New Developments for an Efficient Solution of the Discrete Material Topology Optimization of Composite Structures

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INTRODUCTION

- Development of new renewable energy systems: high performance materials (strength, durability...)

- Sustainability of transportation systems: weight reduction

- Revived interest in composite structures.
  ➔ optimization of composites to take the best of their performances
Goals of this work

- Problems to be addressed:
  - Optimal layout of laminates over the structure
  - Through-the-thickness-optimization of composites: stacking sequence optimization

- Discrete Material Optimization approach
  - Improve and robustify
Goals of this work

- Discrete Material Optimization approach
  - Formulate the optimization problem as a ‘n’ materials selection problem
  - Use an extended topology optimization approach to solve the problem in continuous variables
  - General / global approach to solve optimal layout and stacking sequence

\[ C_i = \sum_{j=1}^{m} w_{ij} C_i^{(j)} \]

\[ 0 \leq w_{ij} \leq 1 \quad \sum_{j=1}^{m} w_{ij} \leq 1 \]

\[ w_{ik} = 0 \quad (k \neq j) \text{ when } w_{ij} = 1 \]
Goals of this work

- Discrete Material Optimization
  - Pioneer work by Stegmann and Lund (2005)
    - Several interpolation schemes (DMO1...5)
  - New approach by Bruyneel (2011) with the Shape Function with Parameterization (SFP)
    - Limited to four materials (0°/90°/-45°/45°) or three materials (0°/90°/ (45°/-45°))

- This work:
  - Generalize the DMO and SFP
  - Investigate the penalization
  - Tailor a robust solution procedure based on the sequential convex programming (approximation + solution using efficient math programming algos)
  - Validate on applications
Outline

- Introduction & motivation
- Discrete Material Optimization
  - Interpolation schemes:
    - DMO,
    - SFP,
    - BVCP
  - Penalization schemes
- Laminate optimization problem
- Numerical applications
  - In-plane laminate optimization
  - Composite shell optimization

Conclusion & Perspectives
**DMO** (*Stegmann and Lund, 2005*)

- **DMO4 interpolation scheme:**
  - Extension of Thomsen (1992) and Sigmund & Torquato (2000) topology optimization schemes
  - Introduces one existence variable ([0,1]) per material
  - Uses a power law (SIMP) penalization of intermediate densities

\[ w_{ij} = x_{ij}^p \prod_{\substack{\xi = 1 \\ \xi \neq j}}^{m_v} \left( 1 - x_{i\xi}^p \right) \quad \text{with} \quad 0 \leq x_{ij} \leq 1 \]

![Graphs showing interpolation](https://via.placeholder.com/150)

- \( w_1 \) with \( p = 3 \)
- \( w_1 \) with \( p = 15 \)
Shape Function with Penalization (SFP)  
Bruyneel (2011)

- SFP scheme takes the Lagrange polynomial interpolation of finite element shape functions
  - For 0°/90°/45°/-45°: four-node finite element
  - For 0°/90°/[45°/-45°]: three-node finite element

\[
\begin{align*}
  w_1 &= \frac{1}{4} (1 - R)(1 - S) \\
  w_2 &= \frac{1}{4} (1 + R)(1 - S) \\
  w_3 &= \frac{1}{4} (1 + R)(1 + S) \\
  w_4 &= \frac{1}{4} (1 - R)(1 + S)
\end{align*}
\]

- Introduces a power penalization (SIMP)

\[
w_i^{SFP} = \left[ \frac{1}{4} (1 \pm R)(1 \pm S) \right]^p
\]
Shape Function with Penalization (SFP) 
Bruyneel (2011)

- SFP shape functions and penalization

\[ w_i^{SFP} = \left[ \frac{1}{4} (1 \pm R)(1 \pm S) \right]^p \]

- Two design (instead of 4) variables ranging in \([-1, 1]\)
- Extension to ‘n’ node finite element theoretically possible, but problem rapidly complex
Bi-valued coded parameterization
(G. Tong et al. 2011)

- Bi-value coding parameterization generalizes the SFP scheme
- Abandon the shape function idea, but keep the idea of coding the materials using bi-value variables (typically [-1,1])

\[ w_{ij} = \left[ \frac{1}{2^m} \cdot \prod_{k=1}^{m_v} \left( 1 + s_{jk} x_{ik} \right) \right]^p \] with \(-1 \leq x_{ik} \leq 1\) and \(k = 1, \ldots, m_v\)

- Number of design variable is \(m_v = \log_2 m\)
  - Possible to interpolate between \(2^{(m_v-1)}\) to \(2^{m_v}\) materials with \(m_v\) variables
  - Introduce a penalization scheme (here power law) to end-up with -1/1 values
**Bi-valued coded parameterization**
*(G. Tong et al. 2011)*

- Visualization: for $m_v=2$ and $m_v=3$, the method recovers 4-node and brick (8-node) elements shape functions.

<table>
<thead>
<tr>
<th>$j$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<td>-1</td>
<td>1</td>
<td>1</td>
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<tr>
<td>2</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1 $s_{jk}$ values ($m_v=2, m=4$)

(a) $m_v=2$, $m=4$

(b) $m_v=3$, $m=8$
Penalization schemes

To come to a solution with one single material, one introduces a penalization schemes:

- SIMP
  \[ f(\chi) = \chi^p \]

- RAMP (Stolpe & Svanberg, 2001)
  \[ f(\chi) = \frac{\chi}{1 + p(1 - \chi)} \]

- Halpin Tsai (Halpin-Tsai, 1969)
  \[ f(\chi) = \frac{r\chi}{(1 + r) - \chi} \]

- Polynomial penalization (Zhu, 2009)
  \[ f(\chi) = \frac{\alpha - 1}{\alpha} \chi^p + \frac{1}{\alpha} \chi \]
Penalization schemes

- Choice of penalization scheme
  - Basically one can find equivalent penalizations of intermediate densities by a proper choice of the parameters
    \[ P=3 \leftrightarrow r=0.269 \leftrightarrow \alpha=16 \]
  - Polynomial / RAMP and Halpin Tsai are necessary when considering body loads (see Bruyneel & Duysinx, 2005)
  - Polynomial is generally the simplest and the most effective in many cases

- Continuation procedure (Bendsoe, 1989) i.e. increasing progressively the penalization parameter \( p \)
  - Should avoid being trapped in local optima
  - In practice, here, the numerical experiments have reported no influence of even detrimental influence.
Optimization Problem Formulation

- **Laminate optimization**
  - Compliance minimization under given load cases
    
    \[ \text{find: } \{x_{ik}\} \quad (i = 1, \ldots, n; k = 1, \ldots, m_v) \]
    
    \[ \text{minimize: } C = F^T u \]
    
    subject to: \( F = Ku \)
  
  - For pure laminate optimization, no resource (volume) constraint is generally necessary

- **Topology optimization of laminate**
  - Selection of ply orientation and layout of the laminate
    
    \[ c^l = (\mu_1)^q \left( \sum_{i=1}^{n'} w_i^l c_i^l \right) \]
    
    \[ \sum_{l=1}^{n_v} \mu_1 V_l \leq V \]

- Introduce a volume constraint of the foam or of the fiber material
Sensitivity analysis

- Sensitivity analysis
  \[
  \frac{\partial C}{\partial x_{ik}} = 2u^T \frac{\partial F}{\partial x_{ik}} - u^T \frac{\partial K}{\partial x_{ik}} u = -u^T \frac{\partial K}{\partial x_{ik}} u
  \]

  Requires the derivatives of the weighting functions
  \[
  \frac{\partial K_i}{\partial x_{ik}} = \sum_{j=1}^{m} \frac{\partial w_{ij}}{\partial x_{ik}} K_i^{(j)}
  \]

- Solution of optimization problem: Sequential Convex Programming
  - Sequence of explicit subproblems
    - CONLIN (Fleury, 1989)
    - GCMMA (Bruyneel et al., 2002)
  - General strategy with efficient capabilities in treating large scale problems
Implementation

- Implementation
  - Analysis carried out in SAMCEF Composites
    - Laminate plate elements
    - Thick composite shells (8-node bricks)
- Optimization
  - Boss Quattro Open Object Oriented platform for Optimization
Design problem

- Design variables (more than 1,000):
  - Composite panels and stringers lay-ups (thicknesses, ply proportions)
  - Stringer dimensions
- Minimize weight
- Constraints (more than 100,000)
  - Reserve factors
  - Buckling, reparable, ...

Illustration of limitation of present solvers

Solution of large scale problem using CONLIN

With courtesy by Samtech and Airbus Industries
Numerical applications: Square plate under vertical force

- Maximum in-plane compliance problem is solved by selecting the optimal orientation of the ply

.Loads and boundary conditions Design model with 4x4 patches

Table 4 Material properties

<table>
<thead>
<tr>
<th></th>
<th>$E_x$</th>
<th>$E_y$</th>
<th>$G_{xy}$</th>
<th>$\nu_{xy}$</th>
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<tr>
<td></td>
<td>146.86GPa</td>
<td>10.62GPa</td>
<td>5.45GPa</td>
<td>0.33</td>
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</table>

Table 3 Orientations

<table>
<thead>
<tr>
<th>Number of material phases ($m$)</th>
<th>Number of design variables for each region ($m_v$)</th>
<th>Discrete orientation angle (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2</td>
<td>90/45/0/-45</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>80/60/40/20/0/-20/-40/-60/-80</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
<td>90/75/60/45/30/15/0/-15/-30/-45/-60/-75</td>
</tr>
</tbody>
</table>
Numerical applications: Square plate under vertical force

(a) DMO ($C=1.220 \times 10^{-4}$)  
$\delta_{v}=4$

(b) SFP ($C=1.182 \times 10^{-4}$)  
$\delta_{v}=2$

(c) BCP ($C=1.182 \times 10^{-4}$)  
$\delta_{v}=2$

Optimization results of the square plate under vertical force ($m=4$)
Numerical applications: Square plate under vertical force

*Iteration histories of the weight for patch 16 (BCP m=4)*
Numerical applications: Square plate under vertical force

Influence of the penalization factor \( p \) of the BCP scheme upon the optimization results
Numerical applications: Square plate under vertical force

- Topology optimization: void + laminate
- Volume constraint: $V < 11/16$

4 orientations
90/45/0/-45

18 orientations
90/80/70/60/50/40/30/20/10/0/
-10/-20/-30/-40/-50/-60/-70/-80
**Numerical applications: Square plate under vertical force**

- Both orthotropic glass-epoxy and isotropic polymer-foam are involved
- Amount of glass-epoxy is limited to 11 patches

Optimization result of the square plate under vertical force with volume constraint:
Glass-epoxy with 4 orientations (90/45/0/-45) and polymer-foam

- Remark: convergence is difficult. CONLIN and MDQA fail. GCMMA-V4 converges after 150 iterations
Numerical applications: Simply support beam (MBB beam)

- A simply-supported beam of one single layer

- Mesh: $240 \times 40$ quadrangular finite elements
- Possible ply orientation: $m=36$
  \(90/85/80/75/70/65/60/55/50/45/40/35/30/25/20/15/10/5/0/-5/-10/-15/-20/-25/-30/-35/-40/-45/-50/-55/-60/-65/-70/-75/-80/-85\)
  - 6 design variables ($mv=6$) are needed for each designable patch
Numerical applications: Simply support beam (MBB beam)

Optimization results using BCP scheme (Patch: 12×4, m=36)
Numerical application: long box

- 4 layers; element size=4
- L1 & L4: glass-epoxy (90/45/0/-45)
- L2 & L3: glass-epoxy (90/45/0/-45) + polymer-foam
- Orientation: 90/45/0/-45 in 1-2 plane for each element axes
Numerical application: long box

- Objective: minimize structural compliance
- 90° /45° /0° /-45° (blue/cyan/yellow/red)

Layer 1 (inner ply)

Layer 2

Layer 3

Layer 4 (outer ply)
Numerical application: long box

- Objective: minimize structural compliance
  s.t. volume constraint: glass-epoxy < 80%

- Results:
Numerical application: long box

- Layer 1
- Layer 2
- Layer 3
- Layer 4

- 90°-45°-0°-45° (blue/cyan/yellow/red)
CONCLUSIONS

- A novel parameterization scheme based on a bi-value coding for solving the discrete material optimization of composite structures

- Reduced number of design variables, the BCP scheme generalizes the SFP scheme by Bruyneel (2011) and is a challenger to the classic DMO for large-scale problems

- BCP formulation provides a well-posed problem for an efficient solution using sequential convex programming algorithms (15-20 iterations necessary)
PERSPECTIVES

- Extend the application of this novel parameterization scheme to larger problems involving industrial composite structures.

- Including compliance, displacement, stress constraints but also buckling and perimeter constraints.
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