Astron. Astrophys. 178, 153–158 (1987)

ASTRONOMY AND ASTROPHYSICS

The possible appearance of a second period in the WN 5 star EZ Canis Majoris

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Received June 13, accepted December 2, 1986

Summary. Previously published photometric data on EZ Canis Majoris are reanalysed. It is shown by different techniques that a sine-wave type periodicity close to one third of the claimed period of $P = 3^{d}.766$ may also be present in what was considered as peculiarities in the lightcurve.

The data available do not allow to know if this second period is precisely equal or only close to one third of P nor if there is some dependence between the two. In any case, the mere possible existence of such a second period is an interesting fact in the discussion concerning the possible existence of non-radial pulsations in some WR stars.

Key words: stars: EZ CMa – stars: variable – stars: Wolf-Rayet

1. Introduction

EZ Canis Majoris (HD 50896, WR 6) is one of those Wolf-Rayet (WR) stars exhibiting periodical variability in photometry and/or in radial velocity which are classically interpreted as due to the presence of a compact companion (Firmani et al., 1980). In a previous paper, one of us (Vreux, 1985) has questioned the interpretation of the variability of most of these so called WR + compact companion systems and has suggested that non-radial pulsations, instead of binarity, could be the cause of some of the observed variabilities. More recently, peculiarities in the periodic behaviour of some WR + c systems have been pointed out, peculiarities which again could be indicative that another cause than binarity could be responsible for some of the observed variabilities (Vreux, 1987). One of the best arguments of course would be a change of the value of the period as has been frequently observed in non-radial pulsating B stars. The lack of evidence of such a change in periodicity has even been considered as a proof of the non-existence of non-radial pulsations by some authors. However, quite recently, Baade (1986) has questioned such a criterion which, according to him, was based on an insufficient sample of non-radial pulsators. Another good argument for the presence of non-radial pulsations is the simultaneous existence of more than one period. Until now the only report of such a situation for a WR star exists in the paper of Smith et al. (1985) on WR 40 = HD 96548. Carefully analysing previously published photometric data of Moffat as well as their own new data, Smith et al. (1985) conclude that neither the period of $4^{d}.762$ (or $2^{d}.381$) claimed by Moffat and Isserstedt (1980) nor the revised period of $4^{d}.158$, suggested as the best by Moffat (1983) on the basis of spectroscopic data, were present in any of the photometric data to which they had access. Instead they show that the photometric data lead to a period of $5^{d}.879$ (or its one-day aliases $1^{d}.204$ or $0^{d}.855$). More important is the fact that in the new discussion they also find that, when the data are prewhitened with respect to the $5^{d}.879$ period or its aliases, then another period of $2^{d}.8396$ seems to dominate in all data, including subsets.

In the present paper we plan to show that EZ CMa may be a second example of such a situation i.e. a second example of a WR star exhibiting a double periodicity.

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2. The star EZ CMa (WR 6 or HD 50896)

The existence of a $3^{d}.76$ periodicity in some of the properties of EZ CMa has been clearly demonstrated and confirmed (McLean, 1980; Cherepashchuck, 1981; Niemela and Mendez, 1982; Lamontagne et al., 1986) since the original paper of Firmani et al. (1980). The stability of this period may have led to disregard the importance of some peculiarities which are also mentioned here and there in the papers. These peculiarities as well as their relation with the ones observed in some other WR + c systems will be discussed in detail elsewhere (Vreux, 1987). Let us just point out a few facts:

– the amplitude of the photometric variations is strongly variable (nearly null in 1975 and 1976; 0.05 mag in 1977; 0.08 mag in 1980 and 1985: Lamontagne et al., 1986);

– the radial velocity of some visible lines shows no phase dependence with the purported $3^{d}.76$ period, depending on the epoch of the observations (Niemela and Mendez, 1982);

– the semi-amplitude K of the radial velocity variations is variable (Firmani et al., 1980; Niemela and Mendez, 1982);

– the strong line profile variations of the N IV lines show no phase dependence in the purported $3^{d}.76$ period neither in the ultraviolet (N IV λ1718; Willis et al., 1986) nor in the visible (N IV λ4058, Niemela and Mendez, 1982; Vreux, 1987).

Altogether this represents a large number of peculiarities even if they do not necessarily mean that EZ CMa is not a WR + c system. In addition, while reviewing the data accumulated on that star, our attention was first drawn upon the photometric data of Cherepashchuk (1981) which show a high level of variation outside the peak exhibited by the lightcurve. This impression was comforted when we had the opportunity to see the new data of Lamontagne et al. (1986) which are more numerous. From our point of view a second periodicity is clearly evident in Fig. 2f of Lamontagne et al. (1986) in what these authors consider as peculiarities in the shape of the lightcurve. To test that idea we have submitted this set of data to a detailed analysis.

3. Analysis of the photometric data

The V magnitudes have been submitted to a Fourier analysis. Figure 1a shows the resulting Discrete Fourier Transform (DFT as defined by Deeming, 1975). One large family of aliases is visible: this result is apparently in good agreement with the conclusions of Lamontagne et al. (1986) whose analysis is based on a Lafler and Kinman (1965) type method. The dominant periodicity corresponds to $\nu_1 \sim 0.25\,\mathrm{d}^{-1}$. The highest one-day alias is $\nu = 1 - \nu_1$ at $\nu \sim 0.76\,\mathrm{d}^{-1}$. A second family is present at $\nu \sim$ 0.5, 1.5, 2.5 d^{-1}: it corresponds to the second harmonic of ν_1 and its presence could be explained by the shape of the light maximum as will be shown subsequently. Being approximately their own reflection through the Nyquist frequency associated with the one-day sampling, those peaks have of course slightly larger widths than the first family. We have also considered the DFT of a pure cosine monochromatic wave of frequency ν_1 sampled in exactly the same way as were the data (see Fig. 1b): this can help in understanding the effects of the spectral window even if this way of doing things is to be taken with caution as the curve is usually non-sinusoidal and, worse, the square of the DFT is *not* the convolution product of the true power spectrum by the power spectral window (Deeming, 1975). By comparing the two parts of Fig. 1, it becomes clear that the DFT of the V magnitudes presents some anomalies. First, the peak at $\nu \sim 0.76\,\mathrm{d}^{-1}$ is markedly too high relative to the one at $0.25\,\mathrm{d}^{-1}$; second, all the peaks of this family of aliases are about 40% wider than the natural width exhibited by the peaks of the DFT of Fig. 1b. Apparently, a phenomenon different from simple power leakage through aliasing has to be invoked. In order to investigate further this problem, we have whitened the data with respect to a periodicity $P = 3^{\mathrm{d}}.766$ (Lamontagne et al., 1986) which corresponds to the frequency ν_1. The DFT of the new data set is given in Fig. 2. Clearly, the same family of aliases is still

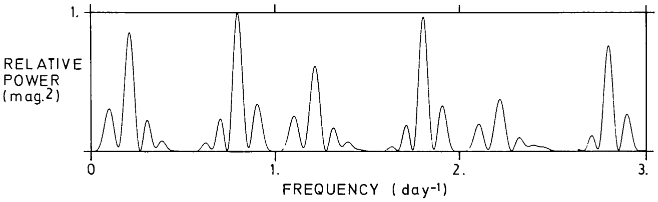

Fig. 2. Same as Fig. 1a for the V data whitened with the fitted function $f(3.766, 2, 2)$

present: the highest peak being now the one at $\nu \sim 1 - \nu_1$. If one removes a periodicity from a function, the relevant peak and all the aliases must disappear from the power spectrum. The persistence of the family of Fig. 2 has three possible explanations: either we have chosen the wrong alias as the progenitor peak, or we are prevented to completely subtract the variation because of its highly non-sinusoidal character or, as a third hypothesis, a second periodicity is present. As we have removed the most powerful alias, the first hypothesis can be disregarded. The rejection of the second one is less straightforward and first requires a more detailed analysis of the data.

In order to disentangle this intricate case, we have performed a harmonic analysis of the lightcurve in the phase diagram. This means that we have fitted to the magnitudes $V(\varphi(P))$ a function of the form

$$f(P, N_c, N_s) = a_0 + \sum_{n_c=1}^{N_c} a_{n_c c} \cos(n_c 2\pi\varphi(P)) + \sum_{n_s=1}^{N_s} a_{n_s s} \sin(n_s 2\pi\varphi(P))$$

where $\varphi(P)$ is the phase computed for the period P. The epoch zero, corresponding to maximum light and taken from Lamontagne et al. (1986), permits inducing small values for the sine terms. The results for the period $P = 3.766$ days are given in Table 1 for different degrees of the function f. Column 2 gives the values of the coefficients taken into account for the harmonic analysis and column 3 their standard deviation. Column 4 gives some relevant statistics: σ_{fit} is the standard deviation of the fit whereas R is the multiple linear correlation coefficient. A general inspection of the latter clearly implies the necessity to include terms up to $N_c = N_s = 3$; the corresponding value of R is by far the only good one and the value of σ_{fit} is the only one of the same order as the expected error deduced from the two comparison stars ($\sigma(\mathrm{C1} - \mathrm{C2}) = 0.003$ (Lamontagne et al., 1986)). Figure 3 illustrates the quality of the fit. We have also tested the advantage of adding the term $n_c = 4$. An F-test for the additional term gives a value $F = 2.5$ with 1 and 34 degrees of freedom leading to a significance level greater than 0.10. There is therefore no statistical constraint to add this term. Of course, it is always possible to go higher and higher in the degree of the harmonic analysis: usually this will lead to lower and lower standard deviation of the fit but the improvement is not always significant. At a certain threshold, even if new terms can still slightly improve the situation, they are not statistically necessary. It appears that we have reached this point with $N_c = N_s = 3$.

Coming back to column 2 of Table 1 an interesting fact clearly shows up: the absolute value of the coefficient a_{3c} (see fourth fit) is greater than any other one. It is remarkable that this statement also holds – even if marginally – for a_{1c}. It is clear, as pointed out by Lamontagne et al. (1986), that the lightcurve is highly

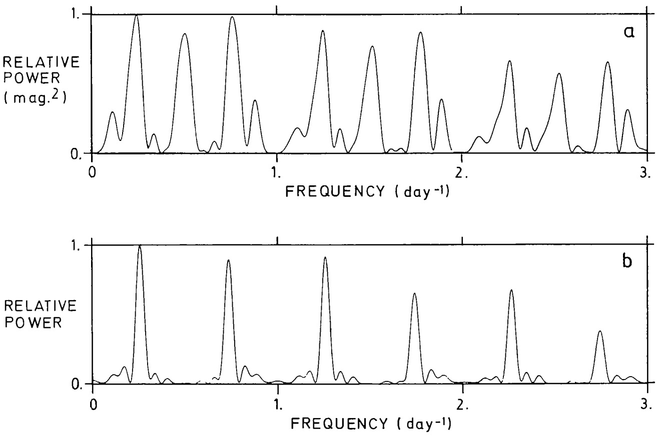

Fig. 1a and b. Fourier analysis of the photometric data. *a* square of the full amplitude of the DFT of the V data; *b* square of the full amplitude of the DFT of a pure cosine monochromatic wave sampled exactly the same way as were the data. Ordinates are in arbitrary units

Table 1. Coefficients of different harmonic analyses of the V magnitudes in a $P = 3\overset{d}{.}766$ phase diagram

Coefficients	Value	Standard deviation	Statistics of the fit
a_0	0.6267	0.0033	$\sigma_{fit} = 0.021$
a_0	0.6272	0.0028	
a_{1c}	−0.0155	0.0039	$\sigma_{fit} = 0.018$
a_{1s}	0.0037	0.0041	$R_{mul} = 0.548$
a_0	0.6279	0.0022	
a_{1c}	−0.0144	0.0031	$\sigma_{fit} = 0.014$
a_{1s}	0.0017	0.0032	
a_{2c}	−0.0173	0.0035	$R_{mul} = 0.763$
a_{2s}	−0.0020	0.0030	
a_0	0.6282	0.0011	
a_{1c}	−0.0170	0.0015	$\sigma_{fit} = 0.007$
a_{1s}	0.0004	0.0016	
a_{2c}	−0.0155	0.0017	$R_{mul} = 0.956$
a_{2s}	−0.0015	0.0014	
a_{3c}	−0.0178	0.0016	
a_{3s}	−0.0036	0.0015	
a_0	0.6279	0.0020	
a_{1c}	−0.0176	0.0029	$\sigma_{fit} = 0.013$
a_{3c}	−0.0188	0.0030	$R_{mul} = 0.806$

non-sinusoidal. In such circumstances, the amplitudes of the different upper harmonics may be large, depending on the shape of the lightcurve. An extreme case would be a signal composed of regularly spaced Dirac functions: all the harmonics have the same amplitude. Concerning other signals, the amplitude of the nth harmonic decreases with n in a way depending on the skewness and the shape of the extremum. For example, a square signal (i.e. a step function in a phase diagram) will lead for the amplitude to a Dirichlet function which decays roughly as $1/n$ (Bloomfield, 1976). In no physical case, may the amplitude of a harmonic reach and, worse, be greater than the amplitude of the fundamental frequency by the sole effect of a narrow extremum. Using different approximations, it is possible to estimate the expected relative amplitudes that can be induced by the shape of the light maximum (to be found near phase 0.0 in Lamontagne et al.'s (1986) lightcurves). Concerning the second and third harmonics, we find respectively 72% and 40% of the first one. The difference between the expected (−0.0122, i.e. 72% of a_{1c}) and the observed (−0.0155, i.e. a_{2c} in the fourth fit of Table 1) amplitudes of the second harmonic is no more than 2σ. Our assertion about its origin cannot be rejected. The same calculation leads, for the frequency $3\nu_1$, to a value of 7σ. Clearly, in such a case, it is impossible to sustain that the presence of such a strong third harmonic can be due to the shape of the light maximum in the $P = 3\overset{d}{.}766$ lightcurve. Moreover the fit with only the terms corresponding to a_{1c} and a_{3c} leads to a slightly better R than when one includes terms up to $N_c = N_s = 2$. All this demonstrates the importance of the third harmonic $\nu_2 \sim 0.79\,\mathrm{d}^{-1}$ ($P \sim 1.255$ days) which can be considered as a second frequency present in the data. It is of great interest to notice that, due to the particular value ($\nu \sim 0.26\,\mathrm{d}^{-1}$) of the first frequency ν_1, its one-day alias $1 - \nu_1$ approximately coincides with its own third harmonic. This particular arrangement of the two frequencies gives a direct and simple explanation to the above-mentioned phenomenon of persistence. After having shown arguments in favour of the third hypothesis, let us now discuss why we have to reject the second one.

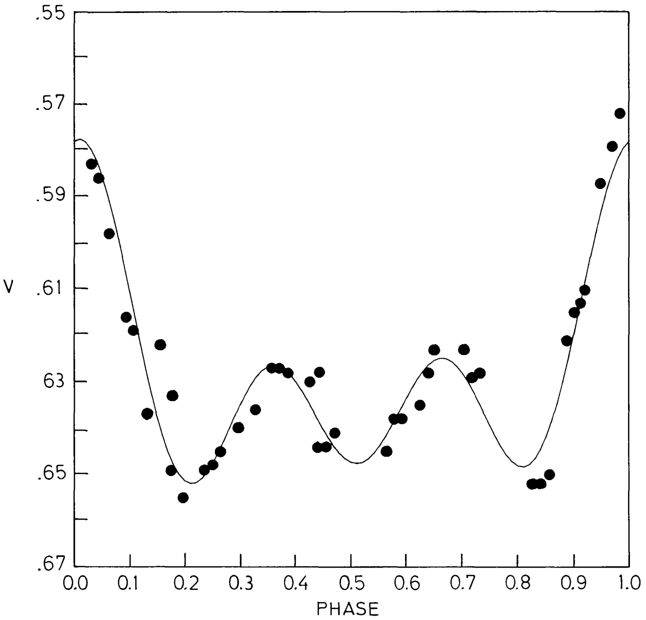

Fig. 3. The V data in a $P = 3\overset{d}{.}766$ phase diagram along with the fitted function $f(3.766, 3, 3)$

To give further evidence to our assertion concerning the second hypothesis, we have investigated the power in the different peaks for several data sets: unwhitened V magnitudes or whitened with respect to ν_1 or ν_2.

In Table 2, we give the normalized power $P(\nu)$ (i.e. power divided by variance as computed from the Power Estimate

Table 2. Some characteristics of the peaks corresponding to ν_1 and ν_2 in the power spectrum of different data sets

Data set	Normalized power in the peak corresponding to		Precise observed position (unit d^{-1}) of the peak corresponding to		Identification of the peak corresponding to	
	ν_1	ν_2	ν_1	ν_2	ν_1	ν_2
V	8.1	7.9	0.25	0.76	ν_1 and $1-\nu_2$	ν_2 and $1-\nu_1$
$V-f(3.766,2,2)$	14.4	16.4	0.21	0.79	$1-\nu_2$	ν_2
$V-f(1.255,1,1)$	12.1	10.6	0.26	0.74	ν_1	$1-\nu_1$

Method of Scargle, 1982). This is a good measure of the tendency of the variations, quantified by the variance, to concentrate in only one periodic phenomenon as opposed to a stochastic process for which the power is best distributed amongst all the frequencies. As already mentioned (Fig. 2), the data whitened with respect to ν_1 (line 2 of Table 2) exhibit high power at frequency ν_2. This power is even higher (16.4) than the one at ν_1 for the data whitened with respect to ν_2 (i.e. 12.1 in line 3 of Table 2). This would even indicate that the most important frequency could be ν_2. In any case, the two frequencies are highly significant. The power at ν_1 and ν_2 in the unwhitened V magnitudes (line 1 in Table 2) are lower: this is a signature of the simultaneous presence of at least two frequencies. Of course, nothing can be said on the possible dependence between the two frequencies. These considerations also put forward the fact that the persistence phenomenon cannot be due to an imperfect way of removing the variability. More explicitly, if one subtracts a sine function from a non-sinusoidal periodic variation, there may remain some signal at the relevant frequency but it must be closer to the noise level than the original one. The exact opposite is observed here: the second hypothesis has therefore to be rejected.

In Table 2, we also give the precise observed positions of the relevant peaks in the power spectrum. It appears that there is a small shift between power spectra of different data sets. This is a simple explanation of the wider width of the peaks of the DFT of Fig. 1a; it is due to a blend of two peaks whose natural width ($0.06\,\mathrm{d}^{-1}$) is of the order of the shift ($0.05\,\mathrm{d}^{-1}$). In such a case, the origin of the peaks would be as suggested in Table 2.

One could argue that our results are dependent on the chosen period search technique. To answer this, all the previous investigations have been carried out again using two trial period methods, i.e. a completely different technique. The phenomena of persistence and of shift of the peaks (dips in this particular case) remain clearly present. We show only the values of θ_1 (Renson, 1978) and of Θ (Lafler and Kinman, 1965), the selected trial methods statistics, relevant to the frequencies ν_1 and ν_2 (see Table 3). Both ν_1 and ν_2 are statistically important. We also note that the dip at ν_2 for the V data set (line 1 of Table 3) is less deep than the corresponding one for the data whitened for ν_2 (line 3). Since the aliasing is a function of the sampling, it is completely abnormal that a dip becomes more visible when the relevant frequency has been removed. The only explanation is that a particular arrangement of the V data in a $P = 1.255$ day phase diagram has induced a diminution of the importance of the relevant dip. The effective presence of the second frequency is responsible for this. Let us notice that in a phase diagram with $P = 1.255$ day, the data points lie on two separated curves. The first one is defined by the points responsible for the largest maximum in a phase diagram with $P = 3.766$ days, the other one by the points delineating the two secondary light maxima at $\varphi \sim 0.33$ and 0.66 in the same phase diagram. Lafler and Kinman's (1965) statistic can be imagined as a broken line linking all the data

Table 3. Some interesting particular values of two trial method type statistics as computed for the different data sets

Data set	Value of the θ_1 statistic (Renson, 1978)[a] relevant to the dip corresponding to the frequency		Value of the Θ statistic (Lafler and Kinman, 1965) relevant to the dip corresponding to the frequency	
	ν_1	ν_2	ν_1	ν_2
V	12.4	41.9	0.24[c]	0.63
$V-f(3.766,2,2)$	22.5[b]	26.8	0.57[b]	0.53
$V-f(1.255,1,1)$	12.0	30.7	0.22	0.48

[a] For the e parameter (see Renson, 1978), we took 0.004 mag
[b] Those values are lower limits and must be taken with caution, the relevant dips disappearing in the noise
[c] This value disagrees by a factor of two with Fig. 1 of Lamontagne et al. (1986). The statistic they used is not the one mentioned in their ordinates label

Table 4. Coefficients of different harmonic analyses of the I magnitudes in a $P = 3\overset{d}{.}766$ phase diagram

Coefficients	Value	Standard deviation	Statistics of the fit
a_0	0.4215	0.0030	$\sigma_{fit} = 0.019$
a_0	0.4230	0.0027	
a_{1c}	−0.0134	0.0037	$\sigma_{fit} = 0.016$
a_{1s}	0.0028	0.0038	$R_{mul} = 0.532$
a_0	0.4235	0.0021	
a_{1c}	−0.0125	0.0029	$\sigma_{fit} = 0.013$
a_{1s}	0.0016	0.0031	
a_{2c}	−0.0149	0.0030	$R_{mul} = 0.773$
a_{2s}	−0.0051	0.0029	
a_0	0.4225	0.0010	
a_{1c}	−0.0131	0.0014	$\sigma_{fit} = 0.006$
a_{1s}	0.0007	0.0015	
a_{2c}	−0.0129	0.0015	$R_{mul} = 0.953$
a_{2s}	−0.0043	0.0014	
a_{3c}	−0.0152	0.0015	
a_{3s}	−0.0039	0.0014	

points and sometimes switching from a curve to the other with, as a consequence, an increase of the value of the statistic. All this indicates that the depth of the dips in such a type of periodogram is an ill-behaved criterion when two periodicities are simultaneously present.

As can be seen many arguments can be found in favour of the simultaneous existence of two frequencies in the photometric data. However, time coverage is insufficient to decide if the second frequency ν_2 is exactly the third harmonic of ν_1 or if the equality $\nu_2 = 3\nu_1$ is only approximate and fortuitous. Whether the equality of the phase of ν_1 and ν_2 is fortuitous and whether the two periodicities are independent or not also remain open questions.

The I magnitudes of Lamontagne et al. (1986) have been submitted to exactly the same investigation as the V ones. We do not want to give details here; we simply give Table 4, which is the equivalent of Table 1 but for the near infrared magnitudes. Our conclusions are in all respects identical.

Finally, it is worth noting that a rapid look at Fig. 1 of Cherepashchuk (1981) gives the impression that the phenomenon of presence of two frequencies is already present at this epoch. Small discrepancies with the present data could be explained by a small phase shift of ν_2 with respect to ν_1. In any case, the characteristics of the variability of EZ CMa are already known to be epoch dependent (Lamontagne et al., 1986; Vreux, 1987).

4. Conclusions

From our analysis of these photometric data, it is clear that they cannot be used to reject the hypothesis of multiperiodicity in the variability of EZ CMa as claimed by Lamontagne et al. (1986). Using different methods, we have shown beyond any reasonable doubt that besides the well known periodicity at $\nu_1 = 0.26\,d^{-1}$, another periodicity at $\nu_2 = 0.79\,d^{-1}$ may also be present. With the available data, it is not possible to determine if the second period is or is not the physical third harmonic of ν_1 (i.e. if ν_1 and ν_2 are or are not dependent) and/or if there is a phase shift between the two periodicities. In other words, we are not able to conclude whether the second periodicity has, in the star, an independent existence or whether it is linked to the presence of ν_1. The answer to these questions is crucial for any definite interpretation of this phenomenon.

In many aspects (see introduction), EZ CMa is indeed a very peculiar object which clearly deserves more attention observationally.

Acknowledgements. We would like to express our thanks to our referee, Dr. A. Moffat, for some constructive criticisms. We are also indebted to Dr. P. Renson for fruitful discussions concerning his method.

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