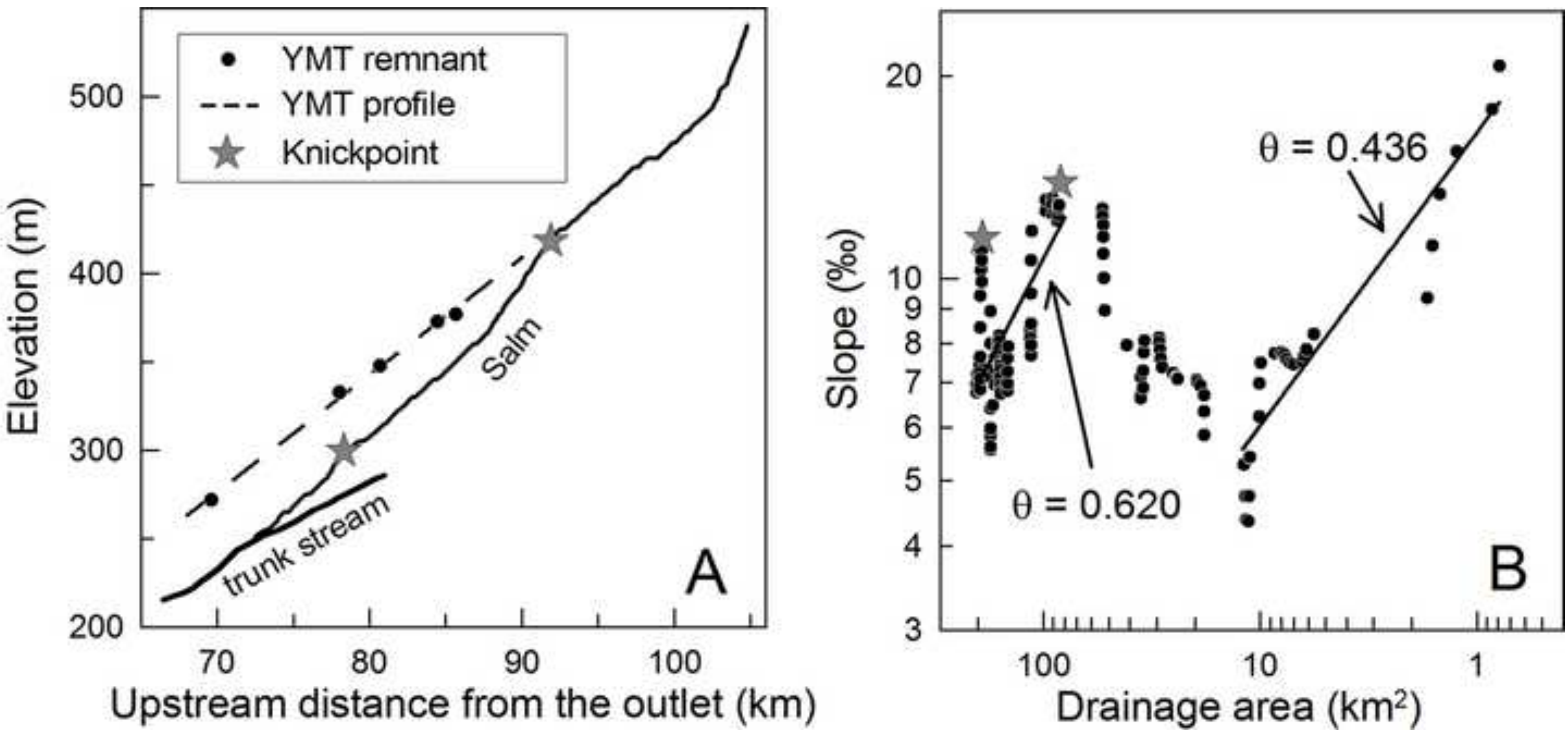

Figure 3
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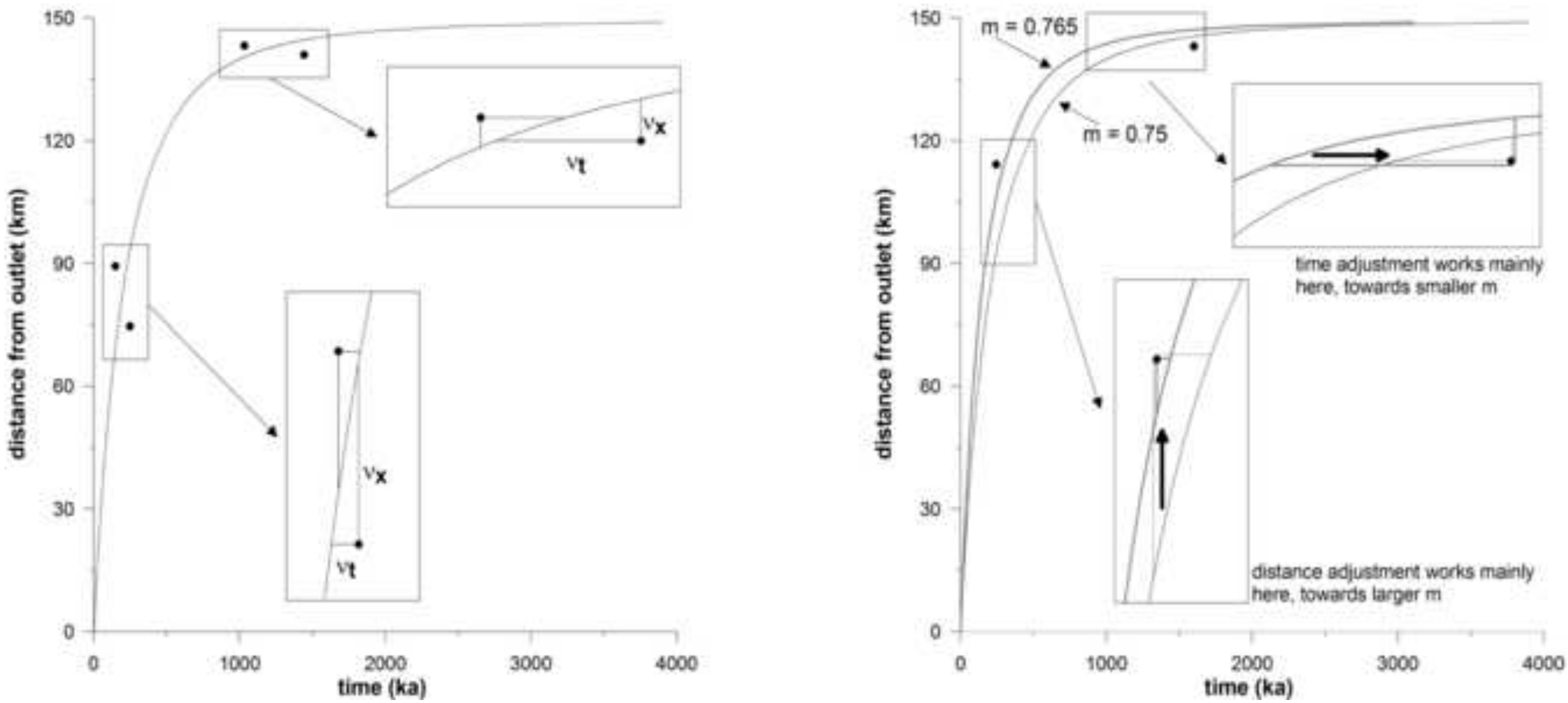

Figure 4
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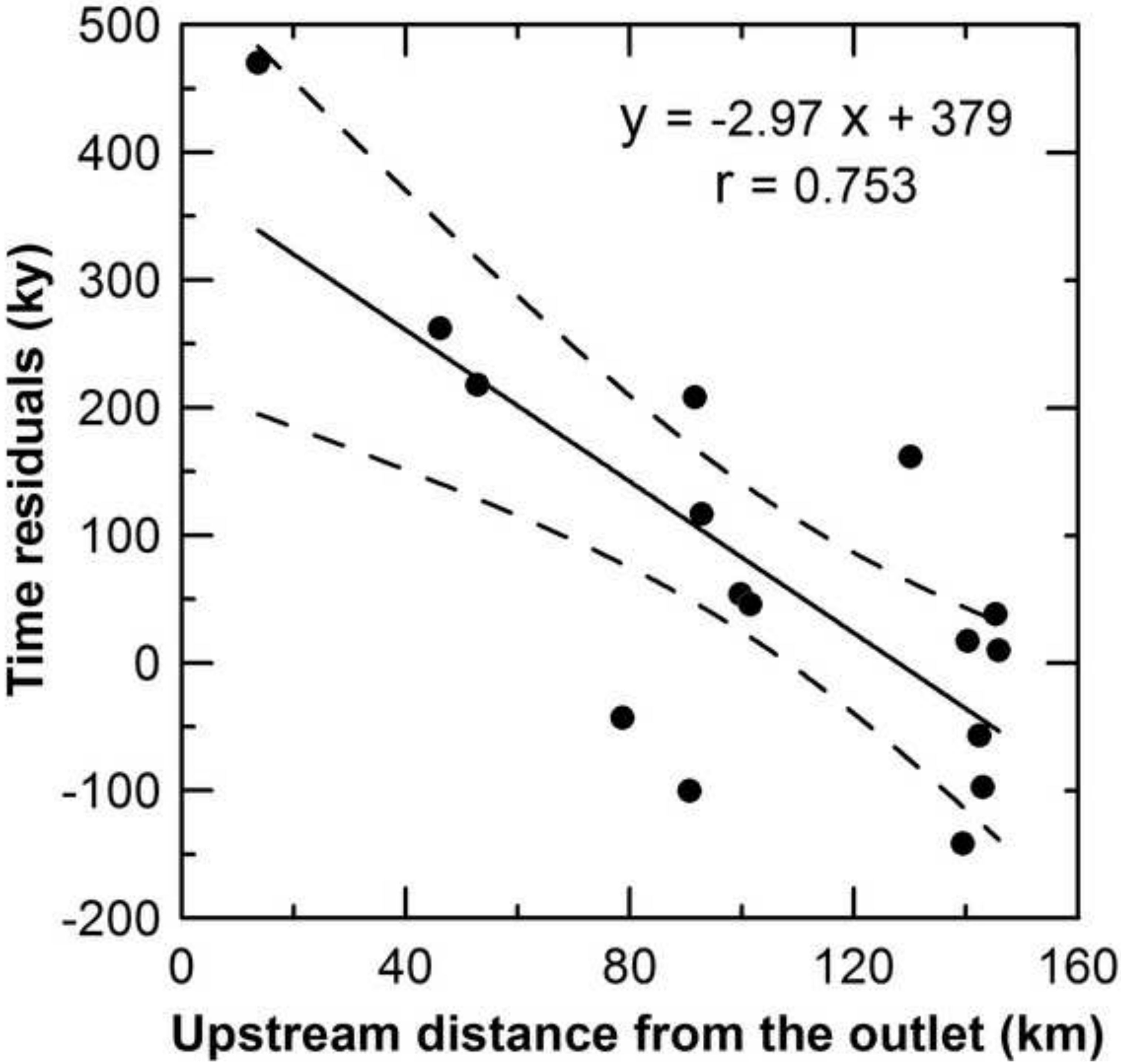

Figure 5
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