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Research Note

The possible biperiodicity of WR 40

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Abstract. Strong arguments are presented in favour of the simultaneous presence of two periodicities in the variability of the Wolf-Rayet star WR 40. The frequencies are $\nu_A \sim 0.16\,\mathrm{d}^{-1}$ ($P_A \sim 6.25$ d or the one-day alias $1-\nu_A$) and $\nu_B \sim 0.40\,\mathrm{d}^{-1}$ ($P_B \sim 2.5$ d or the one-day alias $1-\nu_B$). Some uncertainties remain concerning the choice of the actual one-year alias. On the basis of a detailed statistical analysis, reliable evidence that the variability related to ν_B could be coherent is also presented: this points out ν_B as a possible fundamental frequency linked to the "core" of WR 40. No firm conclusion could be drawn about ν_A, suggesting that it could be a recurrent quasi-periodicity or a second order periodicity (e.g. a beat frequency). The two frequencies are sometimes hidden by the white noise, or slightly correlated, random process that constitutes the third component of the variability of WR 40.

Key words: stars: Wolf-Rayet – HD 96548 = WR 40

1. Introduction

Recently, Gosset et al. (1989) performed a detailed analysis of the photometric variability of the Wolf-Rayet star WR 40 (HD 96548). They suggested that the light variations came from the combination of several phenomena, not necessarily independent; they showed that a periodicity $P_A = 6.250$ d ($\nu_A = 0.160\,\mathrm{d}^{-1}$ or the relevant one-day aliases $1-\nu_A$ or $1+\nu_A$) is present, that a second one $P_B \sim 2.4$ d ($\nu_B \sim 0.41\,\mathrm{d}^{-1}$ or the relevant one-day aliases $1-\nu_B$ or $1+\nu_B$) is possibly present and that the remaining variations are well modelled by a random process. No certainty about the stability of the suspected periodicities could be derived. The semi-amplitude of a fitted sine-curve with the periodicity P_A is about 0.010 mag.

More recently, and in a *completely independent* way, Balona and Egan (1989) and Balona et al. (1989a, b) presented and analyzed a very nice and extensive photometric (Johnson B) data set for the same star WR 40: they reported the possible detection of biperiodicity. Their main period, $P_1 = 2.5$ d ($\nu_1 = 0.40\,\mathrm{d}^{-1}$) or its one-day alias $P_1 = 1.67$ d ($\nu_1 = 0.60\,\mathrm{d}^{-1}$), is considered as identical to P_B of Gosset et al. (1989). Their second period is $P_2 = 7.25$ d ($\nu_2 = 0.138\,\mathrm{d}^{-1}$) and it is considered as significantly different from the one claimed by Gosset et al. (1989) under the name of P_A. They conclude that those periodicities are not stable from season to season and that "the evidence of recurring quasi-periodicities is very strong for this star".

It is clear that the possible biperiodicity of WR 40 deserves more attention. The existence and the stability of the detected frequencies is a highly interesting problem. The definite proof of a lack of stability in the periodicities would be extremely important as it would indicate that the corresponding source of variability is not linked to a completely deterministic physical mechanism deeply rooted in the star (like rotation or pulsation). Such incoherent variations could instead be the signature of the characteristic time-scale(s) of phenomena either linked to the dynamics of the wind or to the appearance of transient inhomogeneities on the surface of the star. On the other hand, the detection of a coherent periodicity would rather indicate the feasibility of extracting information pertaining to the "core" of the star. With the observations of Balona et al. (1989a), the number of measurements available for analysis jumps from 277 (in Gosset et al., 1989) up to 455. We thus start to have enough data to be able to use some statistical tools in a safer way. Consequently, we have decided to analyze both data sets jointly in order to attempt to improve our understanding of the light variation of WR 40.

2. Analysis of the data

As a first step, we performed an analysis of Balona et al.'s data (1989a, hereafter data set β, 178 measurements). Those data usually exhibit a good daily coverage with up to 11 measurements on some nights. Such a coverage allows the search for variations on a rather short time-scale. A rapid look at Fig. 8 of Balona et al. (1989b) clearly indicates that no outstanding peak exists beyond $\nu \sim 1.6\,\mathrm{d}^{-1}$. This result is in good agreement with the conclusions of Gosset et al. (1989). One could even say that most of the power is located at frequencies $\nu \leqq 1.0\,\mathrm{d}^{-1}$. This view is supported by the fact that the night-to-night variance is 25 times greater than the nightly variance. Figure 1 exhibits the power spectrum of those data as well as the power spectral window of the relevant sampling

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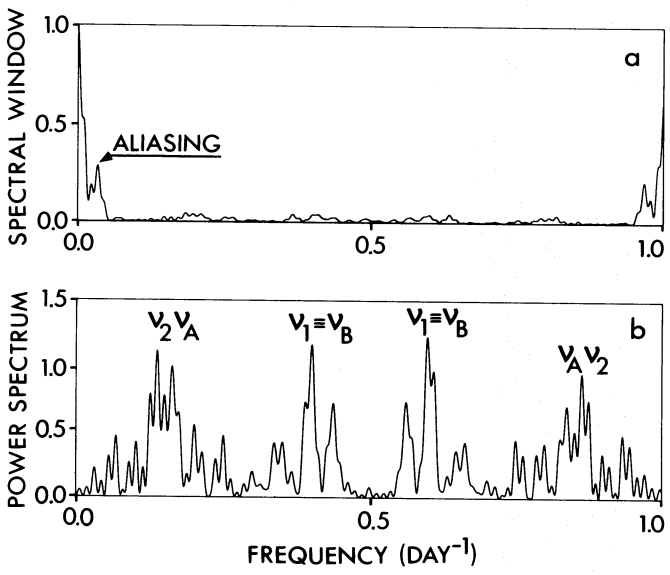

Fig. 1 a and b. Fourier analysis of data set β using Deeming's (1975) method. **a** is the power spectral window whereas **b** is the power spectrum of the data. Ordinates are in arbitrary units.

(Deeming, 1975). The particular sampling generates an additional aliasing phenomenon with a $\Delta\nu \sim 0.03\,\mathrm{d}^{-1}$ (see Fig. 1a). Neglecting the problem of the one-day aliasing, the presence and the value of P_1 makes no doubt when attention is drawn to Fig. 1b. However, the feature corresponding to P_2 is wide and markedly double-peaked. The highest peak is at $\nu = 0.138\,\mathrm{d}^{-1}$ ($P = 7.25$ d) whereas the other one is at $\nu = 0.163\,\mathrm{d}^{-1}$ ($P = 6.13$ d); they are aliases of each other ($0.163 - 0.138 = 0.025 \sim \Delta\nu$) and, even if the choice of the highest one is reasonable, it should be noted that the height of the second one is not significantly different.

We performed non-linear least-squares simultaneous fits of two sine-curves to these data. We clearly found two clean, well-marked principal minima. The first one corresponds to the following frequencies and semi-amplitudes (i.e. the coefficients of the sines):

$$\nu_1 = 0.4017\,\mathrm{d}^{-1} \qquad a_1 = 0.017\,\mathrm{mag}\,,$$
$$\nu_2 = 0.1387\,\mathrm{d}^{-1} \qquad a_2 = 0.019\,\mathrm{mag}$$

with an associated χ^2_r (reduced χ^2) of $0.4311\ 10^{-3}$, whereas the second one leads to

$$\nu_1 = 0.4014\,\mathrm{d}^{-1} \qquad a_1 = 0.018\,\mathrm{mag}\,,$$
$$\nu_2 = 0.1635\,\mathrm{d}^{-1} \qquad a_2 = 0.018\,\mathrm{mag}$$

with $\chi^2_r = 0.4368\ 10^{-3}$. The difference between the two resulting values of χ^2_r is not significant. In addition, the data resulting from a prewhitening (see Gosset et al., 1989, for the explanation of this term) with any of the two pairs are virtually equivalent. As a consequence, we can consider that the data of Balona et al. (1989b) are quite compatible with the periodicity noted P_A and reported by Gosset et al. (1989): P_2 is no more than an alias of P_A. There is no necessity to invoke a changing period although a change of the amplitude may be possible.

Therefore, the Wolf-Rayet star WR 40 could be a truly biperiodic star: the independence of the reports by Gosset et al. (1989) and by Balona et al. (1989b) is indeed in itself good evidence. In any case, in order to test this hypothesis further, we merged the data set I + II + III + IV + V of Gosset et al. (1989, hereafter labelled as data set α, 277 measurements), and the data set β (Balona et al., 1989a). The whole data set $\alpha + \beta$ (455 measurements) was submitted to a period analysis. Two frequencies are clearly outstanding: they are (again)

$$\nu_A \sim 0.16\,\mathrm{d}^{-1} \quad \text{(semi-amplitude: 0.011 mag)},$$

$$\nu_B \sim 0.40\,\mathrm{d}^{-1} \quad \text{(semi-amplitude: 0.010 mag)}$$

and are detected by any Fourier technique. The precise values for ν_A and ν_B are specified below. The normalized power (power over variance of the data as defined by Scargle, 1982) for those two frequencies are, respectively, $\Psi_A = 25.2$ and $\Psi_B = 20.9$. The corresponding significance levels (i.e. the probability of having such a high value of the statistic under the null-hypothesis of white noise) computed using equation 14 of Scargle (1982) are extremely low (i.e. correspond to a rejection of the null-hypothesis). However, the sampling is particular and some related correlations (i.e. aliasing) can appear between different frequencies, invalidating the hypotheses on which the equation is based (Horne and Baliunas, 1986). Therefore, we estimated the correct significance level by generating data sets of white noise sampled in exactly the same way as the data were. Each of the 2000 simulated data sets was Fourier analyzed, and we searched for the highest peak in the normalized periodogram of Scargle (1982). From these simulations, we derived that the 0.001 significance level corresponds to a normalized power somewhere between 13.4 and 14.0, i.e. well below $\Psi_B = 20.9$ and $\Psi_A = 25.2$ which are thus confirmed as highly significant. Nevertheless, such simulations implicitly assume that every measurement is uncorrelated with any other one. The behaviour of the variance (night-to-night compared to nightly, as quoted above) suggests that this assumption could be too restrictive when testing the reality of such low frequency periodic variations as those considered here. Some problems could arise from the high nightly coverage of data set β. We took that into account and, as an extreme case, we reduced data set β to one observation per night. On this basis, new simulations lead to an extreme upper limit of 0.01 for the significance level corresponding to the observed normalized power. However, such an extreme case is to be considered as a border-line case. As a consequence of all these simulations, we can conclude that the null-hypothesis of white noise is rejected (S.L. < 0.01) by each of the two frequencies separately and that the statistical analysis points towards the existence of a biperiodic phenomenon.

When the sampling is very uneven, one usually can cast some doubt on the reality of a frequency if the phase diagram corresponding to this frequency is also very uneven. In order to test for this possibility, we computed a statistic similar to the Completeness-of-Phase statistic introduced by McCandliss (1988). As a result, it turned out that the sampling is rather fair for every frequency except $\nu \sim 0.20\,\mathrm{d}^{-1}$, $\nu \sim 0.33\,\mathrm{d}^{-1}$ and frequencies lower than $0.02\,\mathrm{d}^{-1}$. This strengthens our conclusions about the true existence of two periodicities in WR 40.

It should be mentioned that besides the one-day aliasing phenomenon discussed above, the data also suffer from a one-year aliasing phenomenon as well as from a ten-year one. Essentially, this means that all the main peaks have a complex substructure: they are made of peaks separated by $\Delta\nu \sim 0.0027\,\mathrm{d}^{-1}$ each of them being in turn composed of subpeaks separated by $\Delta\nu \sim 0.0003\,\mathrm{d}^{-1}$. Those aliasing phenomena are strongly marked and it is impossible to choose the good alias (i.e. the actual progenitor), particularly among the ten-year aliases. We studied the detailed substructure of the peaks corresponding to ν_B and it is

Table 1. List of the best candidates for ν_A and ν_B (unit: d^{-1}). Stars indicate the quality of the candidate (*: good; **: the best)

	ν_A		ν_B	
α	0.154775		0.391200	
	0.157525		0.393925	
	0.160025	*	0.396650	
	0.162500		0.399400	
			0.402100	*
$\alpha+\beta$	0.155325		0.404825	**
	0.157850		0.407550	
	0.159725		0.410275	
	0.160600	*	0.413225	*
	0.162225	**	0.415775	
	0.163325			
	0.164750			
α	0.839975		0.587025	
	0.842450	*	0.589750	
	0.845225		0.593400	
	0.847700		0.595450	
			0.598175	**
$\alpha+\beta$	0.833025		0.600700	*
	0.835750	*	0.603625	
	0.838050		0.606375	
	0.839675	**	0.609100	
	0.842425	*		

remarkable that this structure is very similar in the power spectrum of both data sets α and $\alpha+\beta$. Within each peak, the highest subpeak is always the same for both data sets. In Table 1, we gathered some 19 peaks, each being more or less a good candidate for the exact value corresponding to ν_B or $1-\nu_B$. We tried to do the same for ν_A but, in this case, the structure is somewhat different from one data set to the other. For this reason, we give in Table 1 some 20 candidates originating either from data set α or from data set $\alpha+\beta$. Stars are indicative of the quality of the candidate. Provided the good alias is chosen, the precision is better than $0.001\,d^{-1}$.

Although the existence of two periodicities in WR 40 is demonstrated above, it is not yet clear whether we have to deal with true (stable) periodicities sometimes hidden in a powerful random process (which is present anyway; see Gosset et al., 1989, and hereafter) or with recurrent quasi-periodicities whose phases, and even frequencies, are not completely stable. As we know neither the values of the frequencies with an infinite precision nor the relevant expected absolute power, Fourier techniques are unable to distinguish between the two cases. A way to discriminate between the two is to test the coherency of the variations, i.e. to test whether or not the phases of the variations are stable. In the following, we will try to answer this question. This should be possible as data set α corresponds to data from 1975–1976 and from 1985 to 1987, whereas data set β covers some 10 weeks between January 1988 and June 1988. We selected JD 2447000.0, a date somewhere between the data sets α and β, and, by separately fitting the two data sets, we computed the phases of the two periodicities for this epoch. As a matter of fact, we can say that the coherency exists if the difference between the phases computed from both data sets is, within the errors, equal to zero. Any other value of the phase difference would suggest a lack of coherency or, at least, a failure to demonstrate the coherency. The value of the phase is very sensitive to the adopted value of the period and, as a consequence, we have to test the coherency independently for each pair (ν_A, ν_B) of frequency values given in Table 1. A problem arises from the fact that, as pointed by Balona et al. (1989b) and as can be seen from the above χ_r^2 of the fits, the two periodicities are drowned in a random process. Therefore, least-squares techniques are hard to apply and the fit could usually be qualified as marginal. In order to secure ourselves against spurious results, we have used as a phase-matching statistic the phase difference divided by its standard deviation estimated from the fit. This renders the overall test a little more conservative. Essentially, no firm conclusion could be drawn for most of the pairs from Table 1. The general impression is that the phase is not strictly conserved or is very ill-defined for frequency ν_A, whereas the phase difference concerning ν_B has sometimes a marked tendency to be near zero indicating a coherent variation. An attempt is made hereafter in order to quantify those results. In most of the cases, the statistic relevant to ν_B is markedly smaller than 2. If the statistic were normally distributed, this would suggest the rejection of the adopted null-hypothesis of non-coherency. However, the distribution of the statistic is clearly not normalized in the expected way. The main reason is that the selected frequencies correspond to the highest peaks in the power spectrum, therefore leading to a better coherency than frequencies chosen at random. Consequently, the actual distribution of the statistic is skewer than the expected one. In order to take all the effects into account and to properly quantify the results, we performed simulations ($N_{sim}\sim 5000$). One simulation consists in shifting in time data set β by a random value uniformly distributed between 0 and 10 days (this breaks any possible coherency). Then the shifted data set is merged with the α one and the power spectrum of the whole data set is computed. The highest peaks corresponding to ν_A and to ν_B (and/or to their aliases) are spotted and the corresponding values of the frequencies are adopted. Then, least-squares fits of two sine-curves of the relevant frequencies are performed separately on both data sets (α and shifted β). The statistic of phase-matching is computed in the usual way. From these simulations, we can deduce a distribution for the statistic and, consequently, the corresponding significance level of any realization, i.e. the probability to observe at least such a value of the statistic under the null-hypothesis of non-coherency. The observed values of the statistic can therefore be partitioned in four coherency classes in the following way:

class 1 $\quad$ S.L. $\leqq 0.005$

class 2 $\quad 0.005 <$ S.L. $\leqq 0.01$

class 3 $\quad 0.01 <$ S.L. $\leqq 0.05$

class 4 $\quad$ S.L. > 0.05 .

Class 1 suggests of course the existence of some kind of coherency whereas class 4 corresponds to a non-rejection of the null-hypothesis of non-coherency. For several reasons, such a test is conservative. The main reason is that the random amount by which data set β is shifted could sometimes be approximately equal to an integer multiple of one of the periods. In such a case the coherency is not properly broken.

For the frequency ν_A, the statistic belongs most of the time to classes 3 and 4: no conclusion could be drawn and we could thus have to deal with a quasi-periodicity. Concerning the frequency ν_B, classes 1 and 2 are well populated for some combinations of ν_A and ν_B. Some interesting pairs, corresponding to the most likely frequencies, are given as entries to Table 2. Table 2 gives the

Table 2. Some results of the coherency test for ν_B. The class of coherency is given as a function of the frequencies ν_A and ν_B (expressed in d^{-1})

		ν_A							
		α	$\alpha+\beta$			α		$\alpha+\beta$	
		0.160025	0.159725	0.160600	0.162225	0.839975	0.842450	0.839675	0.842425
ν_B	0.404825	3	2	4	2	2	1	1	3
	0.598175	1	2	1	2	2	2	2	2

relevant classes of coherency for ν_B. Clearly, the presence of some coherency is suggested: the probability (if the frequencies are chosen a priori) to obtain such a value being sometimes less than 0.005. As a consequence, this analysis is indicative that some determinism plays a role and that we could have to deal with a true periodicity. No further definitive conclusion can be achieved.

The data have been prewhitened simultaneously for ν_A and ν_B assuming both are true periodicities (this point is not crucial). The resulting variance is of the order of 0.5 10^{-3} (in square mag), well above the observational noise. The power spectrum of the prewhitened data set is quite compatible with a white noise random process or with a random process slightly correlated. By correlated random process, we mean a stochastic process in which some inertial characteristics exist in such a way that observed values of the time series separated by a small time lag, are to some extent correlated. On the opposite of the case of a white noise process, the autocorrelation function of such a time series is therefore different from zero at small lags, and the power spectrum is not flat but presents a relative excess of power at low frequencies. In addition, two insignificant peaks could be noticed in the power spectrum of the prewhitened data set. The highest one is around $\nu \sim 0.32\,d^{-1}$, it was noticed in data set I by Gosset et al. (1989) but, in the present case, it also coincides with a bad phase coverage. The second peak is located at $\nu \sim 0.068\,d^{-1}$ and could have some physical origin.

3. Conclusion

On the basis of the data sets presented by Gosset et al. (1989) and by Balona et al. (1989a), we have established a unified view of the characteristics of the variability of WR 40. We have demonstrated that the periodicity P_A and the possible periodicity P_B reported by Gosset et al. (1989), are quite compatible with the respective quasi-periodicities P_2 and P_1 reported by Balona et al. (1989b). We have shown that the two data sets, when merged together, permit to demonstrate that WR 40 is effectively biperiodic at least at the 0.01 significance level. Such results are clearly not due to a problem linked to the sampling and actually pertain to the star itself. We have presented, in Table 1, a set of candidates for the values of ν_A and ν_B. No firm conclusion can be drawn from the data concerning the phase coherency of ν_A and, therefore, even if we cannot exclude that ν_A is a true periodicity, it is more likely a recurrent quasi-periodicity or has an indirect existence (e.g. it could be a beat frequency). This conclusion about the coherency of ν_A is partly opposite to the relevant statement of Gosset et al. (1989). The latter clearly indicated the fragility of their test due to an indirect analysis of the phase coherency. The conclusions of the present paper have firmer statistical bases (essentially due to the larger number of measurements permitting a direct test of the phase coherency). On the other hand, we have also presented good evidence for the probable existence of some coherency linked to the frequency ν_B which consequently could be a fundamental one. Besides ν_A and ν_B, WR 40 exhibits also some random variations of variance about 0.5 10^{-3} (in square magnitude) compatible with the existence of a white noise, or slightly correlated, random process. The latter is most probably located in the wind of the Wolf-Rayet star. No more could be said on the basis of the data available; anyway, the mere existence of two characteristic time-scales is a challenge for current models of Wolf-Rayet stars. WR 40 is an extremely interesting object. The physical origin of these two frequencies is indeed difficult to ascertain not only because we are dealing with relatively broad band photometry, but also because the variations we observe most probably take place in the dense wind blowing from the Wolf-Rayet star, i.e. we are only able to observe the response of the wind to the physical mechanism we are looking for. Recently, van Genderen et al. (1989) have suggested (in the case of WR 46) that the photosphere could be partially visible. Even in such a case, one can expect some variability originating in the wind and potentially hiding the signal from the photosphere. On the other hand, Balona et al. (1989b) suggest that density enhancements in the wind are responsible for the observed variability but do not elaborate on the reason of such density enhancements. In the case of a non homogeneous stellar surface, one can indeed expect a modulation of the wind characteristics (e.g. density, temperature) by the rotation through the influence of spots and/or magnetic fields; but this leads to one frequency (possibly with harmonics), not to two frequencies. Modulation of the mass-loss rate by the pattern of velocity field generated at the surface by non-radial pulsations has also been suggested and shown to be able to produce a clear signature on line profiles (Scuflaire and Vreux, 1986). However, the predicted frequencies are up to now definitely higher than the ones discussed here. Altogether, the variability of WR 40 remains extremely challenging as it seems that we are not dealing with a simple periodic phenomenon nor with completely erratic pseudo-periodicities generated by the wind dynamics or spuriously induced by an insufficient sampling. More observations are clearly necessary in order to settle definitively several questions put here. One of the aims of this paper is to stimulate observers. Anyway, it will take some time before we can gather a data set equivalent to the one analyzed here.

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