Eulerian Formulation of a Drillstring Constrained inside a Curved Borehole

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***Abstract*— We address the problem of computing the deformed configuration of a drillstring, constrained to deform inside a curved borehole. This problem is encountered in applications such as *torque-and-drag* and *directional drilling*. In contrast to the traditional Lagrangian approach, the deformed drillstring is described by means of the distance from the borehole axis, in terms of the curvilinear coordinate defined along the borehole. This model is further implemented within a segmentation algorithm -where the borehole and the drillstring are divided into segments limited by contacts, which interestingly transforms the problem into a sequence of analogous auxiliary problems. This Eulerian view of the drillstring flow into the borehole resolves in one stroke a series of issues that afflict the classical Lagrangian approach: (i) the contact detection is reduced to checking whether a threshold on the distance function is violated, (ii) isoperimetric conditions are transformed into regular boundary conditions, instead of being treated as external integral constraints, (iii) the method yields a well-conditioned set of equations that does not degenerate with decreasing flexural rigidity of the drillstring and/or decreasing clearance between the drillstring and the borehole. Theoretical developments related to this Eulerian formulation of the drillstring are presented, along with an example illustrating the advantages of this approach.**

I. Introduction

This paper addresses the problem of computing the configuration of a drillstring, constrained to deform inside a curved borehole. A seminal development in this field is due to Johancsik *et al* [8], who designed a simple *torque-and-drag model* based on the assumption that the drillstring takes exactly the same shape as the wellbore. Owing to its simplicity, this model has been extensively used over the last 25 years, for planning purposes, as well as during field operations. Recent observations have indicated, however, some inconsistencies in this model [9], and have highlighted the need for more sophisticated models [1]. The description of a novel approach to solve the torque-and-drag problem is the main scope of this paper.

The keystone of the *torque-and-drag analysis* is the accurate determination of the contact length and contact forces between the drillstring and the borehole, which is required to estimate the loss of power transmitted from the rig to the drilling bit. Combined with a realistic bit-rock interaction model, the torque-and-drag model contains the essential components to predict the directional tendency of the drilling system. Such integrated models exist [10], [6], but they require considerable computational efforts. We attribute the lack of computational efficiency of the currently available models to the inadequacy of a traditional Lagrangian approach. Furthermore, in the directional drilling context, small discrepancies in the estimated forces transmitted to the drilling bit are likely to affect the predicted trajectory of the bit. Accordingly we agree with [1] in that the standard torque-and-drag model provides only a good estimation of the drag effect along the drillstring, but should be used with caution for any other purpose.

There is thus a real need for a torque-and-drag model that is both accurate and numerically efficient. We consider here the problem of the determination of the constrained deformed shape of the drillstring, i.e. the torque-and-drag analysis, but which is also of paramount interest in other applications. Essentially, the problem considered here may be visualized as the insertion of a drillstring into an existing borehole.

Because the deformations of a drillstring are governed by the theory of elasticity, it is appealing to consider a Lagrangian model. Recent investigations [3] have shown, however, that this natural formulation of the problem has a number of weak points when it is applied to the numerical simulation of slender drillstrings, especially under conditions when the clearance within the borehole is small in comparison with the length scales of the problem. Instead, we formulate the torque-and-drag problem in an Eulerian manner, as a flow of an elastic drillstring into the borehole. In each section of the borehole, we introduce a state variable representing the transverse position of the drillstring inside the borehole. In doing so, we restore the well-conditioning of the governing equations, even in case of a narrow borehole and/or long flexible string, and trivialize the detection of new contacts.

II. Eulerian vs. Lagrangian Descriptions

In a reference system $(\mathbf{e_1}, \mathbf{e_2})$, a borehole is represented by the inclination $\Theta(S)$ of its axis with respect to the vertical axis $\mathbf{e_2}$, where $S \in [0; L]$ denotes the (Eulerian) curvilinear coordinate measured along the neutral axis, see Fig. 1. For the sake of simplicity, the borehole is assumed to have a constant width $2A$ and perfectly rigid walls. This is not a restriction of the proposed method, but a way to introduce the basic features of the model in a simple form.

The drillstring is also supposed to have a constant diameter $2a$, with a given bending stiffness EI, and lineic weight w. A (Lagrangian) curvilinear coordinate $s \in [0; l]$ is naturally introduced, as a curvilinear measure along the drillstring in

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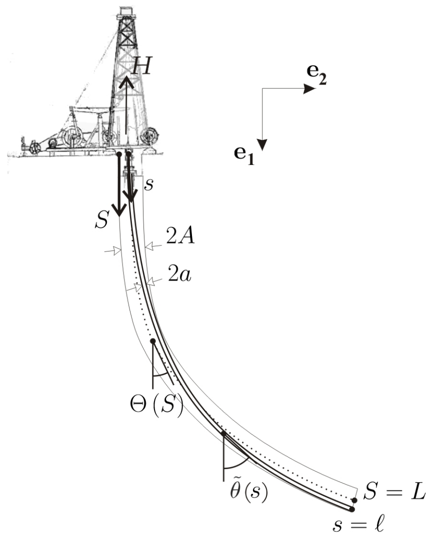

Fig. 1. Description of the borehole with the Eulerian curvilinear abscissa S; description of the drillstring with the Lagrangian curvilinear abscissa s (functions symbolized with a tilde indicate Lagrangian functions). In this paper, the governing equations of the drillstring are written in terms of the Eulerian coordinate S. Sketch is not to scale.

its deformed configuration. The differential equation

$$EI\tilde{\theta}''-\tilde{F}_{0,1}\sin\left(\tilde{\theta}-\tilde{\theta}_0\right)+\tilde{F}_{0,2}\cos\left(\tilde{\theta}-\tilde{\theta}_0\right)+w\,s\,\sin\tilde{\theta}=0, \quad (1)$$

governs the inclination $\tilde{\theta}(s)$ of the drillstring with respect to the vertical axis $\mathbf{e}_2$, see Fig. 1. This equation, known as the nonlinear elastica equation, expresses the shear equilibrium of a nonlinear Bernoulli beam with finite rotations [5]. The axial force $\tilde{F}_{0,1}$ and the shear force $\tilde{F}_{0,2}$ at the rig ($s=0$) are respectively the hook load H and the unknown transverse force on the rotary table.

Reactions take place along the drillstring, as a result of the unilateral penetration constraint with respect to the rigid walls. They are either continuous, i.e. spread along a finite distance, or discrete in which case the contact zone collapses to a single point [11]. Consideration of continuous contacts is unusual in torque-and-drag models. Usually, discrete contacts are rather preferred in order to model the enlargement of the pipe sections where they are connected to each other. Nevertheless, the length scale of an element of pipe is at least one order of magnitude below the one of interest, i.e. that of the average length of continuous contacts. They are therefore not modeled in the proposed method, which results in a significant computational efficiency. In practice, the reactions *pressures* distributed along a continuous contact, as resulting from the proposed model, may be replaced by reactions *forces* at the pipe joints.

Reactions are not accounted for in (1), which makes this equation to be valid between contacts, only.

For simplicity, it is also assumed here that the drillstring rotates during the insertion process; we therefore disregard the friction forces taking place in a direction perpendicular to the plane of analysis. The kinematics of the rotating string are however not embedded in this 2-D model. Neither are the hydraulic pressures.

Various types of boundary conditions are possible. A typical set of limit conditions at the bit would prescribe the bit to be constrained to lie on the neutral axis, and the bit rotation to be free or prohibited to rotate, or to satisfy a bit-rock interaction model, and to be blocked in the axial direction. On the other hand, the upper end of the drillstring at the rig is restrained in rotation and in the transverse direction, but free to slide vertically; furthermore a given hook load H is applied at the rig. The deformed configuration of the drillstring has to be solved not only for these end conditions, but also under the unilateral constraints that express the non-penetration of the drillstring into the rigid walls. The torque-and-drag model presented in this paper allows considering any combination of boundary conditions, as long as the problem is well-posed.

As a part of a more complete directional drilling model, we focus on the problem where the drillstring is assumed to be inserted into an existing borehole, with a suitable selection of the aforementioned boundary conditions.

An interesting outcome of the analysis consists in estimating the actual length of drillstring l inserted into the borehole, as a function of its length L. In Lagrangian approaches, the small relative difference between L and l is closely related to the ill-conditioning of the constrained elastica problem. A reason for the good performance of the Eulerian approach is that the solution methodology does not rely on the small difference $|L-l|/L$. Indeed, the method presented here does not require knowledge about the length of the drillstring; it is actually obtained, if necessary, in a post-processing phase of the analysis results.

III. Segmentation Procedure

A. Lagrangian formulation of the auxiliary problem

A formal introduction of the reactions in (1) would result in a complex differential equation. Instead, and also driven by the idea that the governing equation presents a simple form between contacts in some asymptotic cases, the drillstring is divided into a series of segments separated by the contacts, see Fig. 2. The governing equation is solved sequentially for each segment and some supplementary equations are added afterwards in order to restore the continuity between these segments. This segmentation procedure results in solving a sequence of problems characterized by the same canonical form, hereafter referred to as *the auxiliary problem*, which makes the algorithmic implementation rather straightforward and efficient. An issue related to the segmentation is the varying number of supplementary equations and unknowns; this issue is solved with a hierarchical programming, described in [4]. However, this varying number of segments can actually be interpreted as an adaptive feature of the algorithm, with the number of degrees of freedom naturally matching the complexity of the problem.

The auxiliary problem is written for segment i, running from s_i^+ to s_{i+1}^- along the drillstring and from S_i^+ to S_{i+1}^- along the borehole. In a Lagrangian form, it consists in

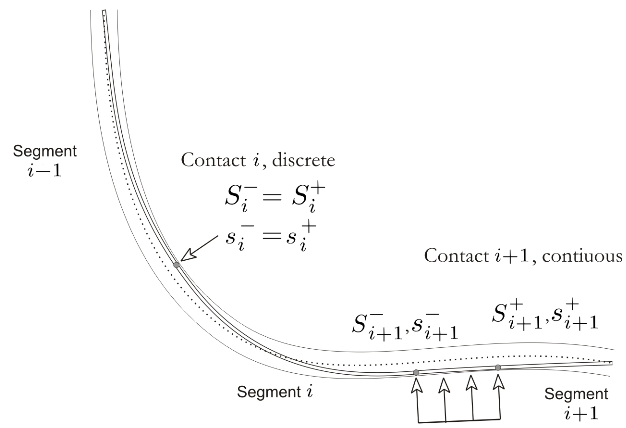

Fig. 2. Segmentation of the drillstring between contacts. Contacts are discrete or continuous; their extents are referenced by $\left[s_i^-; s_i^+\right]$ and $\left[S_i^-; S_i^+\right]$ in the Lagrangian and Eulerian systems. (Notice $s_i^+ = s_{i+1}^-$ and $S_i^+ = S_{i+1}^-$ for discrete contacts)

solving the elastica equation

$$EI\tilde{\theta}'' - \tilde{F}_{i,1}^+ \sin\left(\tilde{\theta} - \tilde{\theta}_i^+\right) + \tilde{F}_{i,2}^+ \cos\left(\tilde{\theta} - \tilde{\theta}_i^+\right) + ws\sin\tilde{\theta} = 0, \tag{2}$$

on the domain $s_i^+ < s < s_{i+1}^-$, where $\tilde{\theta}_i^+$ is the drillstring inclination at $s = s_i^+$ (the upper contact), and with given initial axial force $\tilde{F}_{i,1}^+$, but unknown initial shear force $\tilde{F}_{i,2}^+$. In solving the sequence of auxiliary problems, it is understood that the axial force is an information that may be transported from the first segment (where it is given as the hook load) to the following segment after expressing the global equilibrium of the former segment in the direction of the axis of the borehole. On the contrary, the shear force cannot be carried from segment to segment because it is related to the unknown reaction forces. Equation (2) is solved under the following conditions

$$\tilde{\theta}_i^+ = \Theta_i^+, \quad \tilde{\theta}_{i+1}^- = \Theta_{i+1}^-, \tag{3}$$

$$\int_{s_i^+}^{s_{i+1}^-} \cos\tilde{\theta} ds = \Delta X_i, \quad \int_{s_i^+}^{s_{i+1}^-} \sin\tilde{\theta} ds = \Delta Y_i. \tag{4}$$

The boundary conditions (3) express a tangency condition between the drillstring and the borehole at $s = s_i^+$ and $s = s_{i+1}^-$. (At this stage, the location of the contacts along the borehole S_i and S_{i+1}, and therefore Θ_i^+ and Θ_{i+1}^- are assumed to be given.) The integral conditions (4) simply state that the x- and y-offsets measured along the drillstring correspond to known end positions (the contact positions on the borehole). These conditions provide the necessary information to solve (2) for the inclination $\tilde{\theta}(s)$ in the domain $s_i^+ < s < s_{i+1}^-$ and to determine the segment length $\ell_i = s_{i+1}^- - s_i^+$ as well as $\tilde{F}_{i,2}^+$.

The first and last segments may have boundary conditions different from (3) in order to reflect the boundary conditions of the general problem, i.e. clamped or hinged end, or eventually modeled by a proper bit-rock interaction.

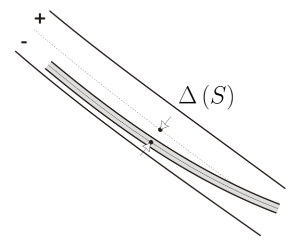

Fig. 3. The signed distance function $\Delta(S)$ is positive on one side of the neutral axis of the borehole and negative on the other side. In our model, the deformed configuration of the drillstring is represented by the signed distance function $\Delta(S)$, rather than $\theta(s)$. The function is one-to-one; we prohibit therefore multiple crossing of the same cross-section.

The auxiliary problem is formulated with a priori given contact locations and therefore known values for Θ_i^+, Θ_{i+1}^- and the offsets ΔX_i and ΔY_i. After the sequential analysis of auxiliary problems, they are actually tuned by restoring the continuity of the bending moment, in the segmented, deformed, drillstring. This is performed thanks to a nonlinear solver wrapped around the sequence of auxiliary problems.

Several limitations of this algorithm are identified. First, several solutions to the nonlinear differential equation (2) exist, but only one is of interest. The other possible solutions involve a curl in the drillstring, a known typical feature of the elastica. In regimes where such solutions are close to the one of interest (e.g. for small flexural rigidity, the differential equation becomes obviously ill-conditioned.) Second, the numerical solution of (2) requires a subtle discretisation of $\left[s_i^+; s_{i+1}^-\right]$, not only because of the possible ill-conditioning of the equation and the existence of high-gradient zones in the response, but also because the limits of this domain are *a priori* unknown. This issue needs to be handled by considering an augmented unknown state collecting also the abscissa of the contacts along the drillstring. Third, for every new position of the drillstring into the borehole, the occurrence of new contacts needs to be checked, which requires the costly computation of the position of multiple sections of the deformed drillstring and the check of their position between the walls of the borehole; this involves a minimum finding optimization problem. At last but not least, the isoperimetric constraints (4) are stiff, and contribute to the ill-conditioning of the system.

These limitations are suppressed by considering an Eulerian formulation of the auxiliary problem, together with the introduction of an Eulerian *signed distance function* $\Delta(S)$ representing the distance between the drillstring and the neutral axis of the borehole, see Fig. 3. It is evident that the determination of this function on $0 \leq S \leq L$ trivializes that contact detection, expressed now as a simple check that $|\Delta| \leq c$, with $c = A - a$.

B. Eulerian formulation of the auxiliary problem

We reformulate next the Lagrangian governing equation (2) in terms of $\tilde{\theta}(s)$, as well as its boundary conditions, into an Eulerian formulation in terms of $\Delta(S)$.

For this purpose, we express the drillstring inclination $\tilde{\theta}(s)$ and its derivatives in terms of the two Eulerian functions,

$\Theta(S)$ and $\Delta(S)$, with the former describing the known geometry of the borehole and the latter the unknown geometry of the deformed drillstring relative to the borehole.

First we introduce the function $\bar{s}(S)$, which maps the Eulerian coordinate onto the Lagrangian coordinate. The Jacobian $\bar{s}'$ of this transformation is expressed as

$$\bar{s}' = \sqrt{(1-\Delta\Theta')^2 + \Delta'^2}, \tag{5}$$

which confirms the existence of a drift between the two curvilinear coordinates since the drillstring does not espouse exactly the borehole of the conduit, i.e., $\Delta(S) \neq 0$ for some S. Any Lagrangian function of s is expressed as an Eulerian function through this mapping; for instance $\tilde{\theta}\left(\bar{s}(S)\right) = \theta\left(S\right)$.

The derivatives of the inverse function $\tilde{S}(s)$ are also required next, expressed as a function of S. Let $J_k(S)$ be defined as

$$J_k = \left.\frac{d^k\tilde{S}}{ds^k}\right|_{s=\bar{s}(S)}. \tag{6}$$

The function $J_k(S)$ can be written explicitly in terms of $\Theta(S)$ and $\Delta(S)$. Indeed, with $J_1 = 1/\bar{s}'$, $J_k(S)$ can be computed recursively according to $J_k = J_1 J'_{k-1}$, $k > 1$.

The inclination $\theta(S)$ of the deformed drillstring is related to $\Delta(S)$ and $\Theta(S)$ by

$$\begin{aligned} \cos\theta &= \left[(1-\Delta\Theta')\cos\Theta - \Delta'\sin\Theta\right] J_1, \\ \sin\theta &= \left[(1-\Delta\Theta')\sin\Theta + \Delta'\cos\Theta\right] J_1. \end{aligned} \tag{7}$$

Furthermore, $\theta'\left(S\right)$ can also be expressed explicitly in terms of $\Delta\left(S\right)$ and $\Theta(S)$ and their derivatives, by first writing

$$\theta' = (\sin\theta)'\cos\theta - (\cos\theta)'\sin\theta \tag{8}$$

which, after taking into account (7), yields

$$\begin{aligned} \theta' = {} & J_1^2\left(\Theta' + \Delta'' - 2\Delta\Theta'^2 + \Delta^2\Theta'^3\right. \\ & \left. + 2\Delta'^2\Theta' - \Delta\Delta''\Theta' + \Delta\Delta'\Theta''\right). \end{aligned} \tag{9}$$

Further derivatives of this expression successively yield higher derivatives of $\theta\left(S\right)$. Finally, the derivatives of the function $\tilde{\theta}(s)$ can be expressed explicitly in terms of the unknown function $\Delta(S)$ and the conduit inclination $\Theta(S)$, using

$$\begin{aligned} \tilde{\theta}' &= \theta' J_1, \\ \tilde{\theta}'' &= \theta'' J_1^2 + \theta' J_2. \end{aligned} \tag{10}$$

The Eulerian formulation of the auxiliary problem is formally obtained by substituting (7) and (10) in (2). More importantly, (2) is now written in terms of the Eulerian coordinate S rather than s, the Lagrangian one,

$$\begin{aligned} & EI\left(\theta'' J_1^2 + \theta' J_2\right) - \tilde{F}^+_{i,1}\sin\left(\theta - \theta_i^+\right) \\ & \quad + \tilde{F}^+_{i,2}\cos\left(\theta - \theta_i^+\right) + w\,\bar{s}(S)\sin\theta = 0 \end{aligned} \tag{11}$$

It is a third order differential equation in $\Delta\left(S\right)$ that needs to be solved along $S_i < S < S_{i+1}$. Four boundary conditions are necessary to solve (11), as the constant $\tilde{F}^+_{i,2}$ is unknown. They are obtained as a formal reformulation of (3)-(4) in an Eulerian context,

$$\begin{aligned} \Delta'\left(S_i\right) &= 0; & \Delta'\left(S_{i+1}\right) &= 0, \\ \Delta\left(S_i\right) &= \pm c; & \Delta\left(S_{i+1}\right) &= \pm c. \end{aligned} \tag{12}$$

upon consideration that the ends of this drillstring segment contact the borehole. Equation (11) is considerably more complicated than its Lagrangian alternative (2), essentially because J_1, J_2 and $\bar{s}\left(S\right)$ are complex expressions of $\Delta\left(S\right)$. Nevertheless, the reformulation has basically resolved the issues associated with the Lagrangian approach. For instance, the introduction of a one-to-one signed distance function prohibits any curl of the drillstring into the borehole, and improves therefore significantly the conditioning of the system of equations. Also, the stiff isoperimetric constraints (4) have been substituted with simple boundary conditions on the function Δ itself.

Equation (11) is written in terms of the dimensionless quantities

$$\alpha = \frac{c}{L_i}; \quad \omega = \frac{wL_i^3}{EI} \tag{13}$$

with $L_i = S_{i+1} - S_i$, the length of the borehole between two successive contacts. The governing equation (11) is naturally written in terms of a dimensionless distance function

$$\delta\left(\xi\left(S\right)\right) = \frac{\Delta\left(S\right)}{c}, \tag{14}$$

with $\xi = \left(S - S^+_{i-1}\right)/L_i \in [0;1]$. After some developments, (11) is written

$$\mathcal{D}[\alpha\,\delta(\xi);\vartheta(\xi);\omega] = 3\mathcal{J}_1^2\mathcal{J}_2\mathcal{K} + \mathcal{J}_1^4\mathcal{K}' + \mathcal{F}_2 = 0. \tag{15}$$

with $\vartheta(\xi) = \Theta\left(\xi L_i\right)$ and with the curvature of the drillstring $\mathcal{K}$ defined as

$$\begin{aligned} \mathcal{K} = {} & \vartheta' + \alpha_i\left(\delta'' - 2\delta\vartheta'^2\right) \\ & + \alpha_i^2\left(\delta^2\vartheta'^3 + 2\delta'^2\vartheta' - \delta\delta''\vartheta' + \delta\delta'\vartheta''\right) \end{aligned} \tag{16}$$

and where $\mathcal{F}_2$ symbolizes the dimensionless shear force. The differential operator $\mathcal{D}$ is nonlinear in $\delta\left(\xi\right)$. It features the same nonlinearities as the elastica equation, i.e. large displacements and rotations, and therefore the ability to model in-plane instabilities. The only difference regards the curl, prohibited in our model. Notice that the segmentation algorithm is convenient for the modeling of a buckling drillstring; indeed, as soon as a new contact appears between the buckled string and the borehole, the segment in question is divided into two parts. If the compressive force is further increased to reach a second critical level, buckling occurs, a new contact takes place and the wavelength is automatically shortened.

IV. Illustration

As an illustration we consider the idealized case of a well with a planar trajectory consisting of a circular segment that changes the orientation of the well from vertical to horizontal and characterized by a radius $R = 750$ m, which is connected to a 500 m long horizontal segment, see Fig. 4. The total

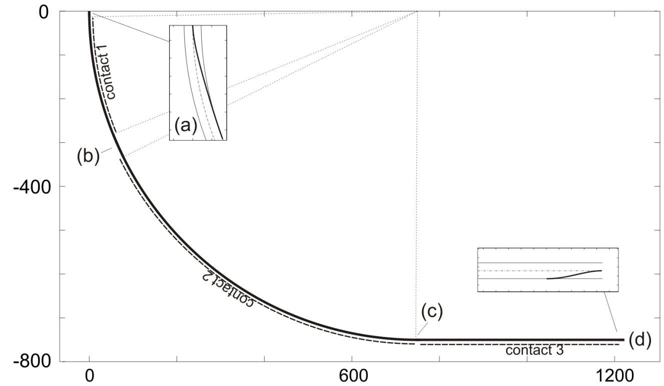

Fig. 4. Example of a drillstring inserted in a curved borehole, idealized by a circular segment and a straight segment. Three zones of continuous contact are identified; they divide the drillstring into four segments (a)-(d). Close-ups in insets are not to scale. Axis units in meters

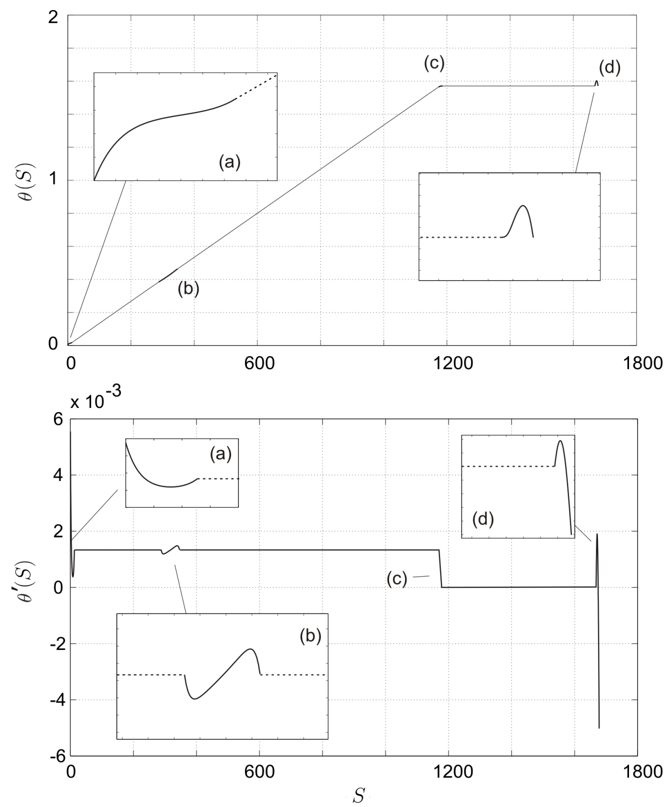

Fig. 5. Inclination $\theta(S)$ (in rad) and Eulerian curvature $\theta'(S)$ (in rad/m) of the drillstring. Because of the small clearance, they can be both considered as small perturbations (in amplitude or extent) of the inclination and curvature of the borehole axis, represented with dashed lines in contact zones.

length of this borehole is thus 1678.1 m and the diameter is taken to be equal to 0.2032 m (8"). The drillstring is assumed to be a continuous pipe with an outer diameter of 0.1143 m (4.5") and an inner diameter of 0.0925 m (3.64"), which is characterized by a weight per unit length $w = 292$ N/m (20 lb/ft) and a bending stiffness $EI = 0.96$ MNm^2. With these values for the outer diameter of the pipe and the diameter of the well, the clearance $c = 0.444$ m. The drillstring is assumed to be clamped at both ends and centered in the borehole: at the rotary table of the rig corresponding to the inlet of the circular segment $\theta = 0$ and $\Delta = 0$; at the bit corresponding to the end of the horizontal segment, $\theta = \pi/2$ and $\Delta = 0$. Finally the system of equations is closed by prescribing the hook load H at the rig and imposing the bit to be blocked in the axial direction.

Any more complex representation of the drillstring could be given, including a massive BHA section with a selection of stabilizers. In that case, the BHA is splitted, in the segmentation process, at the location of stabilizers, simply. Also, the framework of the proposed theory allows any enhancement related to the presence of RS-systems or down-hole pointing system; their model, as complex as desired, just requires being connected to the present model by means of generalized forces, displacements and penetrations. We consider the case when the drillstring is suspended at the rig with an axial force $H = 180$ kN. Figure 4 represents the zones where the drillstring contacts the borehole. In fact, the drillstring is so slender that there are no discrete contacts. The segmentation procedure identifies four free segments, labeled (a)-(d), connected by three continuous contacts. By reference to Fig. 4, where the continuous contacts between the drillstring and the wall (either left or right) of the borehole are represented by dashed lines, the following contact pattern can be observed. Because the boundary condition at the rig is a drillstring centered on the borehole axis, there is a first segment (a) free of contact with the borehole; it stretches over a short length of about 13 m. Due to the large tension force, the drillstring is pressed against the right wall, creating therefore a large continuous contact zone. Figure 4 shows the existence of a segment (b) where the drillstring is not touching the borehole wall over a length of approximately 60 meters. In this segment, the signed distance function varies from $+c$ to $-c$. This is the typical auxiliary problem. Segment (b) is followed by a long continuous contact with the lower part of the borehole. The curvature of the drillstring naturally complies with that of the borehole wall along that continuous contact zone, i.e. $\theta' = 1/(R+c)$. Because the bending moment (and hence the curvature) has to be continuous in the drillstring, the drillstring itself cannot be continuously in contact with the borehole through the transition from the circular to the straight segment. Thus the segmentation procedure naturally generates a short segment (c) where the drillstring is not in contact with the walls. Finally, after a long continuous contact along the straight wall, the drillstring has to separate from the wall in order to satisfy the boundary conditions at the bit. The length of this last segment (d) is about 10 meters.

Figure 5 shows the variation of the drillstring inclination θ and a measure of its Eulerian curvature θ' along the borehole. The Eulerian curvature $\theta'(S)$ is very close to the physical curvature $\tilde{\theta}(s)$ as a result of the small clearance. We may observe that both θ and θ' are expressed as small perturbations of the borehole inclination and curvature, provided we extend the meaning of *small perturbations* to perturbations

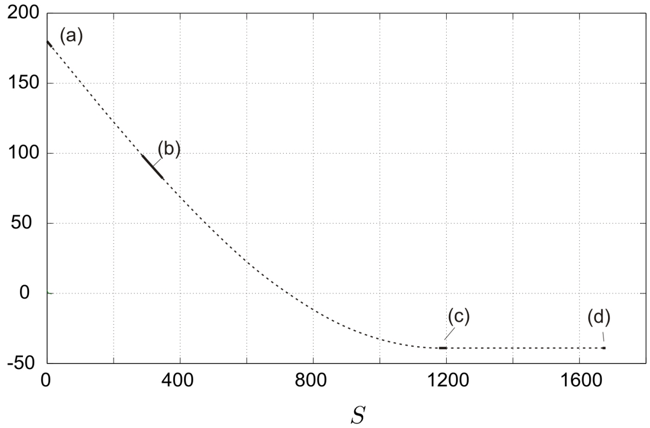

Fig. 6. Axial force in the drillstring $F_1(S)$, in kN.

in very short parts of the domain, but not necessarily with a small amplitude. Observe for instance the curvature in segment (a). This example indicates that torque-and-drag models in which the drillstring sits on the borehole axis provide a poor estimation of the internal forces, especially in segments that are free of contact with the borehole walls. It further suggests that torque-and-drag models in which the deformed configuration of the drillstring is expressed as a small perturbation (in the usual sense) of the borehole axis are questionable. Indeed, although we may agree that the drillstring is "not far" from the borehole axis, its inclination and curvature may differ significantly from those of the borehole.

Of most interest is the variation of θ' in segment (b), where the double curvature response (about the borehole curvature) is typical of a cable with a moderate bending stiffness. For that segment, the dimensionless flexibility is estimated as

$$\omega_i \simeq \frac{wL_i^3}{EI} = \frac{300 \times 60^3}{0.96 \times 10^6} = 67.5. \tag{17}$$

The dimension of the possible boundary layer in such a cable is of order $\omega_i^{-1/2} L_i = 0.12 L_i$, which is a limit below which grid-based numerical methods fail to be efficient. In case of even more slender segments, the gradient of drillstring curvature is so large that it precludes any use of traditional numerical techniques. In this case, asymptotic solutions of (15) may be obtained, see [2] for details.

Finally Fig. 6 illustrates the variation of the axial force in the drillstring from the rig to the bit. It decreases faster where the borehole is vertical, then decreases slowly, as no friction is considered. In particular, the location of the neutral point is calculated to be at $S \simeq 750$m. This indicates that a large portion of the drillstring is in compression. Also, the axial force levels off at $\hat{F}_1 = -39\,kN$. The corresponding buckling length is

$$L_b = \frac{\pi}{2} \sqrt{\frac{EI}{\left|\hat{F}_1\right|}} \simeq 8m. \tag{18}$$

What we observe as segment (d) is nothing but the last wave of a buckling pattern, that would most likely extent all along the horizontal part of the borehole. In this case, this instability is not properly captured by the segmentation algorithm which has provided another equilibrium state, although unstable. This demonstrates the robustness of the proposed algorithm against ill-conditioning.

V. Conclusions

The problem of computing the configuration of a drillstring constrained to deform inside a curved borehole is part of larger class of problems involving a priori unknown contacts between an elastica and a rigid boundary. These problems are computationally challenging, especially in the context of the drilling applications where use of standard numerical tools result in an ill-conditioned system of equations, owing mainly to the narrowness of the borehole compared to its length, but also to the large flexibility of the drillstring and the assumed rigid nature of the borehole walls. In this paper, a number of reasons for which Lagrangian approaches are ill-adapted to tackle this kind of problem have been highlighted.

Taking advantage of a description of the deformed drillstring by means of a signed distance function, we proposed a novel mathematical formulation based on an Eulerian flow of drillstring into the borehole. The model is implemented within an efficient segmentation algorithm, reducing the global analysis of the drillstring to a sequence of simple auxiliary problems (between contacts) having the same canonical form. The a priori unknown number of contact resulting from the segmentation is an issue that is solved with advanced programming techniques, but interestingly this particularity makes also the proposed model adaptive, in the sense that it automatically adapts to the complexity of the deformed shape of the drillstring and of the borehole.

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