ON THE INFLUENCE OF BACKGROUND COMPONENT IN RESONANCE OF CABLES

Thomas CANOR[1,2], Vincent DENOËL[2]

[1]National Funds for Scientific Research of Belgium

[2]Structural Engineering Division, Argenco, University of Liège

Chemin des Chevreuils 1, B52/3, 4000 Liège, Belgium

emails: T.Canor@ulg.ac.be, V.Denoel@ulg.ac.be

Abstract

This work deals with nonlinear dynamical behaviour of cables in the context of random excitations of one of its anchors. Different studies have already been realized, that highlight different cable responses under random excitations such as white noise or narrow band processes. This work can be seen as an extension of these former works. It aims at modeling a more realistic random excitation by associating a background component to a resonant excitation. The background component models the effects of turbulence on the structure and the resonant component results from structural vibrations of the primary system. Any direct excitation on the cable is disregarded in this work.

The first part of this paper describes the model and the random excitation generators. The second part deals with the influence of the background component on the possible resonance of a cable. Results are presented for a given level of energy in the excitation, and as a function of its dispatching between the background and resonant components. This work shows that the background component reduces the vibration in the out-of-plane mode and can avoid resonance from taking place. The on-off intermittency *phenomenon is also observed for the out-of-plane mode and it depends also on the background component.*

INTRODUCTION

Cables are widely used in civil engineering structures as in cable-stayed bridges, suspension bridges or guyed masts. Cables are of great interest for engineers and scientists because of their nonlinear behaviour, often source of *curious* and complex phenomena.

This work deals with nonlinear dynamical behaviour of cables in the context of random excitations. Many studies have already been carried on to better understand the dynamical behaviour cables submitted to internal and external deterministic excitations [2, 1]. Among them, parametric excitation is a challenging problem. It remains an important step in the global understanding of the resonance phenomenon in cables. Nevertheless, such excitations and associated resonance have rarely been encountered apart from lab experiments. A possible explanation for that is the inadequacy of the excitation model . For bridges or masts, excitations are not purely harmonic nor deterministic. For example these structures submitted to wind necessarily evince a random response because of the random nature of the loading.

The works of Ibrahim and Chang are focused on cables submitted to a white noise excitation. The cable is modeled with two [4, 3] or three modes [5]. These works highlight some typical features of the system such as the *on-off intermittency* phenomenon related to the stability of the out-of-plane mode depending on the intensity of the white noise.

More recently, papers written by Zhou, Larsen and Nielsen [6, 7, 8] have extended the results of Chang and Ibrahim by modifying the random excitation from a wide band random process to a narrow band process. By this approach, the influence of the structure on the anchor motions is considered through a second order filter. These work must be seen as a part of an evolution to a more realistic representation

of the indirect excitation of the cable. The cable model is limited to two modes and the results are obtained by Monte Carlo simulations. Georgeakis and Taylor [11] have also made the same kind of research with finite element model of cables. Zhou [7] clearly shows the dependence of the variances of modal coordinates with the damping ratio of the structural filter around the second harmonic of the cable: for high damping, the motion is mainly in the in-plane mode. The present paper is also inspired by the approach of Zhou, Larsen and Nielsen.
A realistic model of the random anchor motion is surely not a white noise (or a wide band process). Because the structure acts as a filter, a narrow band process is more realistic, but it assumes a single-mode structure, which is further subjected to a white noise excitation. Yet, a more realistic model would include the specific features of the turbulent loading. Consequently, the anchor motion discloses a very low frequency content resulting from the quasi-static response to the wind loads (the background component), as well as a narrow-band contribution corresponding to the single-mode structural response. The developments presented next build up on this more realistic excitation model.

NUMERICAL SIMULATIONS

Governing Equations

A set of governing equations for cable motion has been developed by Warnitchai and Fujino [9]. These equations describe the motion of the cable in in-plane and out-of-plane directions. They are coupled in a nonlinear way: a product of some powers of modal coordinates introduces a strong nonlinear behaviour in each mode. Their model allow for direct excitations (which are disregarded in this work) and indirect excitations resulting from the displacement of the anchors. In this study, only a weak cable-structure interaction is considered. Indeed the anchor motion is fixed *a priori*, i.e. without any feedback from the cable response.
The nonlinear coupling between modes introduces an important difficulty because it can induce chaos. The approach adopted here consists in studying the evolution of the statistical characteristics.
In order to simplify the system of equations, two modes are considered as: the first in-plane mode y and the first out-of-plane mode z. The upper anchor of the cable is supposed to be fixed ($u_a = v_a = w_a = 0$) and the cable is supposed to be only excited by a vertical motion of the lower anchor. The set of governing equations becomes

$$\begin{cases} \ddot{y} + 2\xi\omega_y\dot{y} + \omega_y^2 y + \beta_1 yz + \gamma_1 y^3 + \gamma_2 yz^2 + \alpha_1 ye(t)\cos\theta & = 0 \\ \ddot{z} + 2\xi\omega_z\dot{z} + \omega_z^2 z + \beta_2 y^2 + \beta_3 z^2 + \gamma_3 y^2 z + \gamma_4 z^3 + \alpha_2 ze(t)\cos\theta & = \ddot{e}(t)\left(\eta\cos\theta - \dfrac{2}{\pi}\sin\theta\right) \end{cases} \tag{1}$$

where $e(t) = u_b/u_0$ ($u_0 = HL_e/EA$ with L_e an equivalent length [9]) is the random anchor motion. The parameters α_i, β_i and γ_i are adapted from Warnitchai's model to highlight the nonlinear terms.

Background-Resonant Process

The process used to model the motion of the anchor is a third-order filter. First, a Gaussian white noise with zero mean $W(t)$ is filtered through an Ornstein-Uhlenbeck process

$$\dot{f} + \alpha_0 f = \sqrt{2\alpha_0}\sigma_f W(t) \tag{2}$$

where σ_f^2 is the variance of the process and α_0 a characteristic circular frequency. The resulting process f is then filtered through a second-order filter

$$\ddot{e}+2\xi_0\omega_0\dot{e}+\omega_0^2 e \quad = \quad f \tag{3}$$

where ξ_0 and ω_0 are respectively the damping ratio and the natural frequency of the filter (modeling the structure). This third-order filter models a currently observed phenomenon : wind modeled with the Ornstein-Uhlenbeck process, blows on a structure modeled with a single linear mode. Parametric resonance in the cable can be observed when the structural mode has a frequency close to the double of the frequency of the cable $\omega_0 \approx 2\omega_y \approx 2\omega_z$.
This random process can be easily analyzed with the common tools of linear stochastic processes. The power spectral density (PSD) of the Ornstein-Uhlenbeck process is

$$S_f(\omega) = \left|H_f(\omega)\right|^2 = \frac{\alpha_0}{\pi}\frac{\sigma_f^2}{\alpha_0^2+\omega^2} \tag{4}$$

and therefore, the PSD of the complete process is

$$S_e(\omega) = \left|H_f(\omega)H_e(\omega)\right|^2 = \frac{1}{\omega_0^4}\frac{S_f(\omega)}{\left(1-\left(\frac{\omega}{\omega_0}\right)^2\right)^2+\left(2\xi_0\frac{\omega}{\omega_0}\right)^2}. \tag{5}$$

Because the time scales related to the turbulence and the structural resonance are different ($\alpha_0 \ll \omega_0$), the variance of the process $e(t)$ can be approximated by the sum of a background and a resonant contributions, as

$$\sigma_e^2 \quad = \quad |H_e(0)|^2\,\sigma_f^2 + S_f(\omega_0)\frac{\pi}{2\xi_0\omega_0^3} = |H_e(0)|^2\,\sigma_f^2\,(1+\frac{1}{b}) = \frac{\sigma_f^2}{\omega_0^4}\left(\frac{b+1}{b}\right) = \frac{1}{2}e_0^2 \tag{6}$$

where e_0 can be interpreted as the RMS value of the equivalent harmonic signal. The parameter b is the ratio between the background contribution and the resonant contribution (in terms of variance), i.e.

$$b = \frac{|H_e(0)|^2\,\sigma_f^2}{S_f(\omega_0)\frac{\pi}{2\xi_0\omega_0^3}} = \frac{2\xi_0\left(\alpha_0^2+\omega_0^2\right)}{\alpha_0\omega_0} \tag{7}$$

which depends on α_0, ξ_0 and ω_0. If ξ_0 and ω_0 are supposed invariant (because depending on the structure), the process is completely described by e_0 and b.
Various simulations are performed next. They are compared for a fixed value of the parameter e_0 , in order to work under the assumption of constant energy injected in the system. The main scope consists in studying the influence of the background-to-resonant ratio b. Notice that Zhou models the random excitation by a second-order filter (a narrow-band process),

$$\ddot{e}+2\xi_0\omega_0\dot{e}+\omega_0^2 e = \sqrt{2\xi_0\omega_0^3}e_0 W(t) \tag{8}$$

which corresponds to the particular case $b=0$ of the present study.

Numerical Solution

The generation of realizations of a random process is succinctly presented: a white noise is first generated, then it is filtered through a first-order filter and finally the output signal is filtered through a second-order filter.

- *Generation of a white noise W.* A white noise is a mathematical idealization of a random process with infinite energy. The PSD of a Gaussian white noise is a constant with intensity D. It is commonly approached by limiting this frequency content to a large finite band. Samples of such an engineered white noise are generated as follows. Knowing the time step Δt, the associated Nyquist frequency f_{Ny} and the frequency resolution Δf , the constant PSD is sampled from zero to f_{Ny} with a step Δf. A complex random phase is associated to each point. Then using the property of the inverse Fourier transform, a wide band process is generated equivalent to a white noise. According to this method, the noise has a variance equal to $D/\Delta t$.

- *Ornstein-Uhlenbeck process.* An ARMA filter is used to generate a sample of the Ornstein-Uhlenbeck process:
$$f\left(t_i\right) = a\,f(t_{i-1}) + (1-a)W(i) \tag{9}$$
with $a = 1+\beta/2-\sqrt{\beta(1+\beta/4)}$ and $\beta = (\alpha_0 \Delta t)^2$. Then the signal generated is normalized with the given standard deviation σ_f.

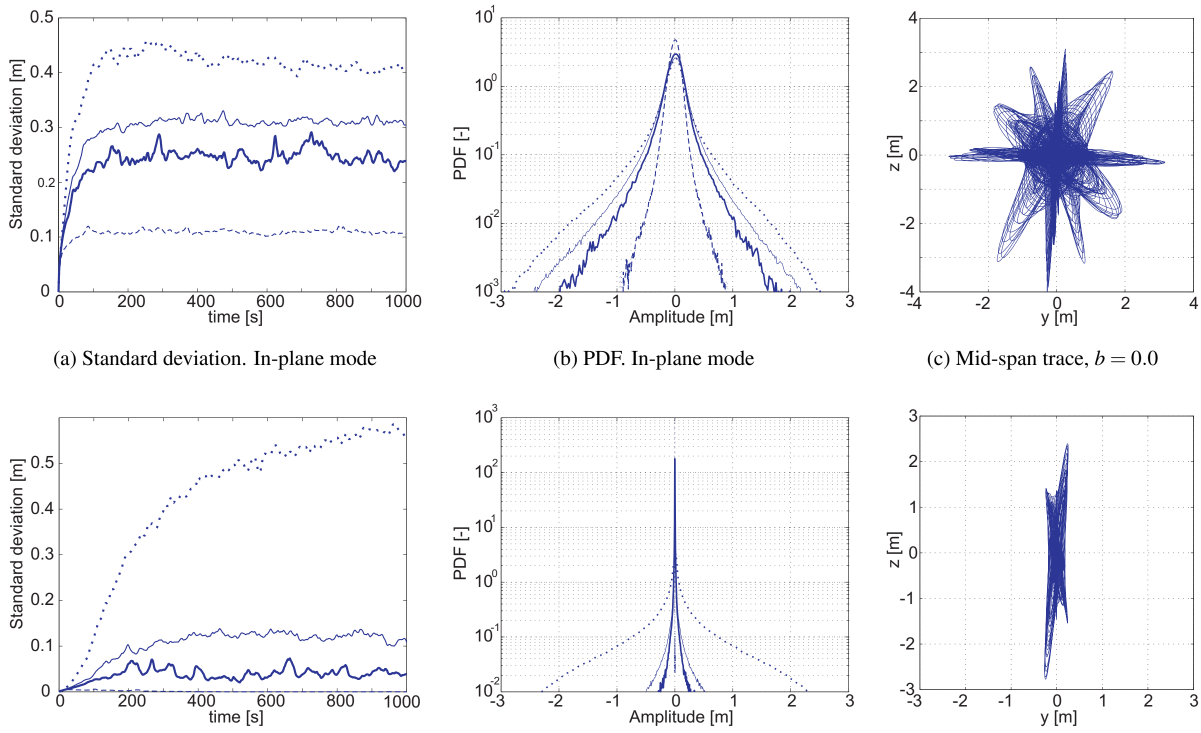
- *Second-order filter.* The signal $f(t)$ is then integrated with a linear Newmark algorithm (constant acceleration).

The frequency resolution is given by $\Delta f = (N\Delta t)^{-1}$ where N is the number of time step. The time step Δt is fixed at $0.02s$ for all the simulations. To improve the frequency resolution and the accuracy of the generated processes, N is fixed at 2^{21} and so Δf is about $2.4 \cdot 10^{-5}$Hz. Afterwards, a window of the generated process is extracted and applied as the excitation of the cable. In a Monte Carlo procedure, these operations are repeated in each simulation.
A nonlinear Newmark scheme is used to solve the governing equations with the sampled anchor motion $e\left(t\right)$.
The cable considered in this work is the longest cable of cable-stayed bridge across the resund between Sweden and Denmark. This cable is also considered by Zhou [7] and Nielsen [8]. The data are: $EA = 2.17 \cdot 10^9$N, $H = 5.5 \cdot 10^6$N, $L = 260.0$m, $m = 81.05$kg/m and $\theta = 30.4°$. The first natural frequencies of the cable is 1.01Hz (in-plane mode) and 1.00Hz (out-of-plane mode) and the value of u_0 is about 0.66m. In [7], the damping ratios ξ are set to 1.0% for each mode, which is also accepted in this study.

INFLUENCE OF BACKGROUND COMPONENT ON CABLE RESONANCE

Statistical Properties

There are a lot of difficulties to describe a transient phase of a random process, because of the assumptions on the initial conditions and because the level of excitation (related to parameter e_0) is obviously not constant in time. Nevertheless, in this example, the excitation is supposed to be stationary and the level of excitation constant in time.
The results are obtained with statistical post-processing of time histories obtained with Monte Carlo simulations. Different levels of excitation and background-resonant ratios b are chosen. So, for a given level of energy e_0, different dispatching of energy between background and resonant components are considered. The number of simulations realized for each parameters are above 6000.
Fig.1 shows an example of time histories of standard deviations for in-plane and out-of-plane modes for $e_0 = 0.10$. For a purely resonant excitation, standard deviation of the in-plane mode reaches quite fast a stationary regime, contrary to the out-of-plane for which the variance has not properly reached stationary regime after 1000s. Same observations have been made for e_0 equal to 0.15 and 0.20 but are not reported here. For the given damping ξ_0 of 1% [7], cable vibrations at resonance associate large amplitude vibrations of in-plane and out-of-plane modes with a predominance of the out-of-plane.



(a) Standard deviation. In-plane mode

(b) PDF. In-plane mode

(c) Mid-span trace, $b = 0.0$

(d) Standard deviation. Out-of-plane mode

(e) PDF. Out-of-plane mode

(f) Mid span trace, $b = 0.4$

Figure 1: Cable excited by a background-resonant process and $e_0 = 0.10$, $\xi_0 = 1\%$. Time evolutions of standard deviation (a,d), stationary pdf (b,e) and mid span trace of cable vibrations (c,f). Legend: $b = 0$ ($\cdots$), $b = 0.4$ (—), $b = 0.6$ (**—**), $b = 1.8$ (- - -)

Now, the aim is to illustrate how the background component prevents the convergence to a limit cycle or how it modifies significantly the cable resonance. The background-resonant ratio b depends necessarily on the considered structure. For the famous viaduct of Millau, the ratios related to the modes with frequencies less than 1Hz are between 0.2 and 1. The relative importance between background and resonant effects has crucial consequences on cable vibrations.

Fig.1 shows time evolutions of standard deviation for the different values of e_0 and b. Background components can induce non-negligible effects, and especially on the out-of-plane bifurcation. Generally, the greater b, the shorter the transient phase.

The ratio b represents the dispatching of energy between background frequencies and the frequencies around resonance. Naturally, if the energy is more concentrated around a resonance frequency of the cable, vibration amplitudes become greater. The cable behaviour gets closer to the behaviour at resonance.

The effect of background component is more significant on the out-of-plane mode. For $e_0 = 0.10$, the probability density functions (Fig.1-b,e) also show that the amplitude of the out-of-plane mode becomes smaller than the in-plane mode amplitude when b increases. When b increases, the whirling motion does not occur and the displacements of the cable are mainly in the in-plane mode. The sharpness of pdf's explains the difficulty to use semi-analytical methods to solve this problem and strengthens the statistical approach. In Fig.1, the in-plane mode has not a symmetric pdf. Indeed, for a same probability, related positive and negative amplitudes are not the same and the skewness is not equal to zero. However, the symmetry of pdf's is observed for the out-of-plane mode.

The background component influences the dispatching of energy between the two modes of the cable and the energy is concentrated in the in-plane mode. Fig.2 shows the dependence of the stationary standard deviations of both modal coordinates with the parameter b and the excitation e_0. Fig.2 highlights the previous observations: the reduction of amplitudes because of the background component is more significant for the out-of-plane mode. The observed phenomenon is not only caused by a reduction of the energy allocated around resonance frequency. For instance, the case $e_0 = 0.25$ and $b = 1.8$ and the case $e_0 = 0.15$ and $b = 0$ have the same level of resonant energy. The difference between these two cases is that the resonance in the out-of-plane mode is canceled and the energy is concentrated in the in-plane mode.

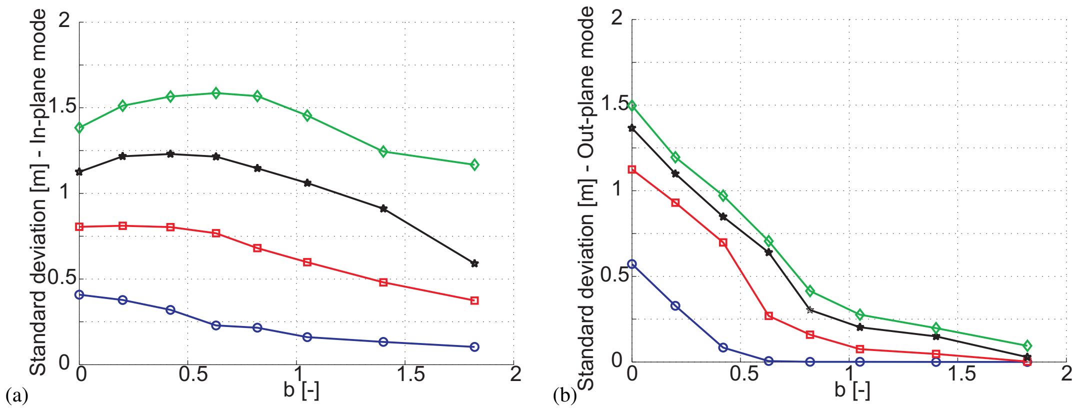
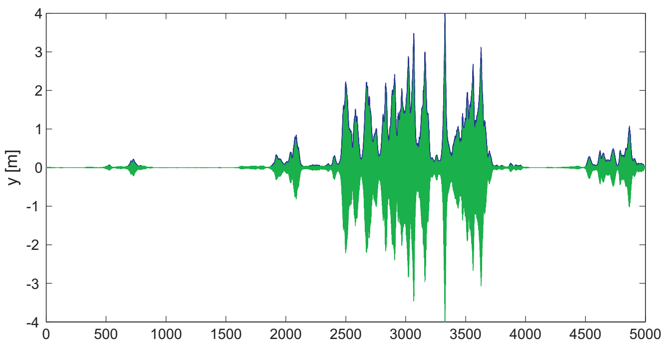


Figure 2: Stationary standard deviation of in-plane (a) and out-of-plane (b) modal coordinates with respect to b for $\xi_0 = 1\%$. Legend: e_0=0.10 (O), 0.15 (□), 0.20 ($\star$), 0.25 (◇).

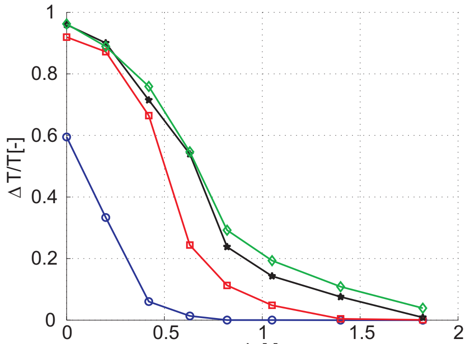
For higher levels of excitation (e_0 =0.20 or 0.25), the effect of background can produce responses greater than the responses for the purely resonant excitation. By limiting vibrations and bifurcations in the out-of-plane mode, the energy in the excitation is concentrated in the in-plane mode.
These results can be compared with the results obtained by Zhou [7] by modifying the damping ratio ξ_0 of the second-order filter of the narrow-band excitation. The background component has the same effect of damping by concentrating vibration in the in-plane mode.

On-Off Intermittency

The concept of *on-off intermittency* is a well-known mechanism used to describe signal alternating laminar phase and bursty ones [10]. The system can oscillate around a weakly unstable position and then bifurcate into a high amplitude and unstable motion for a short time. This phenomenon has already been described in the theoretical studies about a cable submitted to a white noise. The method proposed to quantify this phenomenon in this system (especially for out-of-plane mode) consists in calculating the Cramer-Leadbetter envelop of the signal with the Hilbert transform and then in calculating the ratio $\Delta T/T$ representing the percentage of time spent over a given threshold. The threshold should not be too small, otherwise the whole envelop is over the threshold. A signal and its envelop over the threshold 0.05m is given in Fig.3-a.



(a)



(b)

Figure 3: Example of on-off intermittency phenomenon for the out-of-plane modal coordinate (a). Ratio $\Delta T/T$ with respect to b (b). Legend: e_0=0.10 (O), 0.15 (□), 0.20 ($\star$), 0.25 (◇).

Fig.3-b shows the evolution the ratio $\Delta T/T$ with respect to b. These curves are obtained by averaging ratios calculated on twenty signals of 500,000 points. The time over the threshold drops down with the ratio b . The response of the cable is unimodal if the ratio $\Delta T/T$ is almost equal to zero. There is no range of b where the out-of-plane vibrations are fully developed ($\Delta T/T$=0) for the chosen threshold. The case $e_0 = 0.25$ and $b = 1.8$ and the case $e_0 = 0.15$ and $b = 0$ have the same level of resonant energy. However, the *on-off intermittency* phenomenon in the out-of-plane mode is canceled because of the background component.

CONCLUSIONS

This paper focuses on stochastic dynamics of cables due to random anchor displacements. The aim is to evaluate and quantify the influence of a background component on the resonance of taut cables. The cables are modeled with two modes and the background-resonant excitation is generated by a second-order filtering an Ornstein-Uhlenbeck process.
The main result obtained with simulations states that the background component limits the vibrations in the out-of-plane mode. Thus, the energy of excitation is concentrated in the in-plane mode and therefore vibrations greater than in the case of an exclusively resonant process can occur. The background component of the excitation has a noticeable influence on the bifurcation in the out-of-plane mode. A more important background component is comparable to an increase of damping in the context of a purely resonant excitation.
In the considered model, if there is no direct excitation in the out-of-plane mode, a phenomenon of *on-off intermittency* is notable. This appearance of chaos in the out-of-plane mode is also influenced by the background component and the level of energy in the excitation. The time spent over a given threshold is drastically reduced by the background component.
The results ensued from a two-mode model illustrate the complex behaviour of cables submitted to random excitations. The statistical treatment highlights tendencies which could be confirmed further by experimental investigations. The *on-off intermittency* phenomenon is more related to the mathematical model and is more sensitive to out-of-plane perturbations. In future work, we hope to be able to make confrontations between presented numerical results and experiments.

BIBLIOGRAPHY

[1] J.L. Lilien and A. Pinto da Costa. Vibration amplitudes caused by parametric excitation of cable-stayed structures. *Journal of Sound and Vibration*, 1994.

[2] J.H.G. Macdonald, D.J. Wagg, and al. Generalised modal stability of inclined cables subjected to support excitations. *Journal of Sound and Vibration*, 329:4515–4533, 2010.

[3] W.K. Chang and R.A. Ibrahim. Multiple internal resonance in suspended cables under random in-plane loading. *Nonlinear Dynamics*, 1997.

[4] W. K. Chang, R. A. Ibrahim, and A. A. Afaneh. Planar and non-planar non-linear dynamics of suspended cables under random in-plane loading–i. single internal resonance. *International Journal of Non-Linear Mechanics*, 31:837–859, 1996.

[5] R. A. Ibrahim and W. K. Chang. Stochastic excitation of suspended cables involving three simultaneous internal resonances using monte carlo simulation. *Computer Methods in Applied Mechanics and Engineering*, 168:285–304, 1999.

[6] J. W. Larsen and S. R. K. Nielsen. Non-linear stochastic response of a shallow cable. *International Journal of Non-Linear Mechanics*, 41:327–344, 2006.

[7] Q. Zhou, J. W. Larsen, S. R. K. Nielsen, and W. L. Qu. Nonlinear stochastic analysis of subharmonic response of a shallow cable. *Nonlinear Dynamics*, 48:97–114, 2007.

[8] S. R. K. Nielsen and M. T. Sichani. Stochastic and chaotic sub- and superharmonic response of shallow cables due to chord elongations. *Probabilistic Engineering Mechanics*, 26:44–53, 2011.

[9] P. Warnitchai, Y. Fujino, and Susumpow T. A nonlinear dynamic model for cables and its application to a cable-structure system. *Journal of Sound and Vibration*, 187:695–712, 1995.

[10] N. Platt, E.A. Spiegel, and C. Tresser. On-off intermittency : A mechanism of bursting. *Physical Review Letters*, 70:279–282, 1993.

[11] C.T. Georgeakis and C.A. Taylor. Nonlinear dynamics of cable stays. part2: stochastic cable support excitation. *Journal of Sound and Vibration*, 281:565–591, 2005.

Acknowledgments

The authors would like to acknowledge the National Funds for Scientific Research of Belgium for its support.