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New Developments for an Efficient Solution of the Discrete Material Topology Optimization of Composite Structures

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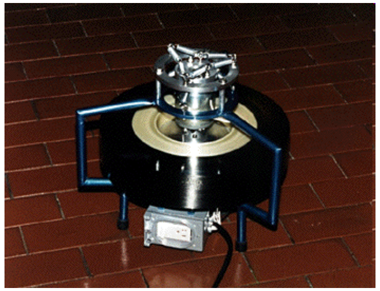
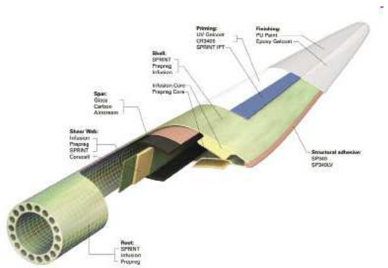
INTRODUCTION

- Development of new renewable energy systems: high performance materials (strength, durability…)
- Sustainability of transportation systems: weight reduction

- Revived interest in composite structures.

➔ optimization of composites to take the best of their performances



Goals of this work

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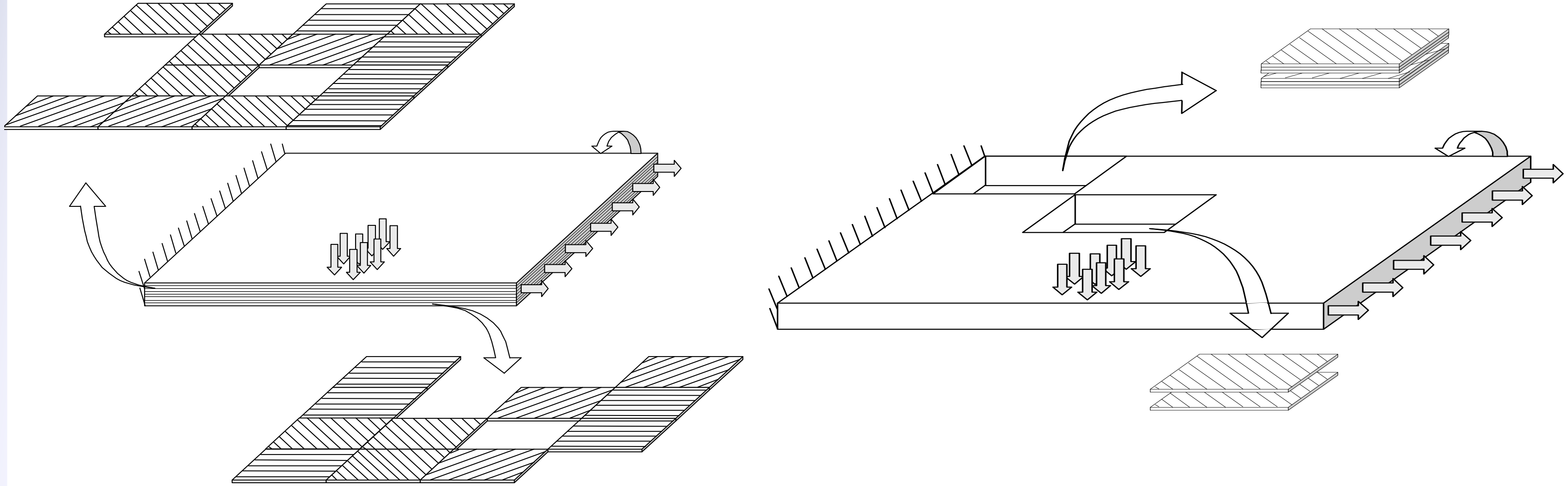
- Problems to be addressed:
  - Optimal layout of laminates over the structure
  - Through-the thickness-optimization of composites: stacking sequence optimization



- → Discrete Material Optimization approach
  - Improve and robustify

Goals of this work

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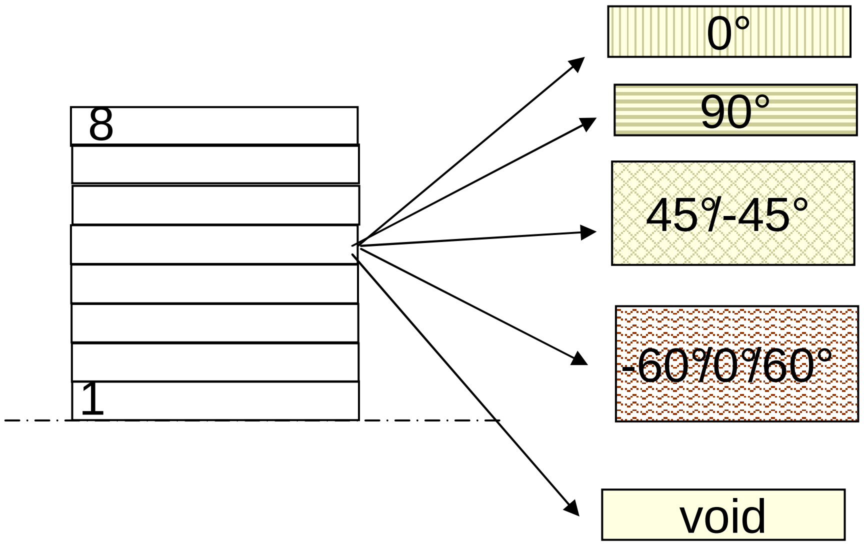
- Discrete Material Optimization approach
  - Formulate the optimization problem as a 'n' materials selection problem
  - Use an extended topology optimization approach to solve the problem in continuous variables
  - General / global approach to solve optimal layout and stacking sequence

$$\mathbf{C}_i = \sum_{j=1}^{m} w_{ij} \mathbf{C}_i^{(j)}$$

$$0 \le w_{ij} \le 1 \qquad \sum_{j=1}^{m} w_{ij} \le 1$$

$$w_{ik} = 0\,(k \ne j) \text{ when } w_{ij} = 1$$



Goals of this work

- Discrete Material Optimization
  - Pioneer work by Stegmann and Lund (2005)
    - Several interpolation schemes (DMO1…5)
  - New approach by Bruyneel (2011) with the Shape Function with Parameterization (SFP)
    - Limited to four materials (0°/90°/-45°/45°) or three materials (0°/90°/ (45°/-45°))

- This work:
  - Generalize the DMO and SFP
  - Investigate the penalization
  - Tailor a robust solution procedure based on the sequential convex programming (approximation + solution using efficient math programming algos)
  - Validate on applications

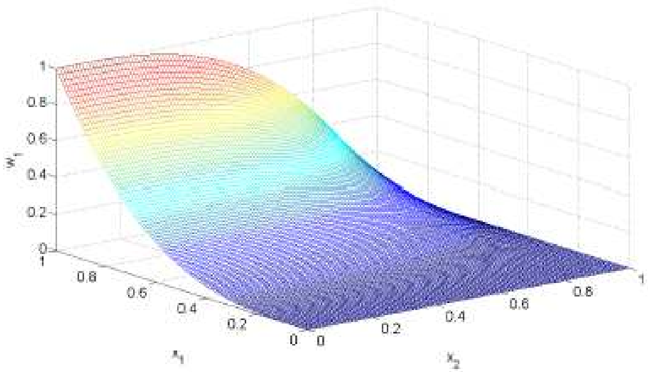
Outline

- Introduction & motivation
- Discrete Material Optimization
  - Interpolation schemes:
    - DMO,
    - SFP,
    - BVCP
  - Penalization schemes
- Laminate optimization problem
- Numerical applications
  - In-plane laminate optimization
  - Composite shell optimization
- Conclusion & Perspectives

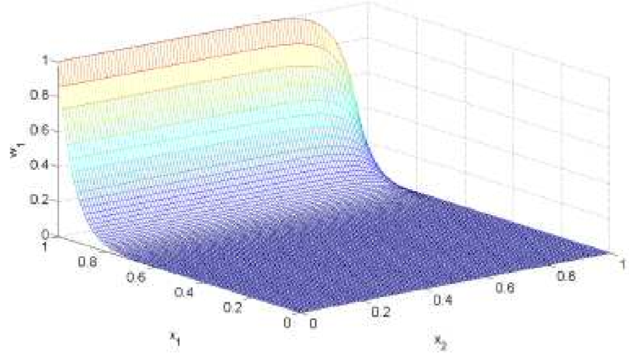
DMO (Stegmann and Lund, 2005)

- DMO4 interpolation scheme:
  - Extension of Thomsen (1992) and Sigmund & Torquato (2000) topology optimization schemes
  - Introduces one existence variable ([0,1]) per material
  - Uses a power law (SIMP) penalization of intermediate densities

$$w_{ij} = x_{ij}^{p} \prod_{\substack{\xi=1 \\ \xi \neq j}}^{m_v} \left(1 - x_{i\xi}^{p}\right) \quad \text{with} \quad 0 \leq x_{ij} \leq 1$$



w_1 with $p=3$



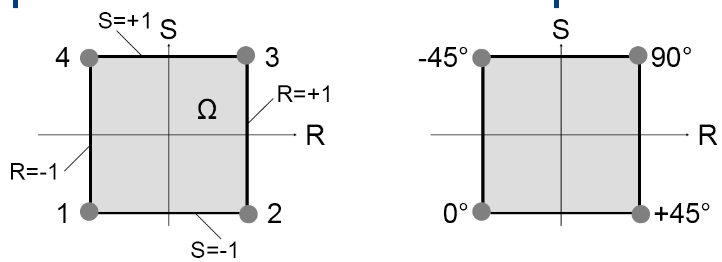
w_1 with $p=15$

Shape Function with Penalization (SFP)
Bruyneel (2011)

- SFP scheme takes the Lagrange polynomial interpolation of finite element shape functions
  - For 0°/90°/45°/-45°: four-node finite element
  - For 0°/90°/[45°/-45°]: three-node finite element

$$w_1 = \frac{1}{4}(1-R)(1-S) \quad w_2 = \frac{1}{4}(1+R)(1-S)$$

$$w_3 = \frac{1}{4}(1+R)(1+S) \quad w_4 = \frac{1}{4}(1-R)(1+S)$$

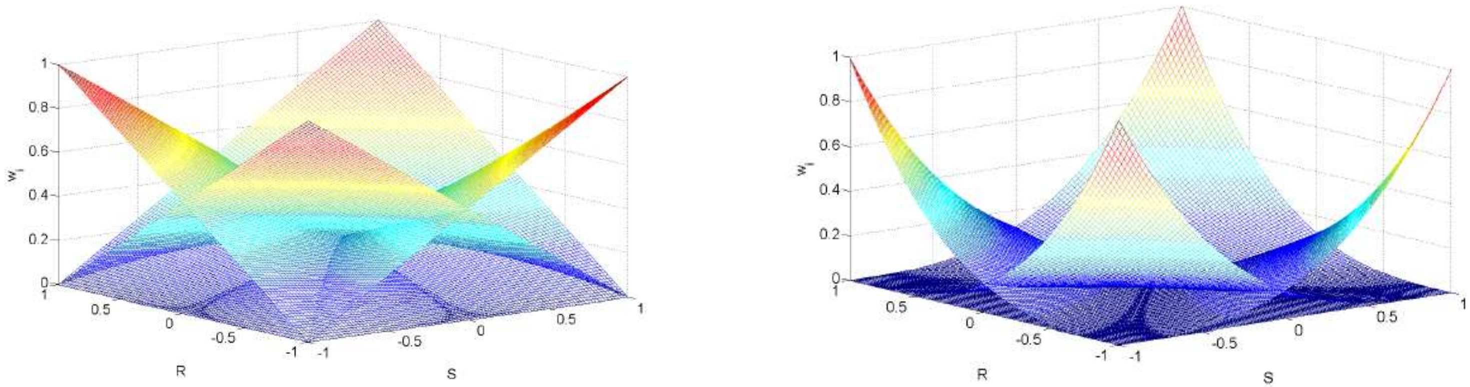


- Introduces a power penalization (SIMP)

$$w_i^{SFP} = \left[\frac{1}{4}(1 \pm R)(1 \pm S)\right]^p$$

Shape Function with Penalization (SFP)
Bruyneel (2011)

- SFP shape functions and penalization



$$w_i^{SFP} = \left[\frac{1}{4}(1 \pm R)(1 \pm S) \right]^p$$

- Two design (instead of 4) variables ranging in [-1,1]
- Extension to 'n' node finite element theoretically possible, but problem rapidly complex

Bi-valued coded parameterization (G. Tong et al. 2011)

- Bi-value coding parameterization generalizes the SFP scheme
- Abandon the shape function idea, but keep the idea of coding the materials using bi-value variables (typically [-1,1])

$$w_{ij} = \left[\frac{1}{2^{m_v}} \cdot \prod_{k=1}^{m_v} \left(1 + s_{jk} x_{ik} \right) \right]^p \text{ with } -1 \le x_{ik} \le 1 \text{ and } k = 1, \cdots, m_v$$

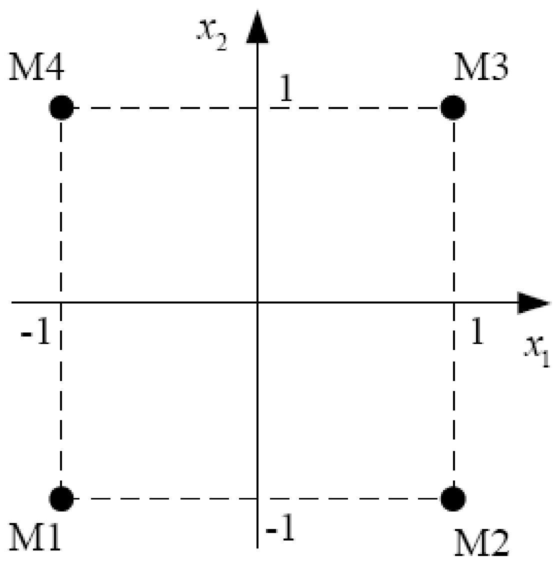
- Number of design variable is $m_v = \log_2 m$
  - Possible to interpolate between $2^{(mv-1)}$ to 2^{mv} materials with m_v variables
- Introduce a penalization scheme (here power law) to end-up with -1/1 values

Bi-valued coded parameterization (G. Tong et al. 2011)

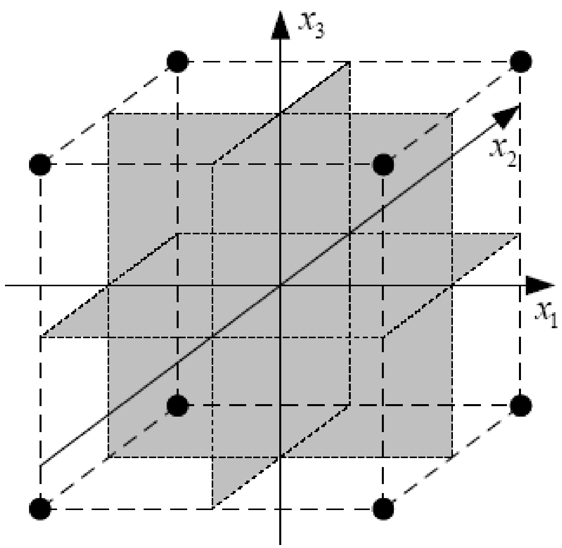
- Visualization: for m_v=2 and m_v=3, the method recovers 4-node and brick (8-node) elements shape functions.

Table 1 s_{jk} values (m_v=2, m=4)

k \ j	1	2	3	4
1	-1	1	1	-1
2	-1	-1	1	1



(a) m_v=2, m=4



(b) m_v=3, m=8

Penalization schemes

- To come to a solution with one single material, one introduces a penalization schemes:
  - SIMP

$$f(\chi) = \chi^p$$

  - RAMP (Stolpe & Svanberg, 2001)

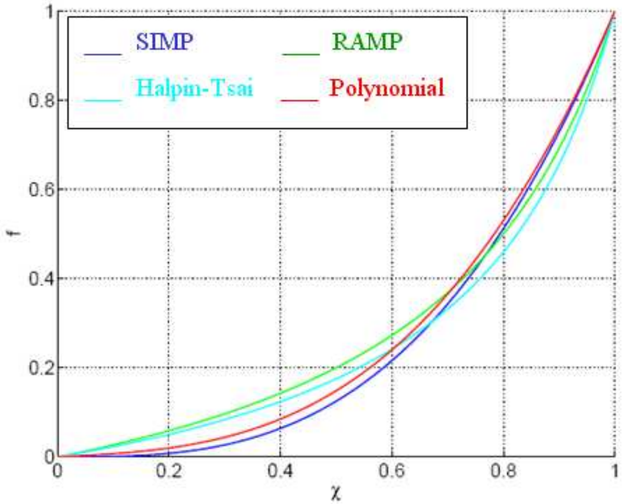
$$f(\chi) = \frac{\chi}{1 + p(1-\chi)}$$

  - Halpin Tsai (Halpin-Tsai, 1969)

$$f(\chi) = \frac{r\chi}{(1+r)-\chi}$$

  - Polynomial penalization (Zhu, 2009)

$$f(\chi) = \frac{\alpha - 1}{\alpha}\chi^p + \frac{1}{\alpha}\chi$$



Penalization schemes

- Choice of penalization scheme
  - Basically one can find equivalent penalizations of intermediate densities by a proper choice of the parameters

    P=3 ←→ r=0,269 ←→ α=16
  - Polynomial / RAMP and Halpin Tsai are necessary when considering body loads (see Bruyneel & Duysinx, 2005)
  - Polynomial is generally the simplest and the most effective in many cases

- Continuation procedure (Bendsoe, 1989) i.e. increasing progressively the penalization parameter p
  - Should avoid being trapped in local optima
  - In practice, here, the numerical experiments have reported no influence of even detrimental influence.

Optimization Problem Formulation

- Laminate optimization
  - Compliance minimization under given load cases

$$\text{find:} \quad \{x_{ik}\} \quad (i=1,\cdots,n;\ k=1,\cdots,m_{\mathrm{v}})$$

$$\text{minimize: } C = \mathbf{F}^{\mathrm{T}}\mathbf{u}$$

$$\text{subject to: } \mathbf{F} = \mathbf{K}\mathbf{u}$$

  - For pure laminate optimization, no resource (volume) constraint is generally necessary

- Topology optimization of laminate
  - Selection of ply orientation and layout of the laminate

$$c^l = (\mu_l)^q \left(\sum_{i=1}^{n^l} w_i^l c_i^l \right) \qquad \sum_{l=1}^{n_{\mathrm{v}}} \mu_l V_l \le \overline{V}$$

  - Introduce a volume constraint of the foam or of the fiber material

Sensitivity analysis

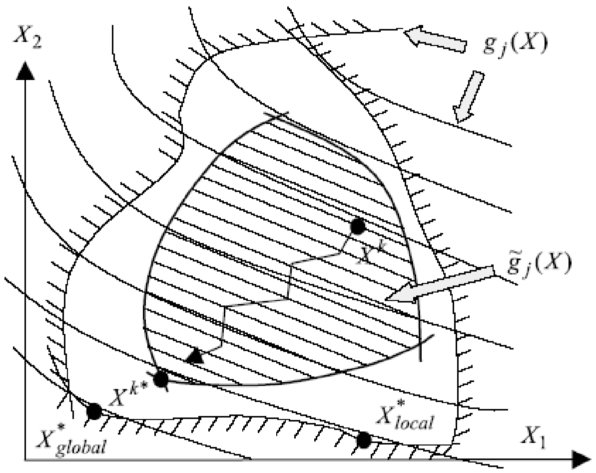
- Sensitivity analysis

$$\frac{\partial C}{\partial x_{ik}} = 2\mathbf{u}^{\mathrm{T}}\frac{\partial \mathbf{F}}{\partial x_{ik}} - \mathbf{u}^{\mathrm{T}}\frac{\partial \mathbf{K}}{\partial x_{ik}}\mathbf{u} = -\mathbf{u}^{\mathrm{T}}\frac{\partial \mathbf{K}}{\partial x_{ik}}\mathbf{u}$$

  - Requires the derivatives of the weighting functions

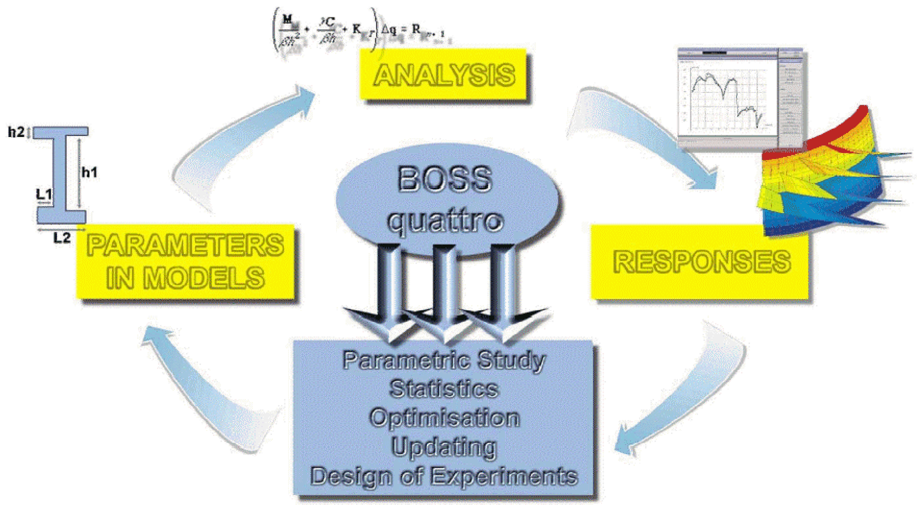
$$\frac{\partial \mathbf{K}_i}{\partial x_{ik}} = \sum_{j=1}^{m}\frac{\partial w_{ij}}{\partial x_{ik}}\mathbf{K}_i^{(j)}$$

- Solution of optimization problem: Sequential Convex Programming
  - Sequence of explicit subproblems
    - CONLIN (Fleury, 1989)
    - GCMMA (Bruyneel et al., 2002)
  - General strategy with efficient capabilities in treating large scale problems

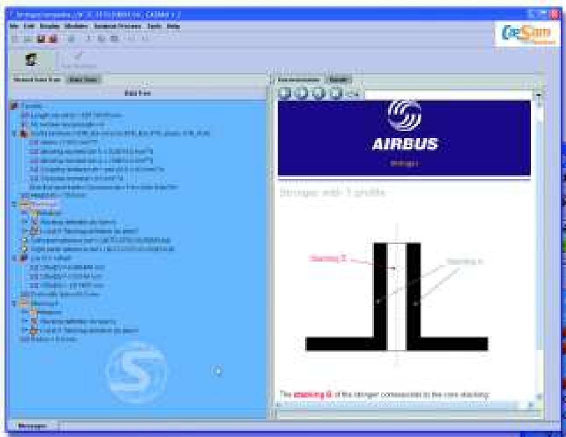


Implementation

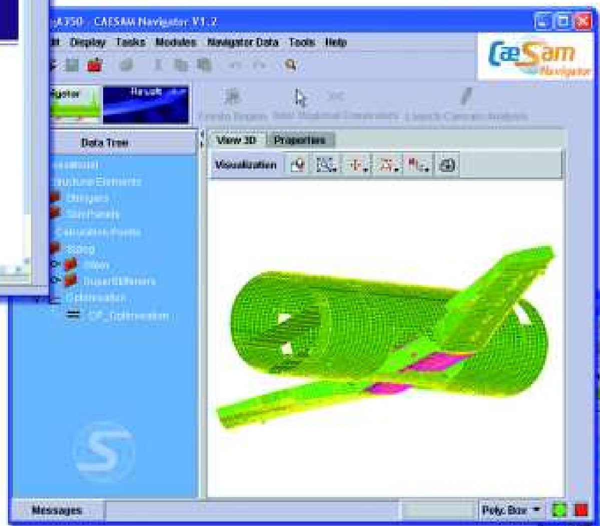
- Implementation
  - Analysis carried out in SAMCEF Composites
    - Laminate plate elements
    - Thick composite shells (8-node bricks)
  - Optimization
    - Boss Quattro Open Object Oriented platform fro Optimization



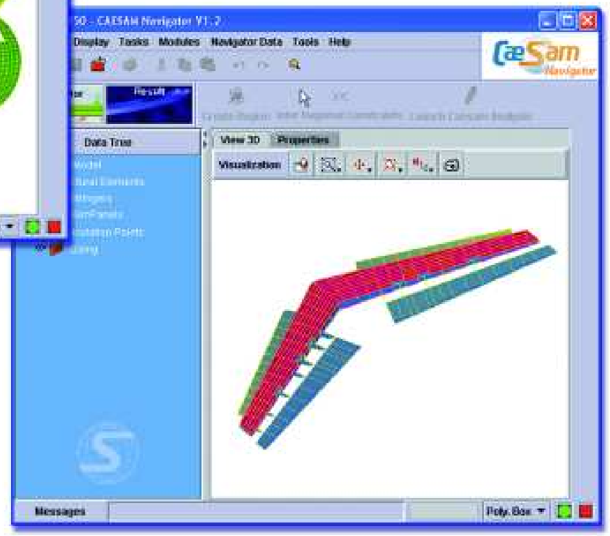
OPTIMIZATION OF LARGE SCALE SYSTEMS



- Illustration of limitation of present solvers



- Solution of large scale problem using CONLIN

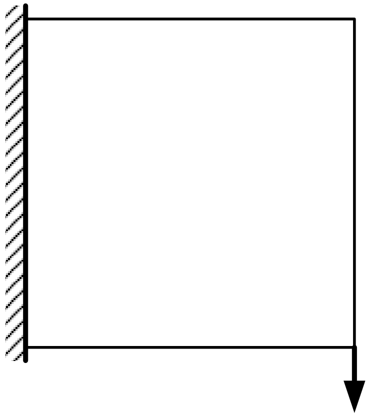


- **Design problem**
  - Design variables (more than 1.000):
    - Composite panels and stringers lay-ups (thicknesses, ply proportions)
    - Stringer dimensions
  - Minimize weight
  - Constraints (more than 100.000)
    - Reserve factors
    - Buckling, reparability, …

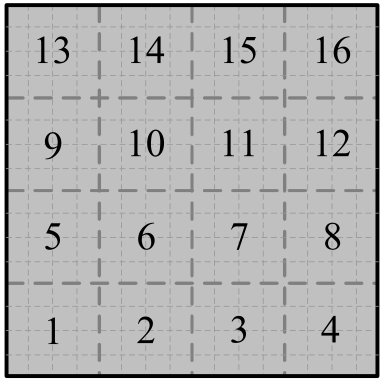
With courtesy by Samtech and Airbus Industries

Numerical applications: Square plate under vertical force

- Maximum in-plane compliance problem is solved by selecting the optimal orientation of the ply



Loads and boundary conditions



Design model with 4×4 patches

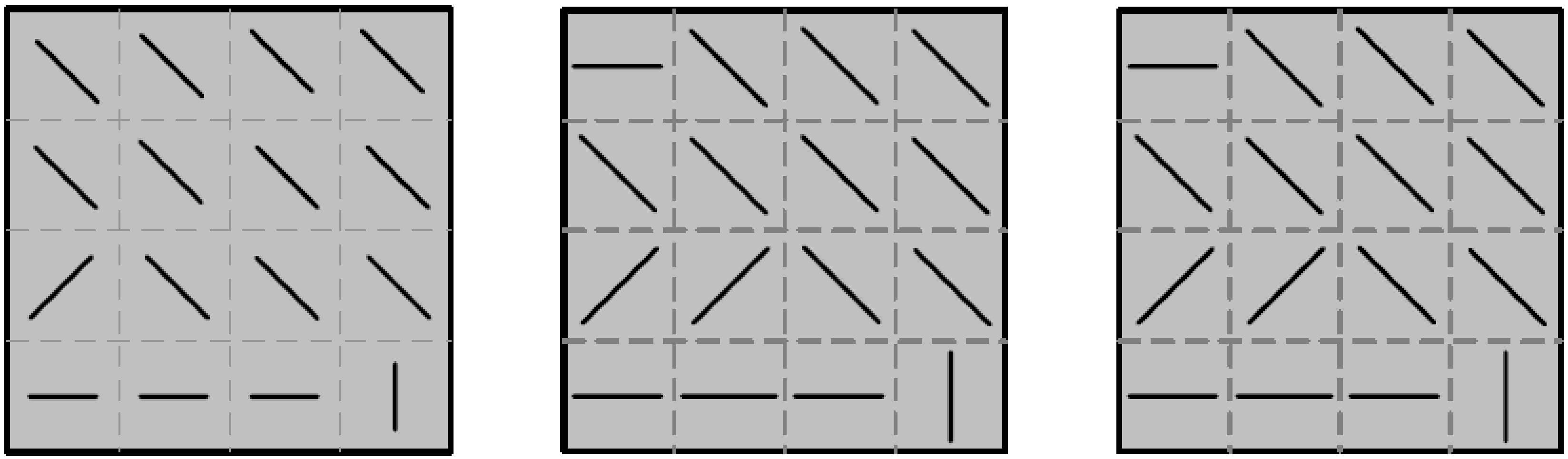
Table 4 Material properties

E_x	E_y	G_{xy}	v_{xy}
146.86GPa	10.62GPa	5.45GPa	0.33

Table 3 Orientations

Number of material phases (m)	Number of design variables for each region (m_v)	Discrete orientation angle (°)
4	2	90/45/0/-45
9	4	80/60/40/20/0/-20/-40/-60/-80
12	4	90/75/60/45/30/15/0/-15/-30/-45/-60/-75

Numerical applications: Square plate under vertical force



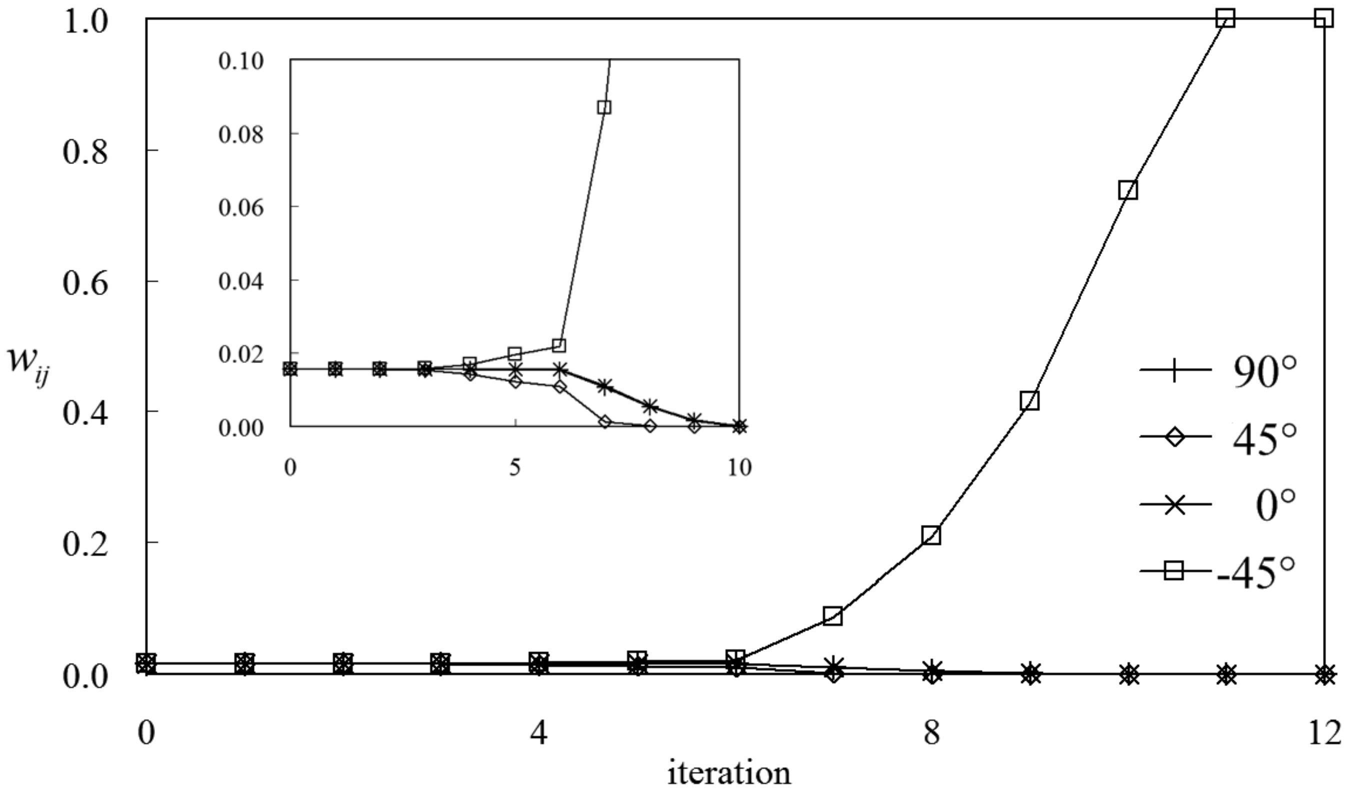
(a) DMO ($C=1.220\times10^{-4}$) $m_v=4$

(b) SFP ($C=1.182\times10^{-4}$) $m_v=2$

(c) BCP ($C=1.182\times10^{-4}$) $m_v=2$

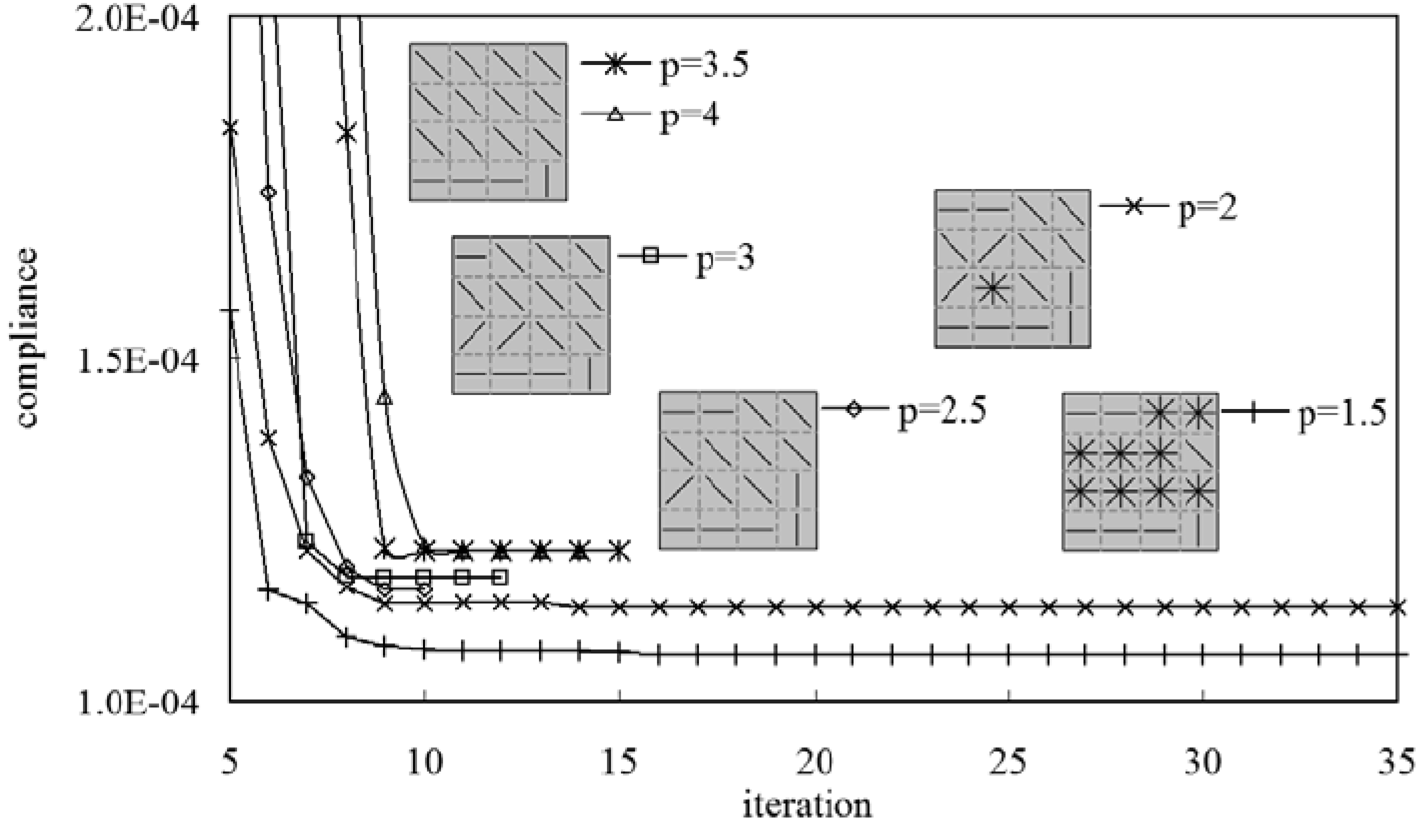
Optimization results of the square plate under vertical force (m=4)

Numerical applications: Square plate under vertical force



Iteration histories of the weight for patch 16 (BCP m=4)

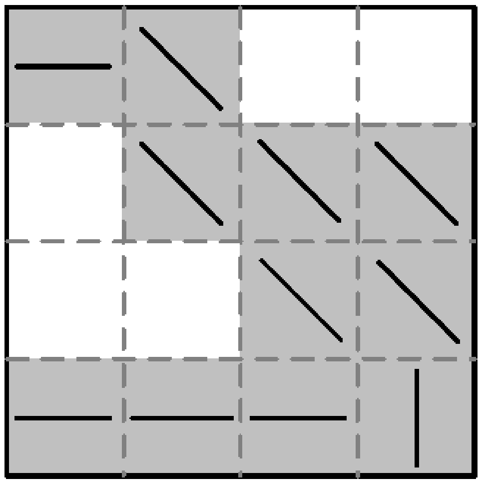
Numerical applications: Square plate under vertical force



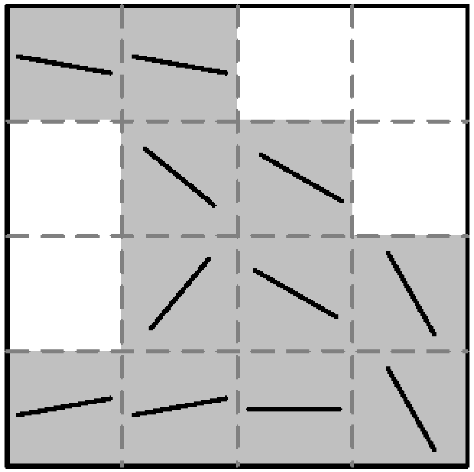
Influence of the penalization factor p of the BCP scheme upon the optimization results

Numerical applications: Square plate under vertical force

- Topology optimization: void + laminate
- Volume constraint: V < 11/16



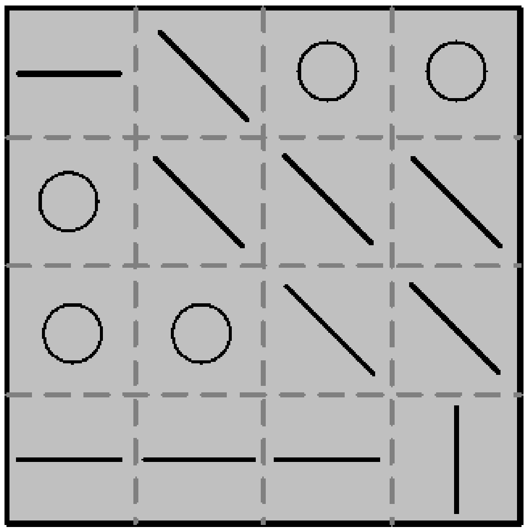
4 orientations
90/45/0/-45



18 orientations
90/80/70/60/50/40/30/20/10/0/
-10/-20/-30/-40/-50/-60/-70/-80

Numerical applications: Square plate under vertical force

- Both orthotropic glass-epoxy and isotropic polymer-foam are involved
- Amount of glass-epoxy is limited to 11 patches

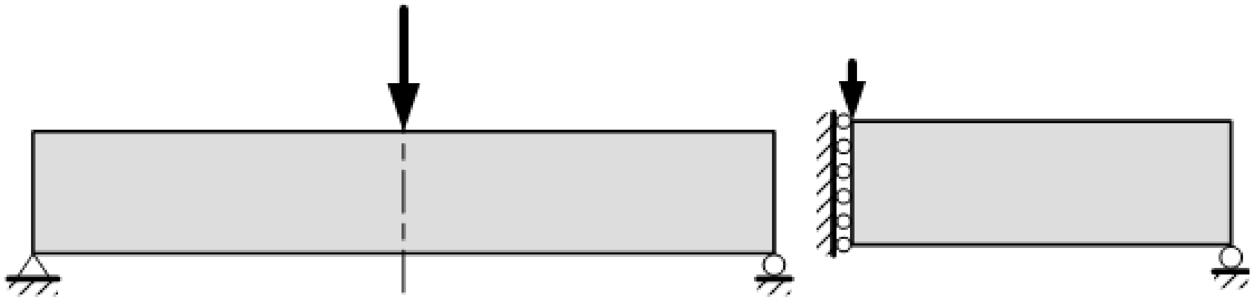


Optimization result of the square plate under vertical force with volume constraint
Glass-epoxy with 4 orientations (90/45/0/-45) and polymer-foam

- Remark: convergence is difficult. CONLIN and MDQA fail. GCMMA-V4 converges after 150 iterations

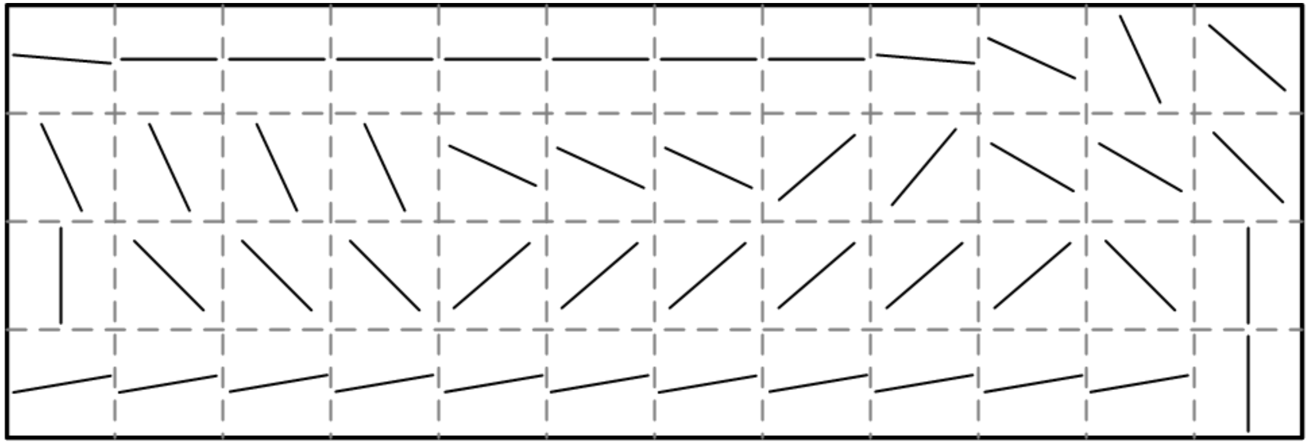
Numerical applications: Simply support beam (MBB beam)

- A simply-supported beam of one single layer



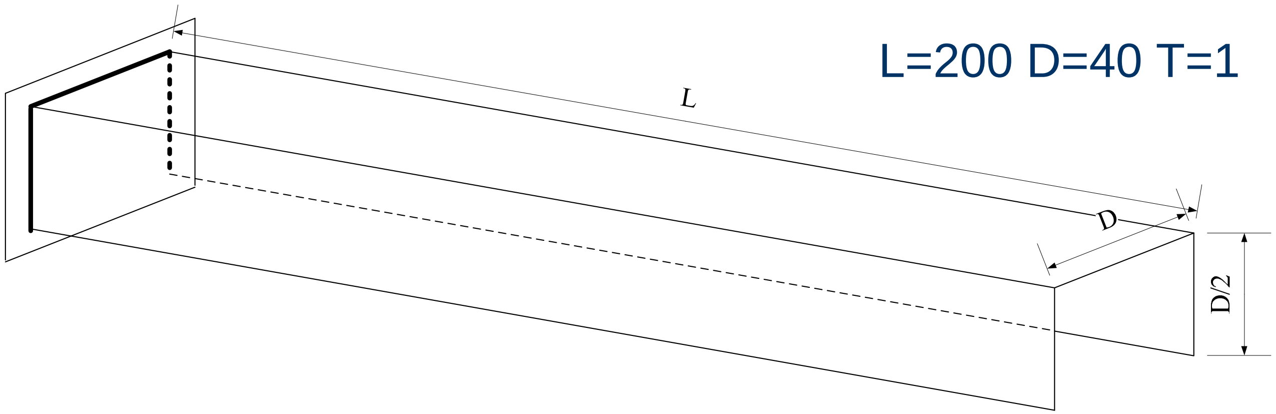
- Mesh: 240×40 quadrangular finite elements
- Possible ply orientation: m=36 (90/85/80/75/70/65/60/55/50/45/40/35/30/25/20/15/10/5/0 /-5/-10/-15/-20/-25/-30/-35/-40/-45/-50/-55/-60/-65/-70/-75/-80/-85)
  - 6 design variables (mv=6) are needed for each designable patch

Numerical applications: Simply support beam (MBB beam)



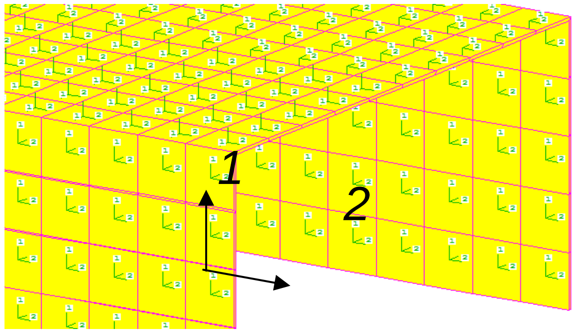
Optimization results using BCP scheme (Patch: 12×4, *m=36)*

Numerical application: long box



L=200 D=40 T=1

- 4 layers; element size=4
- L1 & L4: glass-epoxy (90/45/0/-45)
- L2 & L3: glass-epoxy (90/45/0/-45) + polymer-foam
- Orientation: 90/45/0/-45 in 1-2 plane for each element axes



Numerical application: long box

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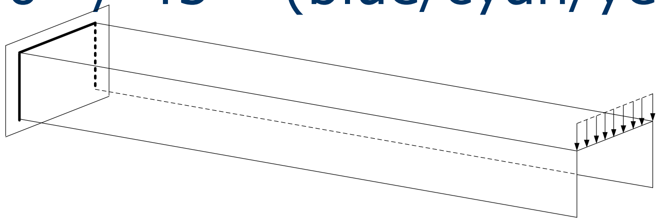
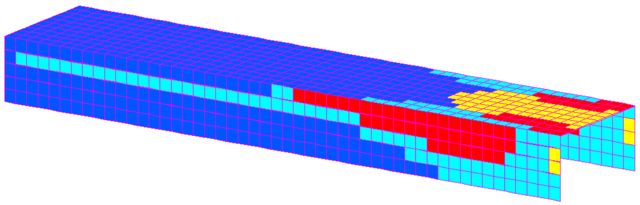
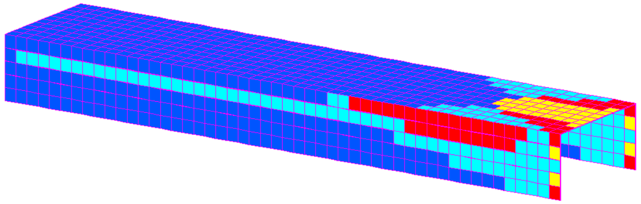
- Objective: minimize structural compliance
- 90° /45° /0° /-45° (blue/cyan/yellow/red)

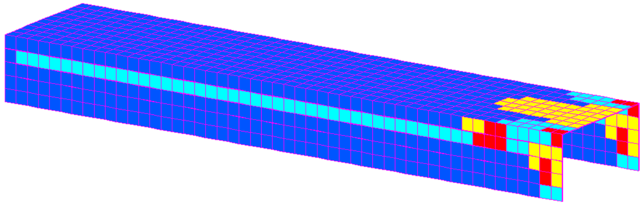




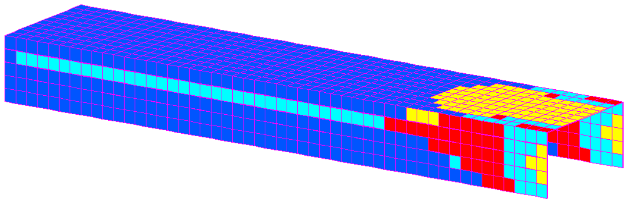
Layer 1 (inner ply)



Layer 2



Layer 3

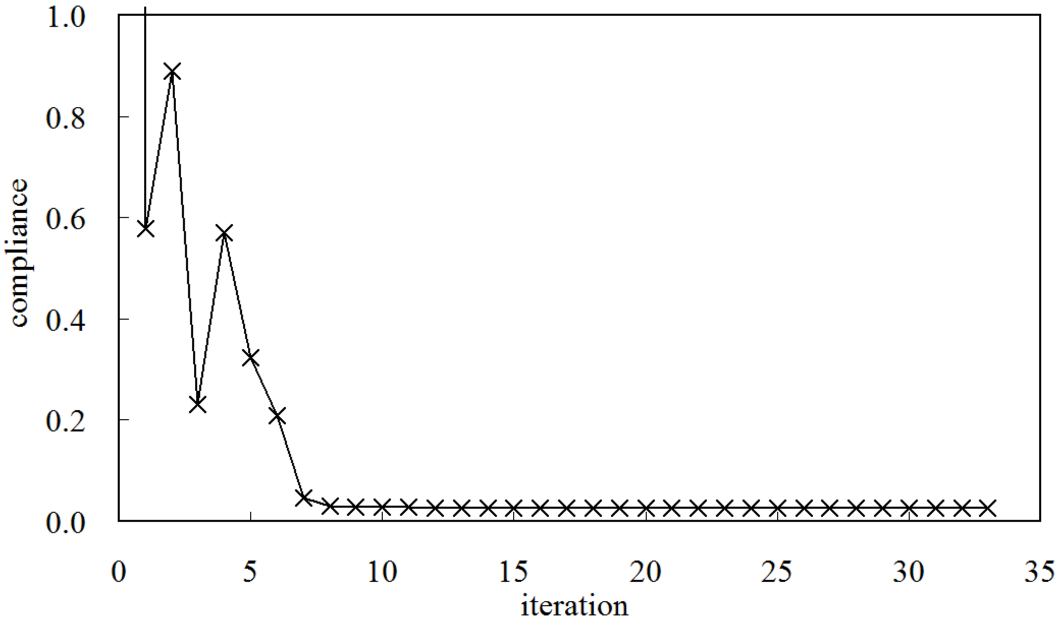


Layer 4 (outer ply)

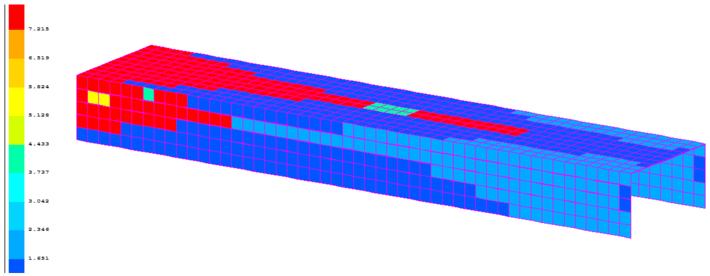
Numerical application: long box

- Objective: minimize structural compliance s.t. volume constraint: glass-epoxy<80%

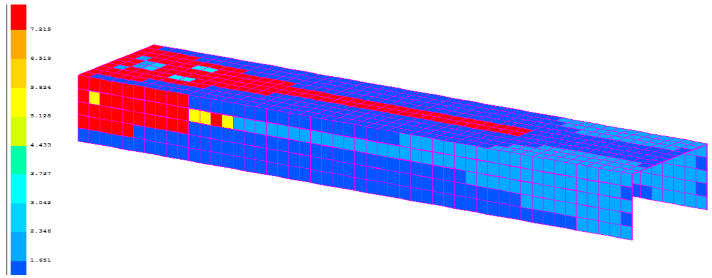
- Results:



Numerical application: long box



Layer 1



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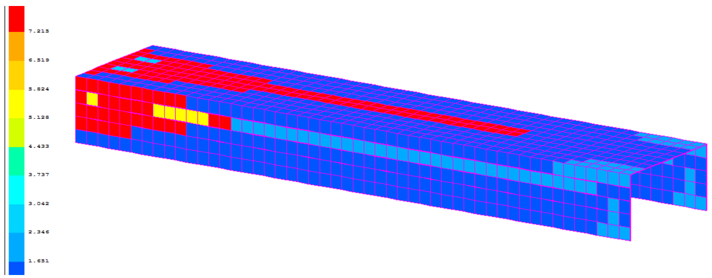
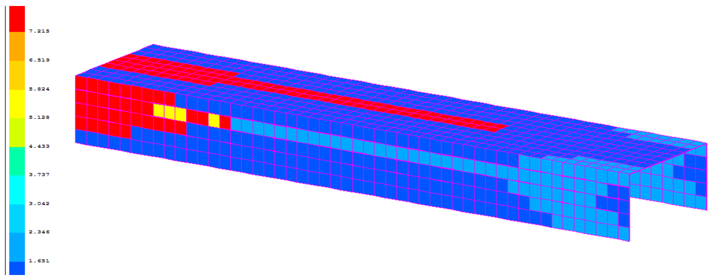


Layer 3



Layer 4

- 90°/45°/0°/-45° (blue/cyan/yellow/red)

CONCLUSIONS

- A novel parameterization scheme based on a bi-value coding for solving the discrete material optimization of composite structures
- Reduced number of design variables, the BCP scheme generalizes the SFP scheme by Bruyneel (2011) and is a challenger to the classic DMO for large-scale problems
- BCP formulation provides a well-posed problem for an efficient solution using sequential convex programming algorithms (15-20 iterations necessary)

PERSPECTIVES

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- Extend the application of this novel parameterization scheme to larger problems involving industrial composite structures
- Including compliance, displacement, stress constraints but also buckling and perimeter constraints.

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THANK YOU VERY MUCH FOR YOUR ATTENTION

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